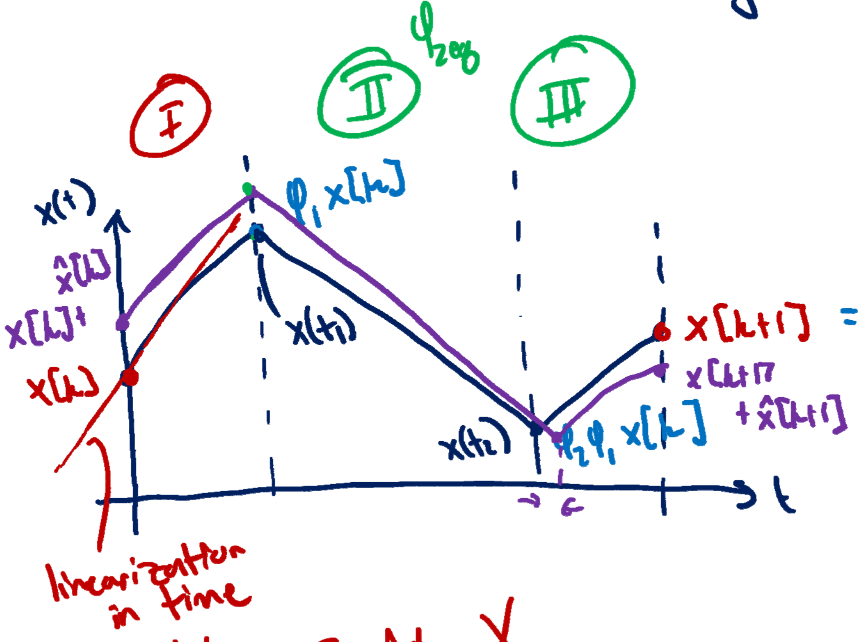
State Transition Matrix

Assume a nonlinear homogeneous system

$$x[k+1] = \Phi(x)\, x[k]$$

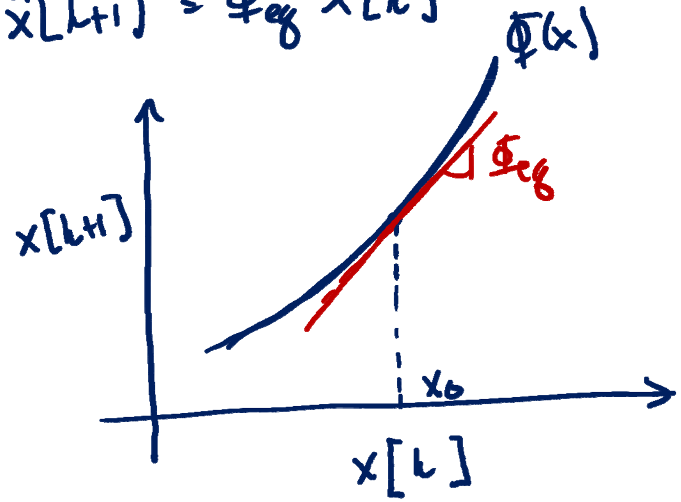
Generalized state space linearization

$$\hat{x}[k+1] = \Phi_{eq}\, \hat{x}[k]$$



$$x[k+1] = \varphi_3\varphi_2\varphi_1\, x[k] = \Phi(x)\, x[k]$$

$$\varphi_1 = e^{A_1 t_1} \approx \underline{I + A t_1} \quad \times$$



$$\Phi(x) = \prod_{i=k}^{1} \varphi_i$$

$$\varphi_i = e^{A_i t_i}$$

and $t_i = f(x)$ is possible

$$\Phi_{eq} = \underbrace{\begin{bmatrix} \Phi_{11} & \Phi_{12} & \cdots & \Phi_{1n} \\ \Phi_{21} & \ddots & & \\ \vdots & & \ddots & \\ \Phi_{n1} & & & \Phi_{nn} \end{bmatrix}}_{\text{Jacobian Matrix}}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\hat{x}_1[k+1] = \Phi_{11}\hat{x}_1[k] + \Phi_{12}\hat{x}_2[k] + \cdots \Phi_{1n}\hat{x}_n[k]$$

$$\vdots$$

$$\hat{x}_n[k+1] = \Phi_{n1}\hat{x}_1[k] + \cdots + \Phi_{nn}\hat{x}_n[k]$$

$$\Phi_{11} = \frac{\partial x_1[k+1]}{\partial x_1[k]} \qquad \Phi_{12} = \frac{\partial x_1[k+1]}{\partial x_2[k]}$$

$$\Phi_{21} = \frac{\partial x_2[k+1]}{\partial x_1[k]}$$

Linearization of States

Can also linearize just select subintervals. Instead of Φ_{eq}, find $\varphi_{i,eq}$

