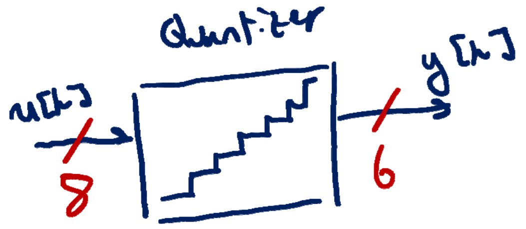
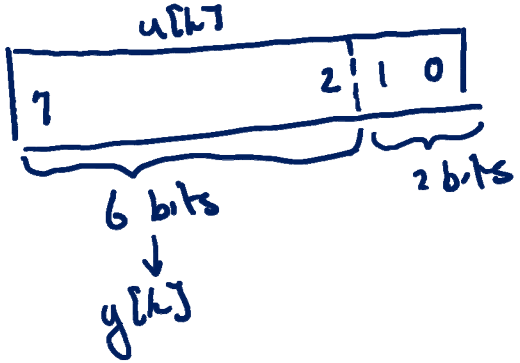
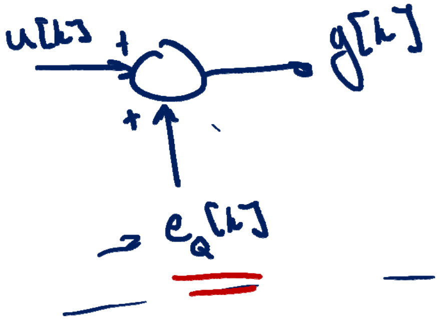
Quantization Noise

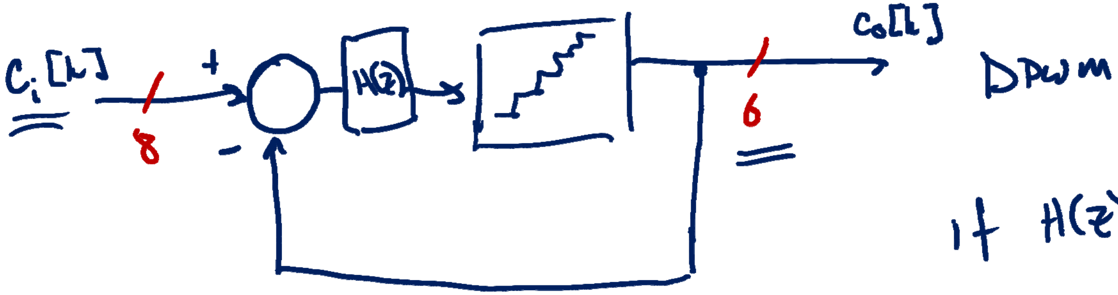
Lets assume we generate a c[k] at higher resolution than DPWM will allow





