State Space Representation

For any Linear, Time-Invariant system, the behaviors are fully characterized by

notation $\underline{\underline{X}} \to$ matrix

$\underline{x} \to$ vector

$$\underline{\dot{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{\underline{B}}\,\underline{u}(t)$$

$\underline{\underline{A}}$ & $\underline{\underline{B}}$ are matrices which define system (circuit topology)

$\underline{u}(t) \to$ ($\underline{v}(t)$ in textbook) $\to$ vector of independent inputs

$\underline{x}(t) \to$ vector of states of the system

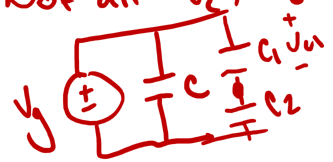
$\underline{\dot{x}}(t) \to \frac{d}{dt}$ of $\underline{x}(t)$

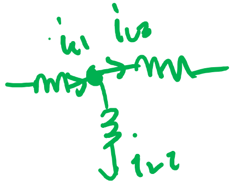
states: A complete set of information @ t_0 that, with $u(t \geq t_0)$, fully characterize the system for $t > t_0$

informally: commonly most or all of the cap voltages & inductor currents

states of a system are not unique

Not all v_c, i_L





$$\underline{y}(t) = \underline{\underline{C}}\,\underline{x}(t) + \underline{\underline{D}}\,\underline{u}(t)$$

$\underline{y}(t) \longrightarrow$ vector of outputs that we select

$\underline{\underline{C}}$ & $\underline{\underline{D}}$ associated matrices to find our selected outputs

For a system w/ n_s states, m_o outputs, & p_i inputs

$\underline{\underline{A}} \in \mathbb{R}^{n_s \times n_s}$

$\underline{\underline{B}} \in \mathbb{R}^{n_s \times p_i}$

$\underline{x}$, $\underline{\dot{x}} \in \mathbb{R}^{n_s \times 1}$

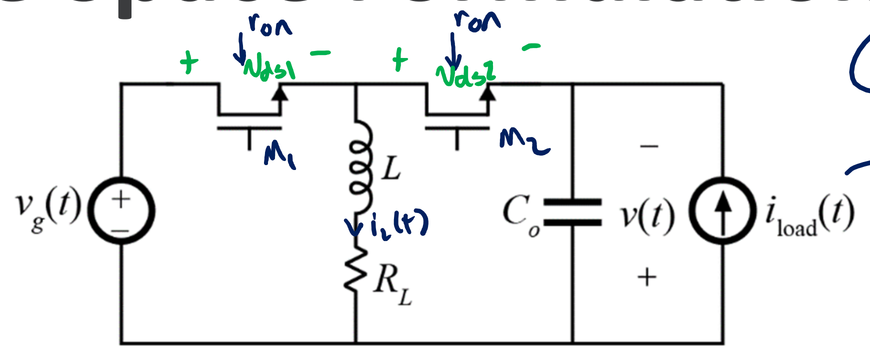
$\underline{\underline{C}} \in \mathbb{R}^{m_o \times n_s}$

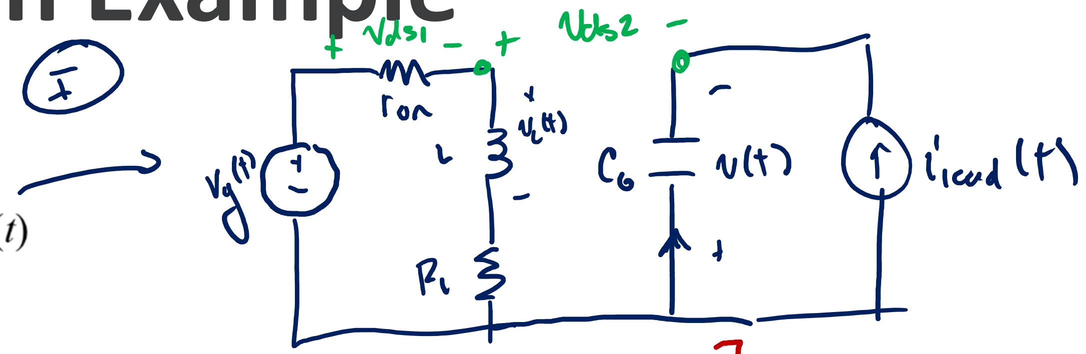
$\underline{\underline{D}} \in \mathbb{R}^{m_o \times p_i}$