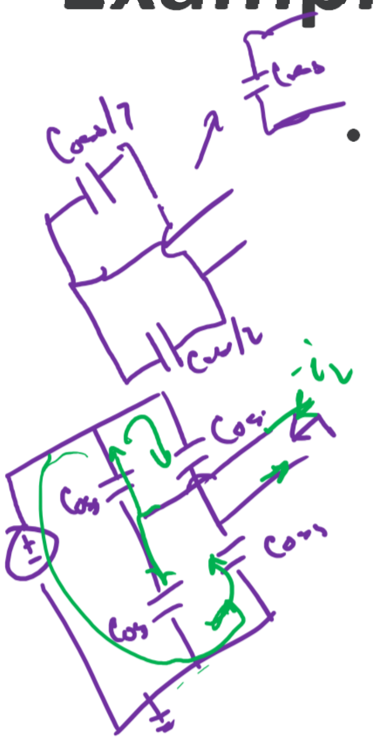
$$\varphi_{i,eq} = \begin{bmatrix} \frac{\partial x_1(t_{i+1})}{\partial x_1(t_i)} & \frac{\partial x_1(t_{i+1})}{\partial x_2(t_i)} & \cdots \\ \frac{\partial x_2(t_{i+1})}{\partial x_1(t_i)} & & \\ \vdots & & \end{bmatrix}$$

Options to deal with nonlinear state-dependent switching

1. Per-period Generalized State Space
2. Generalized State Space in select subintervals

   2b. use circuit analysis to directly find $\varphi_{i,eq}$

Example: DAB Model



- DT model is

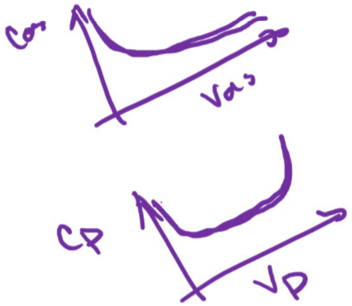
$$\hat{\boldsymbol{x}}[n] = \boldsymbol{\Phi}\hat{\boldsymbol{x}}[n-1] + \boldsymbol{\Gamma}\hat{\varphi}_{ab}[n-1]$$