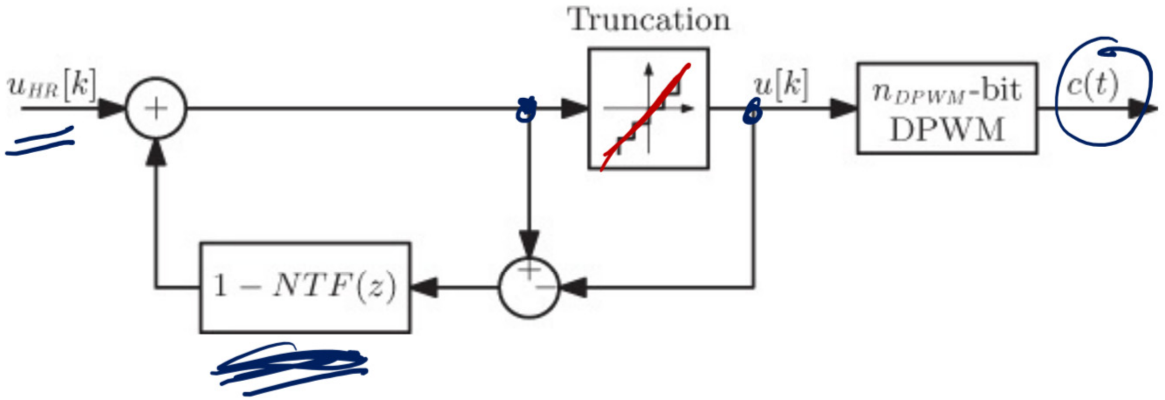


$e_Q[k]$ = u[k] bits 1 → 0

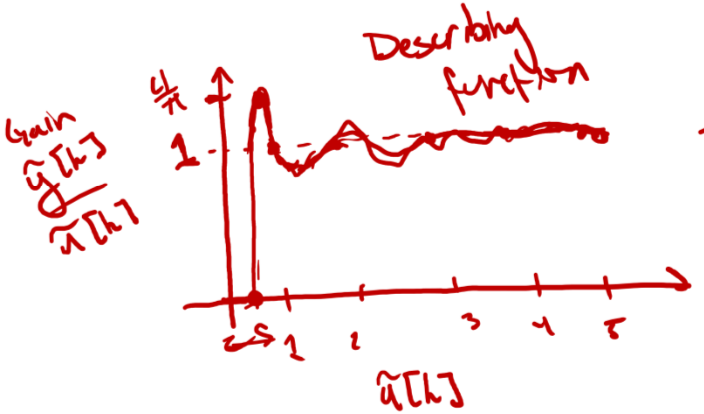


If H(z) is an integrator

ΣΔ Modulation



Alternate view of Quantizer



→ worst-case gain is $\frac{4}{\pi}$

Dynamic Limit cycling if Quantizer Gain deviation from 1 causes an instability

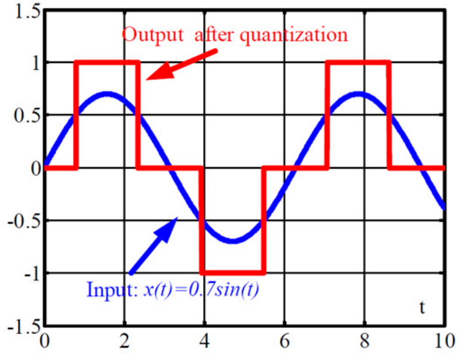
if Both DPWM & ADC have worst-case simultaneous quantization Gain deviation

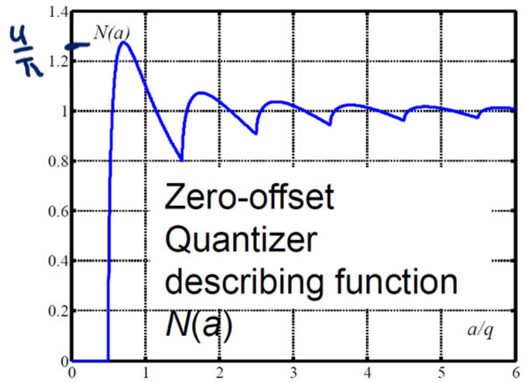
$\left(\frac{4}{\pi}\right)^2 = (1.27)^2 = 4.2\,dB$