$\underline{u} \in \mathbb{R}^{p_i \times 1}$

$\underline{y} \in \mathbb{R}^{m_o \times 1}$

State Space Formulation Example





$$\begin{cases} L\dfrac{di_L(t)}{dt} = v_g(t) - i_L(t)\left(r_{on} + R_L\right) \\[2ex] C_o\dfrac{dv_o(t)}{dt} = -i_{load}(t) \end{cases}$$

select $\underline{x}(t) = \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix}$

$\underline{u}(t) = \begin{bmatrix} v_g(t) \\ i_{load}(t) \end{bmatrix}$

$$\underline{\dot{x}}(t) = \frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{r_{on}+R_L}{L} & \emptyset \\ \emptyset & \emptyset \end{bmatrix}}_{\underline{\underline{A}}_1} \underbrace{\begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix}}_{\underline{x}(t)} + \underbrace{\begin{bmatrix} 1/L & \emptyset \\ \emptyset & -1/C_o \end{bmatrix}}_{\underline{\underline{B}}_1} \underbrace{\begin{bmatrix} v_g(t) \\ i_{load}(t) \end{bmatrix}}_{\underline{u}(t)}$$

$$\underline{\underline{K}}\,\underline{\dot{x}}(t) = \underline{\underline{A}}^*\underline{x}(t) + \underline{\underline{B}}^*\underline{u}(t)$$

$$\underbrace{\begin{bmatrix} L & 0 \\ 0 & C_o \end{bmatrix}}_{\underline{\underline{K}}_1} \underline{\dot{x}}(t) = \underbrace{\begin{bmatrix} -(r_{on}+R_L) & \emptyset \\ \emptyset & \emptyset \end{bmatrix}}_{\underline{\underline{A}}^*} \underline{x}(t) + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\underline{\underline{B}}^*} \underline{u}(t)$$

$$\underline{\underline{A}} = \underline{\underline{K}}^{-1}\underline{\underline{A}}^*$$

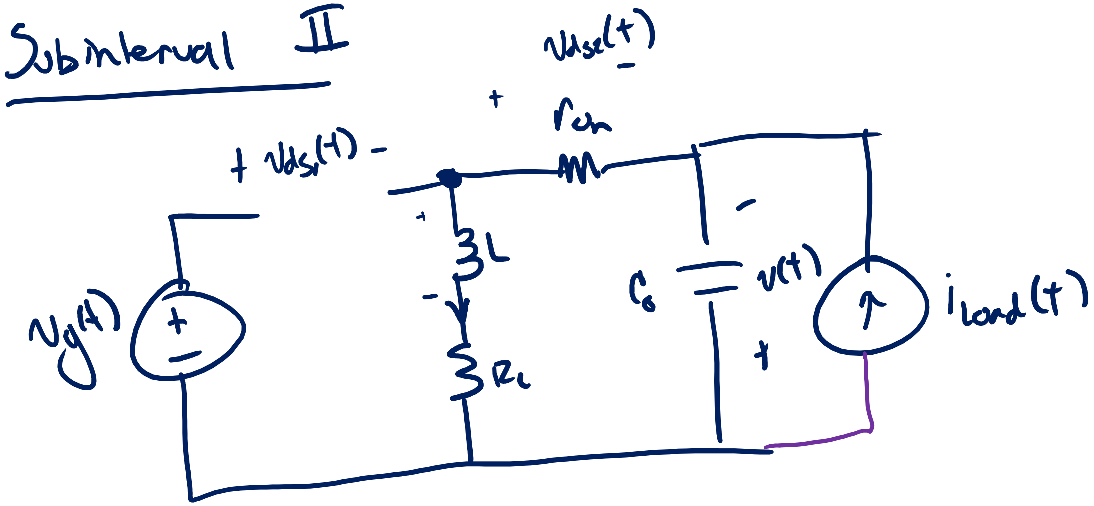
$$\underline{\underline{B}} = \underline{\underline{K}}^{-1}\underline{\underline{B}}^*$$