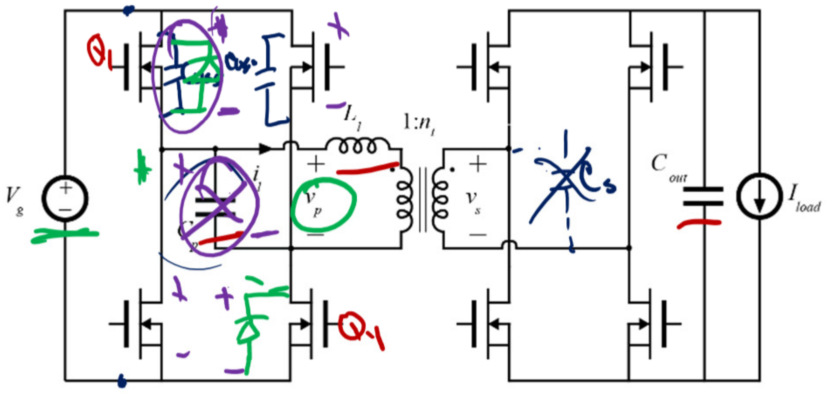
with

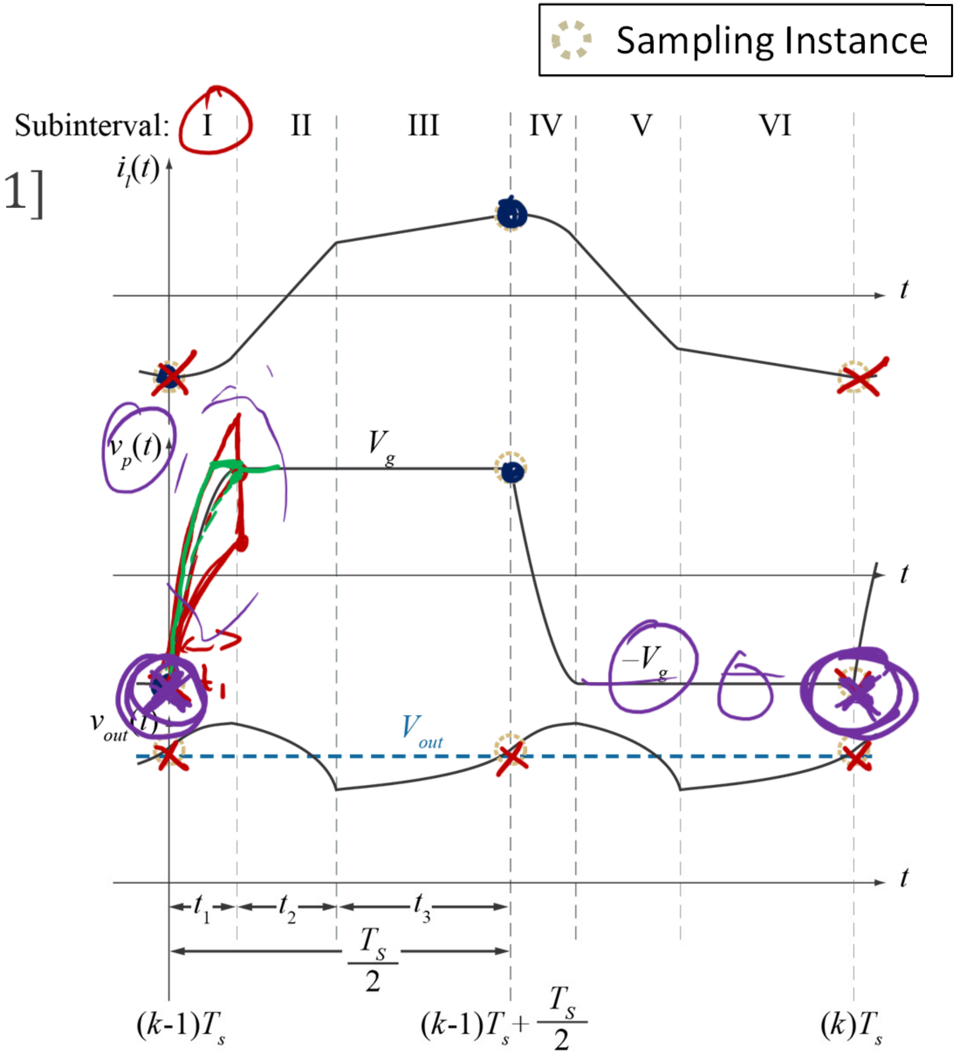
$$\boldsymbol{\Phi} = e^{\mathrm{A}_3 t_3} e^{\mathrm{A}_2 t_2} e^{\mathrm{A}_1 t_1} \boldsymbol{I}_{HC}$$

$$\boldsymbol{\Gamma} = e^{\mathrm{A}_3 t_3}(\mathbf{A}_2\text{-}\mathbf{A}_3)\mathbf{X}_0$$

where $\boldsymbol{\Phi} \in \Re^{3\mathrm{x}3}$ and $\boldsymbol{\Gamma} \in \Re^{3\mathrm{x}1}$







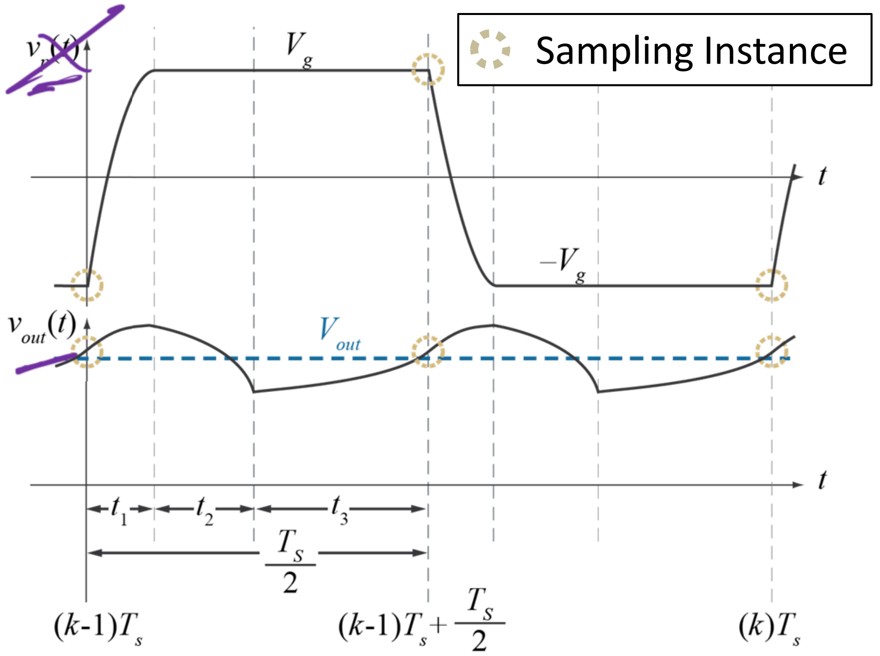
D. Costinett, "Reduced Order Discrete Time Modeling of ZVS Transition Dynamics in the Dual Active Bridge Converter", APEC 2015