If Gain margin is > 4.2dB no dynamic limit cycle

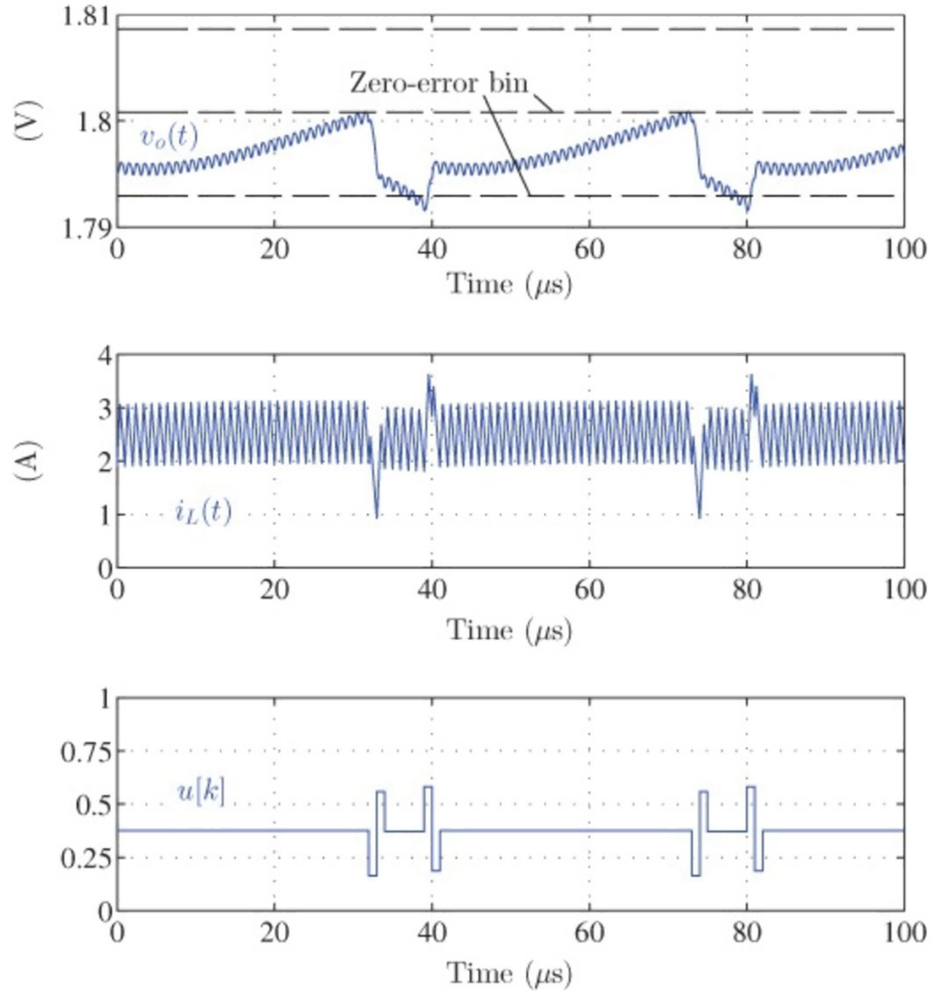
Dynamic Limit Cycling Condition

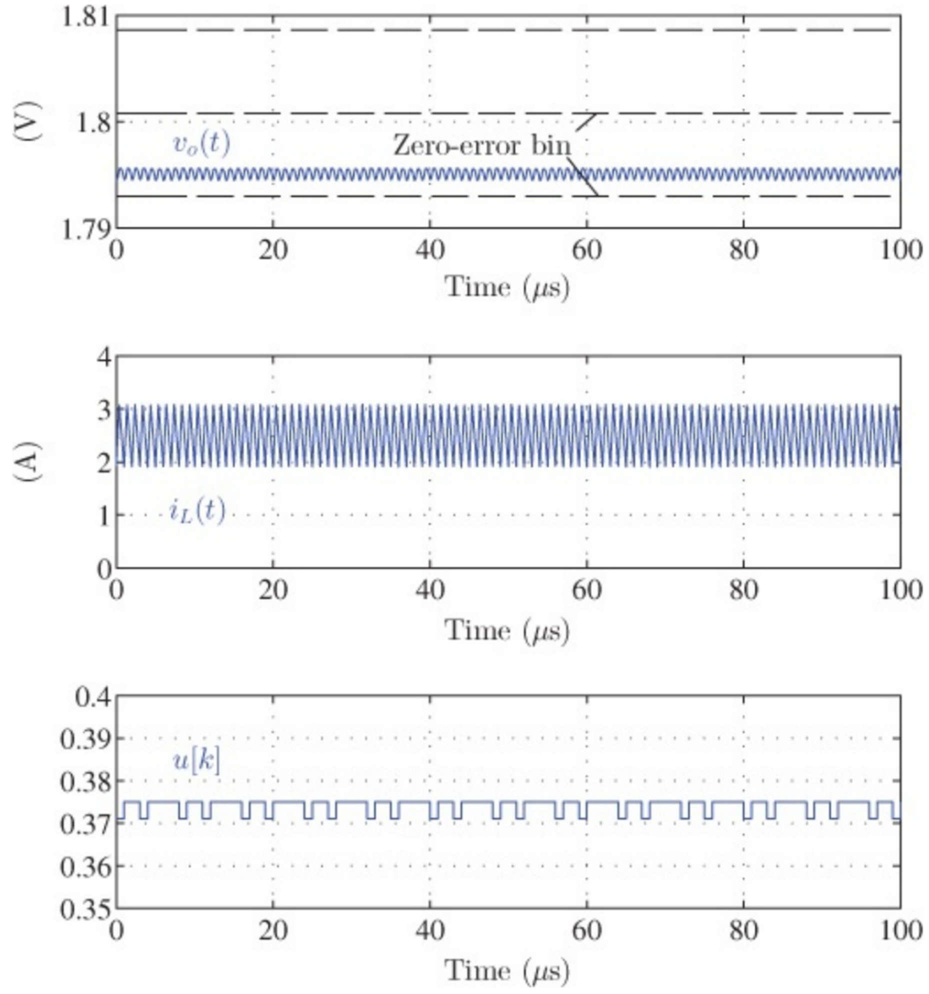
$$N(a) = \frac{\text{output first harmonic amplitude}}{\text{input amplitude } a} = \text{amplitude - dependent "gain"}$$