let's pick $y(t) = \begin{bmatrix} v_{ds1}(t) \\ v_{ds2}(t) \\ v(t) \end{bmatrix}$

$$\begin{cases} v_{ds1}(t) = i_L(t)\, r_{on} \\ v_{ds2}(t) = v_g(t) - i_L(t)\, r_{on} + v(t) \\ v(t) = v(t) \end{cases}$$

$$\underbrace{\begin{bmatrix} v_{ds1}(t) \\ v_{ds2}(t) \\ v(t) \end{bmatrix}}_{\underline{y(t)}} = \underbrace{\begin{bmatrix} r_{on} & 0 \\ -r_{on} & 1 \\ 0 & 1 \end{bmatrix}}_{\underline{\underline{C}}} \underbrace{\begin{bmatrix} i_L(t) \\ v(t) \end{bmatrix}}_{\underline{x(t)}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}}_{\underline{\underline{D}}} \underbrace{\begin{bmatrix} v_g(t) \\ i_{Load}(t) \end{bmatrix}}_{\underline{u(t)}}$$

Subinterval II



$$\begin{cases} L\dfrac{di_L(t)}{dt} = -v(t) - i_L(t)\left(r_{on} + R_L\right) \\ C\dfrac{dv_0(t)}{dt} = i_L(t) - i_{Load}(t) \end{cases}$$

$$\begin{cases} v_{ds1}(t) = v_g(t) + v(t) + i_L(t)\, r_{on} \\ v_{ds2}(t) = -i_L(t)\, r_{on} \\ v(t) = v(t) \end{cases}$$

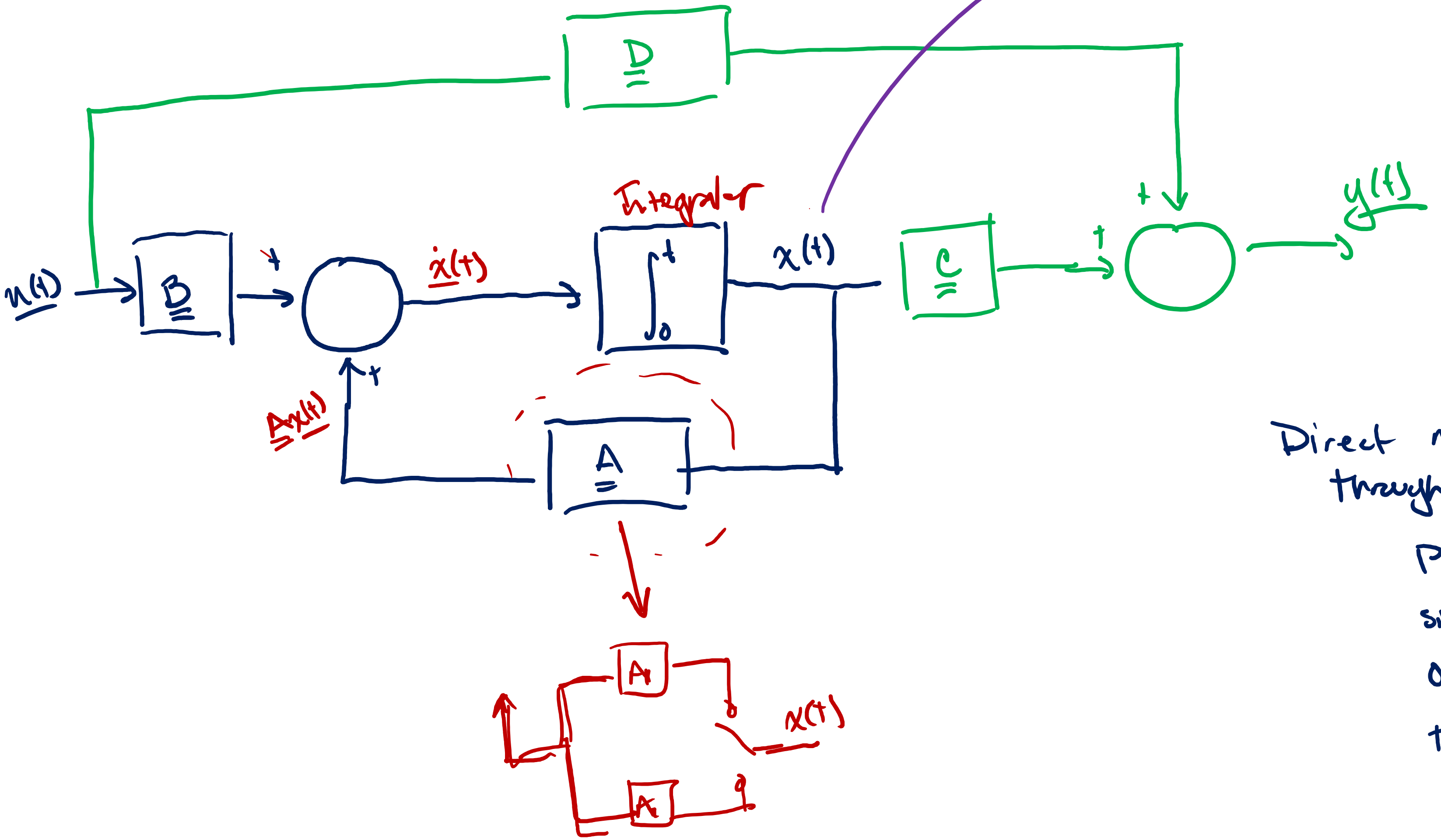
$$\underline{\dot{x}}(t) = \underbrace{\begin{bmatrix} -\frac{r_{on}+R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & \phi \end{bmatrix}}_{\underline{\underline{A_2}}} \underline{x}(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ \phi & -\frac{1}{C} \end{bmatrix}}_{\underline{\underline{B_2}}} \underline{u}(t)$$