Order of System

- Including $v_p(t)$ as a state results in 3rd-order model
- Resulting transfer function is of the form

$$G_{v\varphi}(z) = G_{v\varphi 0}\frac{1-q_1 z^{-1}}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$$

- Outside of resonant transition $|v_p|=V_g$ is constant, resulting in reduced order
- Goal: eliminate constant state, while maintaining effect of resonant transition on $i_l(t)$, $v_{out}(t)$



Model ZVS as Disturbance

- Consider how model takes transition dynamics into account:

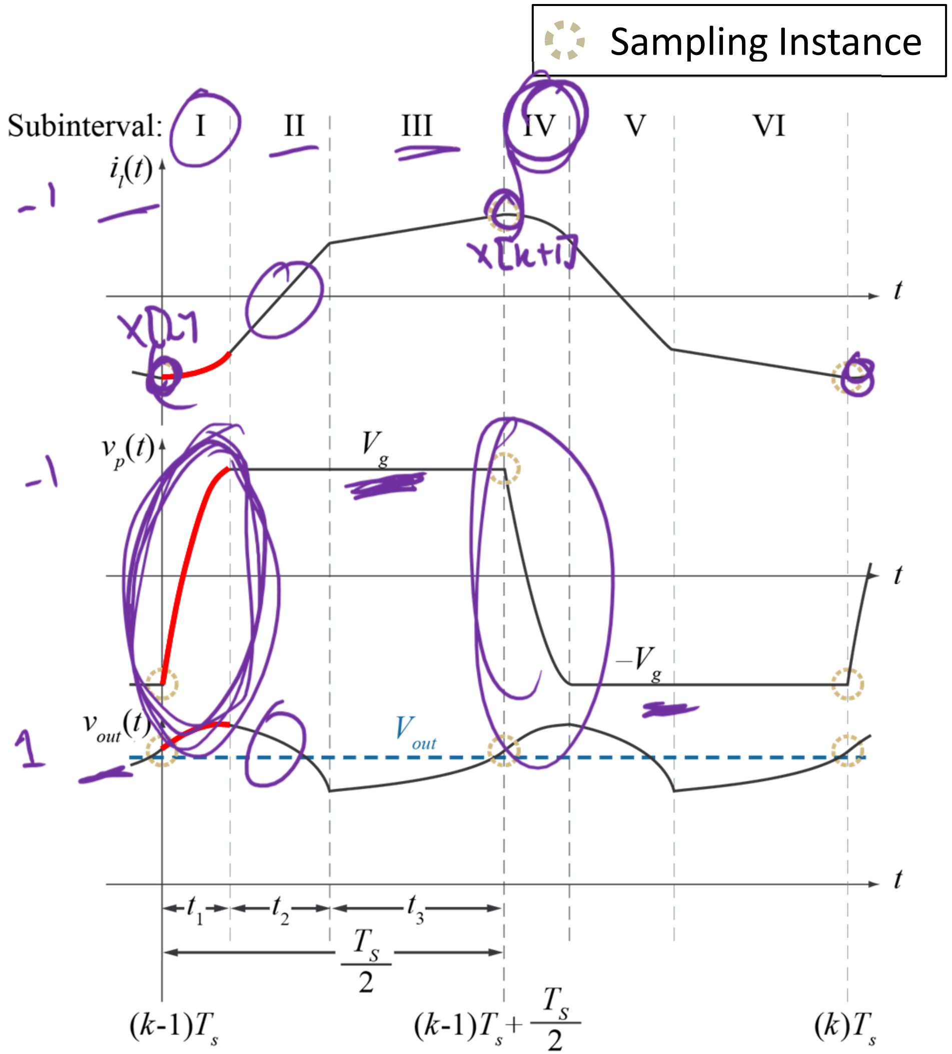
$$\boldsymbol{\Phi} = e^{\mathbf{A}_3 t_3} e^{\mathbf{A}_2 t_2} e^{\mathbf{A}_1 t_1} \boldsymbol{I}_{HC}$$