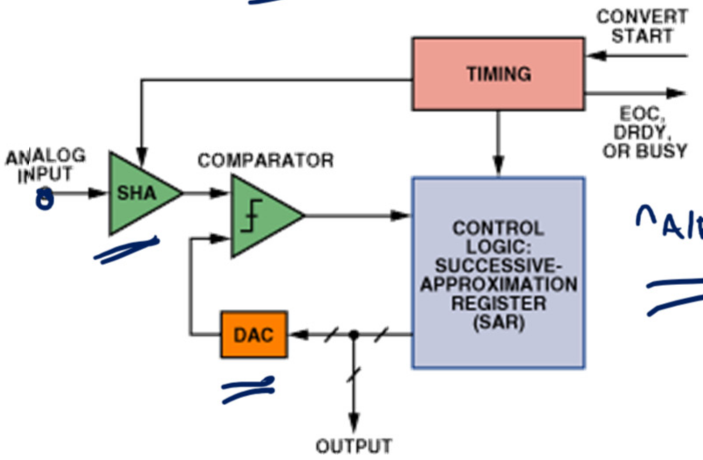


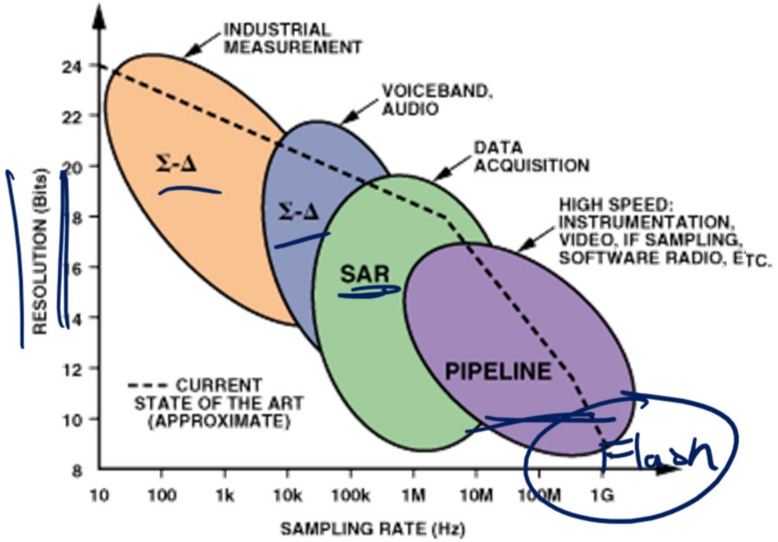
H.Peng, D.Maksimovic, A.Prodic, E.Alarcon, “Modeling of quantization effects in digitally controlled DC-DC converters,” IEEE PESC 2004, pp.4312-4318.

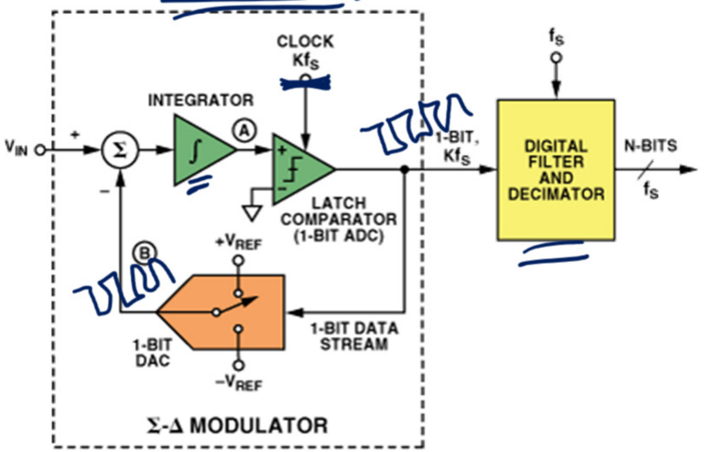

1.81
1.8
1.79
(V)
$v_o(t)$
Zero-error bin
0 20 40 60 80 100
Time (μs)
4 3 2 1 0
(A)
$i_L(t)$
0 20 40 60 80 100
Time (μs)
1 0.75 0.5 0.25 0
$u[k]$
0 20 40 60 80 100
Time (μs)