$$\underline{y}(t) = \underbrace{\begin{bmatrix} r_{on} & 1 \\ -r_{on} & \phi \\ \phi & 1 \end{bmatrix}}_{\underline{\underline{C_2}}} \underline{x}(t) + \underbrace{\begin{bmatrix} 1 & \phi \\ \phi & \phi \\ \phi & \phi \end{bmatrix}}_{\underline{\underline{D_2}}} \underline{u}(t)$$

Block Diagram

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{\underline{B}}\,\underline{u}(t)$$
$$\underline{y}(t) = \underline{\underline{C}}\,\underline{x}(t) + \underline{\underline{D}}\,\underline{u}(t)$$

Across different switching subintervals, states remain continuous



Direct numerical simulation through timestepping integration

PLECS

Simulink

Opal-RT

Typhoon

Solution by Laplace Transform

$$\mathcal{L}\{\underline{\dot{x}(t)}\} = \mathcal{L}\{\underline{\underline{A}}\,\underline{x(t)} + \underline{\underline{B}}\,\underline{u(t)}\}$$

$$s\underline{X(s)} - \underline{x(0^-)} = \underline{\underline{A}}\,\underline{X(s)} + \underline{\underline{B}}\,\underline{U(s)}$$

$$s\underline{X(s)} - \underline{\underline{A}}\,\underline{X(s)} = x(0^-) + \underline{\underline{B}}\,\underline{U(s)}$$

$$(s\underline{\underline{I}} - \underline{\underline{A}})\,\underline{X(s)} = x(0^-) + \underline{\underline{B}}\,\underline{U(s)}$$

$$\mathcal{L}^{-1}\{\underline{X(s)}\} = \mathcal{L}^{-1}\{(s\underline{\underline{I}} - \underline{\underline{A}})^{-1} x(0^-) + (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{\underline{B}}\,\underline{U(s)}\}$$

$$x(t) = \mathcal{L}^{-1}\{(sI - A)^{-1}\}x(0^-) + \mathcal{L}^{-1}\{(sI - A)^{-1}\underline{\underline{B}}\,\underline{U(s)}\}$$

$$\boxed{x(t) = \underline{\Phi}(t)\,x(0^-) + \int_0^t \underline{\Phi}(\tau)\,\underline{\underline{B}}\,u(t-\tau)\,d\tau}$$

$$\mathcal{L}^{-1}\{(sI - A)^{-1}\} = \underline{\Phi}(t) \rightarrow \text{resolvent of } A$$