- Red term models relation between states at start/end of ZVS subinterval

$$\hat{x}(t_1) = e^{\mathbf{A}_1 t_1} \hat{x}(t_0)$$

with $\boldsymbol{A}_1 \in \Re^{3\text{x}3}$

- Using circuit analysis, solve new matrix $\boldsymbol{A}_{res} \in \Re^{2\text{x}2}$ which models how ZVS transition affects states at t=t_1



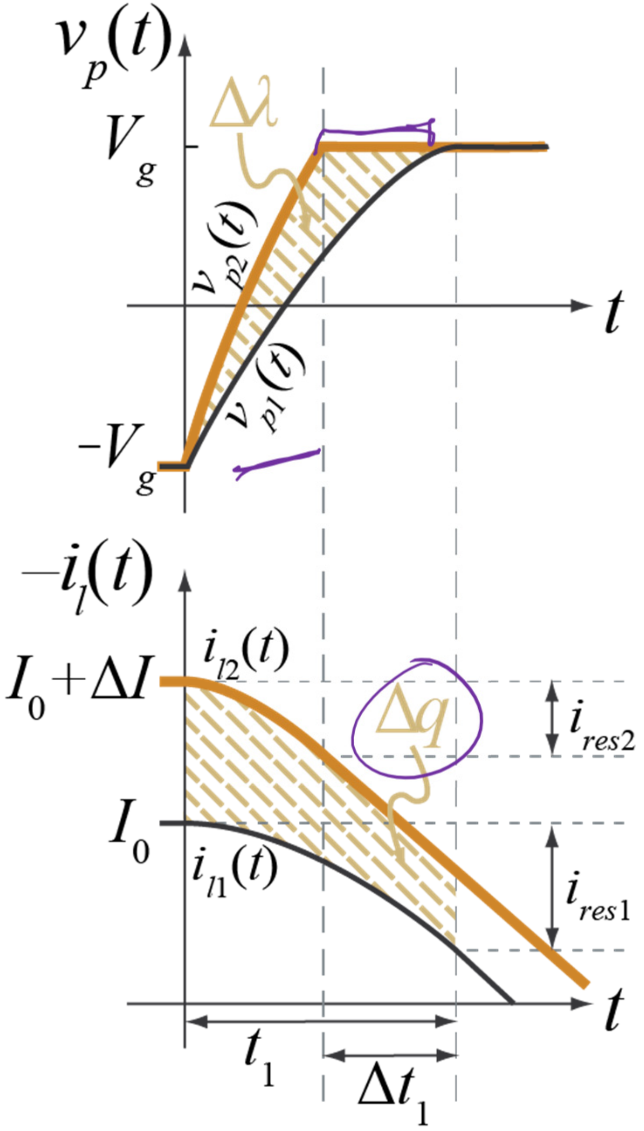
Resonant Transition Matrix

- Maintaining second order, $\boldsymbol{x} = [\, i_l \;\; v_{out}]^T$
- New matrix takes the linear form

$$\boldsymbol{A}_{res} = \begin{bmatrix} \frac{\partial v_{out}(t_1)}{\partial v_{out}(0)} & \frac{\partial v_{out}(t_1)}{\partial i_l(0)} \\ \frac{\partial i_l(t_1)}{\partial v_{out}(0)} & \frac{\partial i_l(t_1)}{\partial i_l(0)} \end{bmatrix} = \varphi_{i,eq}$$