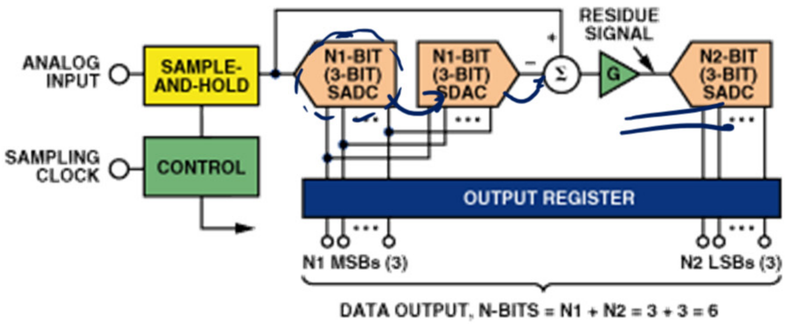

1.81
1.8
1.79
(V)
$v_o(t)$
Zero-error bin
0 20 40 60 80 100
Time (μs)
4 3 2 1 0
(A)
$i_L(t)$
0 20 40 60 80 100
Time (μs)
0.4 0.39 0.38 0.37 0.36 0.35
$u[k]$
0 20 40 60 80 100
Time (μs)

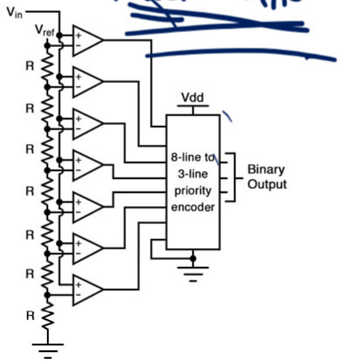
ADC implementation











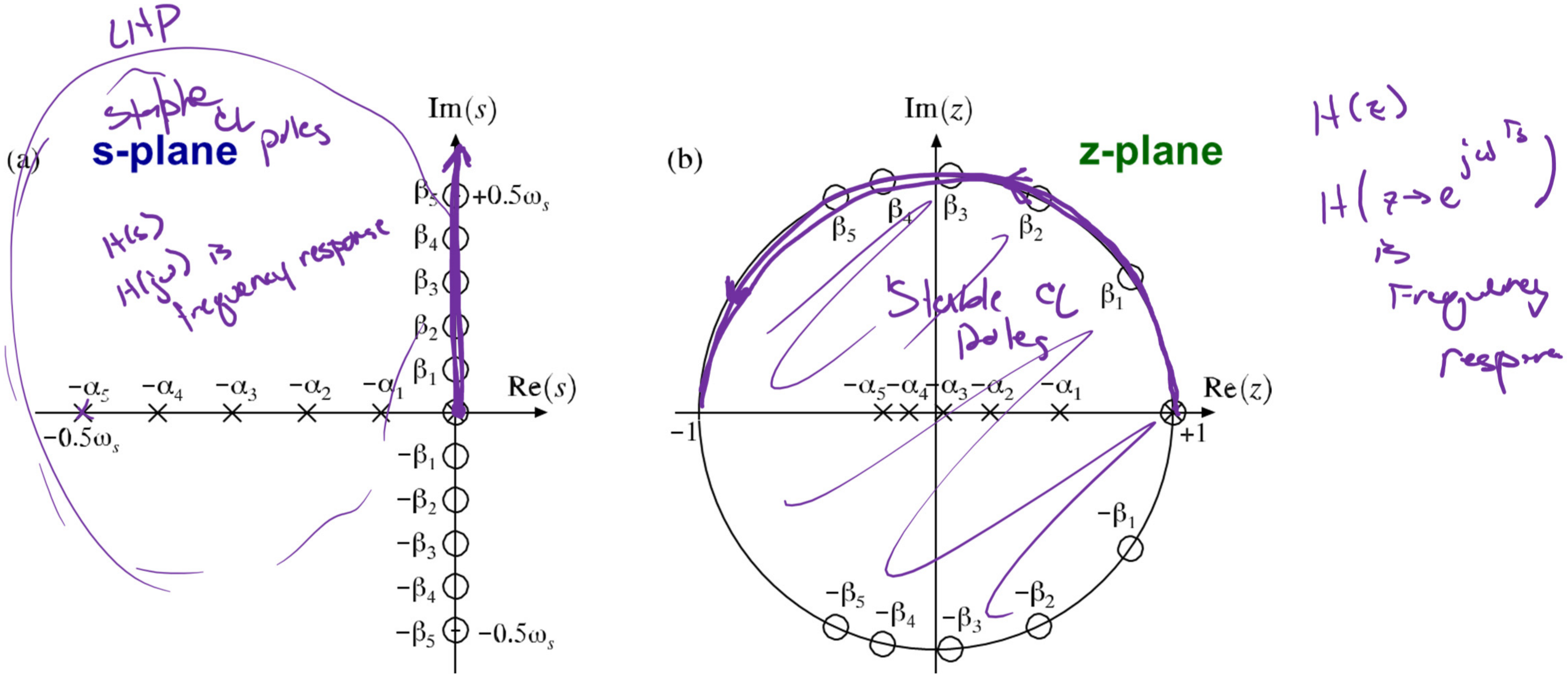
Z-transform

$$Z\{h\} \triangleq \sum_{k=0}^{+\infty} h[k]z^{-k} = H(z)$$

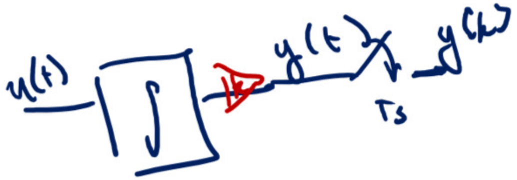
Properties:

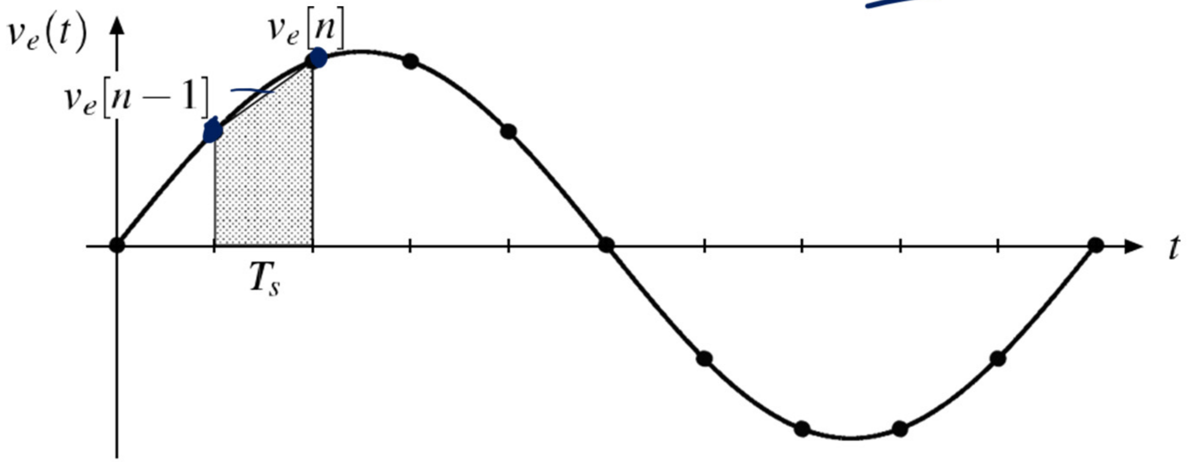
1. Linearity
2. Delay: z^{-1} represents a one-sample delay
3. Discrete convolution of two signals is multiplication of their z-transforms
4. The frequency response of any stable system is equal to $H(z)$ evaluated with $z=e^{j\theta}$, $\theta = \omega T_S$ → Ts is sampling period

Pole Mapping



DT Integrator





Continuous time

$$y[k] = y[k-1] + k\int_{T_s} u(\tau)\,d\tau$$

$$y(t) = k\int_0^t u(\tau)\,d\tau$$

$$\frac{Y(s)}{U(s)} = k\frac{1}{s} = G_I(s)$$

↳ $s = j\omega$

Discrete Time : use trapezoidal approx

$$y[k] = y[k-1] + k T_s \frac{u[k] + u[k-1]}{2}$$

↓ $Z\{\cdot\}$

$$Y(z) = z^{-1}Y(z) + \frac{kT_s}{2}\left(U(z) + z^{-1}U(z)\right)$$

$$\frac{Y(z)}{U(z)} = \frac{kT_s}{2}\,\frac{1+z^{-1}}{1-z^{-1}} = G_I(z)$$

→ $z = e^{j\omega T_s}$