Example Calculation

$$\boldsymbol{A}_I = \begin{bmatrix} -\dfrac{(r_{on} + R_L)}{L} & 0 \\ 0 & 0 \end{bmatrix}$$

State Transition Matrix

Properties of the Matrix Exponential

Derivative

$$\frac{\mathrm{d}}{\mathrm{d}t} e^{\mathrm{A}t} = A e^{\mathrm{A}t}$$

Nonvanishing Determinant

$$\det e^{\mathrm{A}t} \neq 0$$

Same-Matrix Product

$$e^{\mathrm{A}t} e^{\mathrm{A}s} = e^{\mathrm{A}(t+s)}$$

Inverse

$$\left(e^{\mathrm{A}t}\right)^{-1} = e^{-\mathrm{A}t}$$

Commutative Product (1)

$$\mathrm{AB} = \mathrm{BA} \implies e^{\mathrm{A}t}\mathrm{B} = \mathrm{B}e^{\mathrm{A}t}$$

Commutative Product (2)

$$\mathrm{AB} = \mathrm{BA} \implies e^{\mathrm{A}t} e^{\mathrm{B}t} = e^{(\mathrm{A}+\mathrm{B})t}$$

Series Expansion

$$e^{\mathrm{A}t} = \sum_{n=0}^{\infty} \frac{t^n}{n!} \mathrm{A}^n$$

Decomposition

$$e^{\mathrm{PBP}^{-1}} = \mathrm{P}e^{\mathrm{B}}\mathrm{P}^{-1}$$

Matrix Exponential

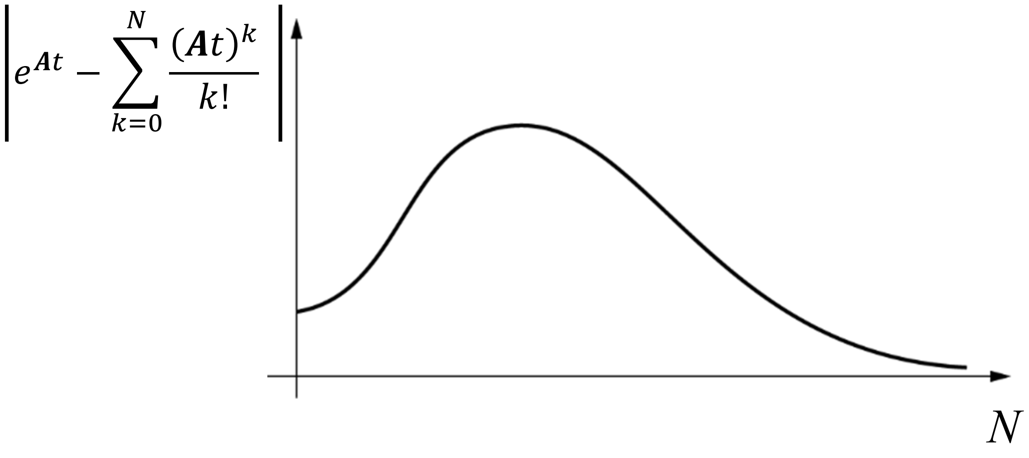
Matrix exponential defined by Taylor series expansion

$$e^{\boldsymbol{A}t} = I + \boldsymbol{A}t + \frac{(\boldsymbol{A}t)^2}{2!} + \cdots + \frac{(\boldsymbol{A}t)^N}{N!} = \sum_{k=0}^{N} \frac{(\boldsymbol{A}t)^k}{k!}$$

Summation always converges and is invertible,

$$e^{\boldsymbol{A}} e^{-\boldsymbol{A}} = \boldsymbol{I}$$

Well-known issue with convergence in many cases



In MATLAB, expm(·) calculates matrix exponential

– Using exp(·) will compute the element-by-element exponential

C. Moler and C. V. Loan, "Nineteen dubious ways to compute the exponential of a matrix," SIAM Review, vol. 20, pp. 801–836, 1978.

Time-Domain Derivation

Solution With DC Inputs