- Linearized with respect to *x*(0), rather than time

Case I: Above ZVS



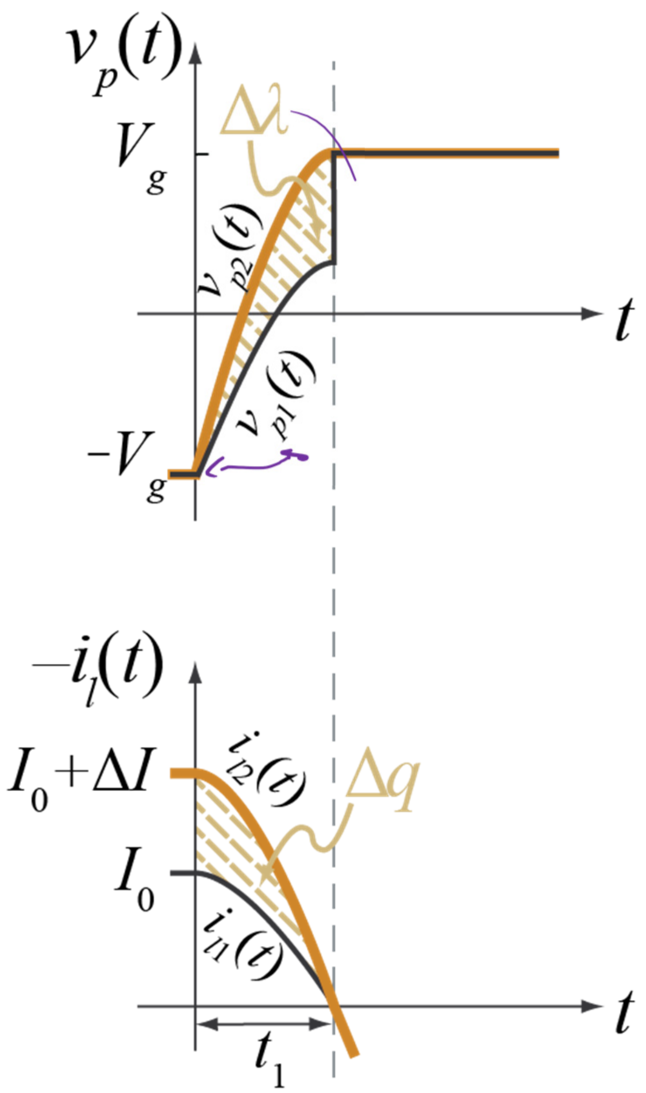
- Linearized relations solved using circuit analysis

$$\frac{\Delta\lambda}{\Delta I} = L_l - L_l\sqrt{1-\left(\frac{2V_g}{R_0 I_0}\right)^2}\ ,$$

$$\frac{\Delta Q}{\Delta I} = \frac{2V_g C_p}{I_0}\ ,$$

D. Costinett, D. Seltzer, R. Zane, and D. Maksimovic, "Analysis of inherent volt-second balancing of magnetic devices in zero-voltage switched power converters," in Proc. Appl. Power Electron. Conf. (APEC), arch 2013, pp. 9–15.

Case II: Below ZVS



- Solution different when partial hard switching occurs

$$\frac{\Delta\lambda}{\Delta I} = L_l - L_l\cos(\omega_0 t_1) \quad ,$$

$$\frac{\Delta Q}{\Delta I} = \frac{\sin(\omega_0 t_1)}{\omega_f} \ .$$

Linearized Result

- Assuming balanced operation and small ripple on V_{out}

$$\boldsymbol{A}_{res} = \begin{bmatrix} \dfrac{\partial v_{out}(t_1)}{\partial v_{out}(0)} & \dfrac{\partial v_{out}(t_1)}{\partial i_l(0)} \\ \dfrac{\partial i_l(t_1)}{\partial v_{out}(0)} & \dfrac{\partial i_l(t_1)}{\partial i_l(0)} \end{bmatrix} = \begin{bmatrix} \cancel{1} & \dfrac{\Delta Q}{\Delta I}\dfrac{1}{C_{out}} \\ \cancel{0} & 1 - \dfrac{\Delta\lambda}{\Delta I}\dfrac{1}{L_l} \end{bmatrix}$$

- Resulting model is now

$$\boldsymbol{\Phi} = e^{\mathbf{A}_3 t_3} e^{\mathbf{A}_2 t_2} \mathbf{A}_{\mathrm{res}} \boldsymbol{I}_{HC}$$

$$\boldsymbol{\Gamma} = e^{\mathbf{A}_3 t_3} (\mathbf{A}_2 - \mathbf{A}_3) \mathbf{X}_0$$

where $\boldsymbol{\Phi} \in \Re^{2x2}$ and $\boldsymbol{\Gamma} \in \Re^{2x1}$

Model Accuracy