Singular A Matrix

if A_i singular

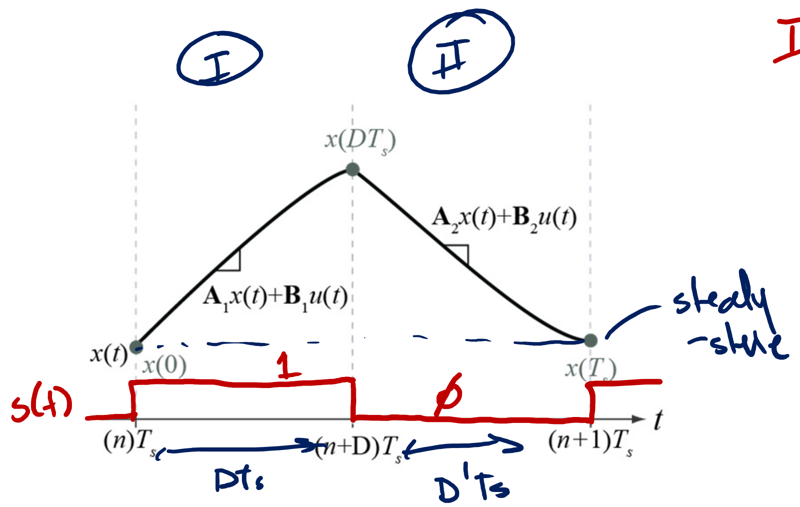
① Make A_i Non-singular

- Change circuit model
- change A_i directly to make it more stable

② calculate $x(t)$ a different way

- Numerically calculate convolution integral
- Augmented State Space

Application to Switching Converters



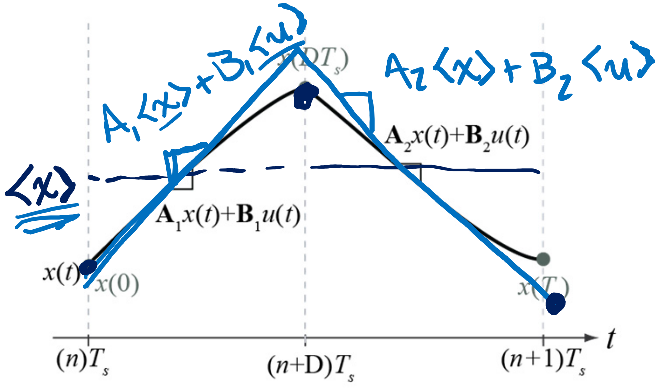
In both I & II converter is modeled by LTI subcircuit

$$\text{Define } s(t) = \begin{cases} 0 & (n+D)T_s < t < (n+1)T_s \\ 1, & nT_s < t < nT_s + DT_s \end{cases}$$

$$\dot{x}(t) = \underbrace{\left[A_1 s(t) + A_2\left(1-s(t)\right)\right]} x(t) + \left[B_1 s(t) + B_2\left(1-s(t)\right)\right] u(t)$$

$$\dot{x}(t) = A_{eq}(t)\, x(t) + B_{eq}(t)\, u(t) \longrightarrow \text{time-varying system}$$

State Space Averaging



Lets just average the two state spaces together

$$A_{av} = DA_1 + D'A_2$$

$$B_{av} = DB_1 + D'B_2$$

$$\langle \dot{x}_{av} \rangle|_{T_s} = A_{av}\langle x\rangle|_{T_s} + B_{av}\langle u\rangle|_{T_s}$$

In steady-state $\langle \dot{x}_{av} \rangle = \phi$

so, in steady-state

$$\phi = A_{av}\langle x\rangle + B_{av}\langle u\rangle$$

$$\langle x\rangle = -A_{av}^{-1} B_{av}\langle u\rangle$$

$$\langle x\rangle = -\left(\sum_{i=1}^{k} A_i \frac{t_i}{T_s}\right)^{-1}\left(\sum_{i=1}^{k} B_i \frac{t_i}{T_s}\right)\langle u\rangle$$

for k switching subintervals

Exact Solution

in I $\quad x(DT_s) = e^{A_1 DT_s} x_0 + A_1^{-1}\left(e^{A_1 DT_s} - I\right) B_1 u$

then, in II $\quad x(T_s) = e^{A_2 D' T_s} x(DT_s) + A_2^{-1}\left(e^{A_2 D' T_s} - I\right) B_2 u$

then, combining both

$$x(T_s) = e^{A_2 D' T_s}\left[e^{A_1 DT_s} x_0 + A_1^{-1}\left(e^{A_1 DT_s} - I\right) B_1 u\right] + A_2^{-1}\left(e^{A_2 D' T_s} - I\right) B_2 u$$

$$x(T_s) = e^{A_2 D' T_s} e^{A_1 DT_s} x_0 + e^{A_2 D' T_s} A_1^{-1}\left(e^{A_1 DT_s} - I\right) B_1 u + A_2^{-1}\left(e^{A_2 D' T_s} - I\right) B_2 u$$

for k subintervals

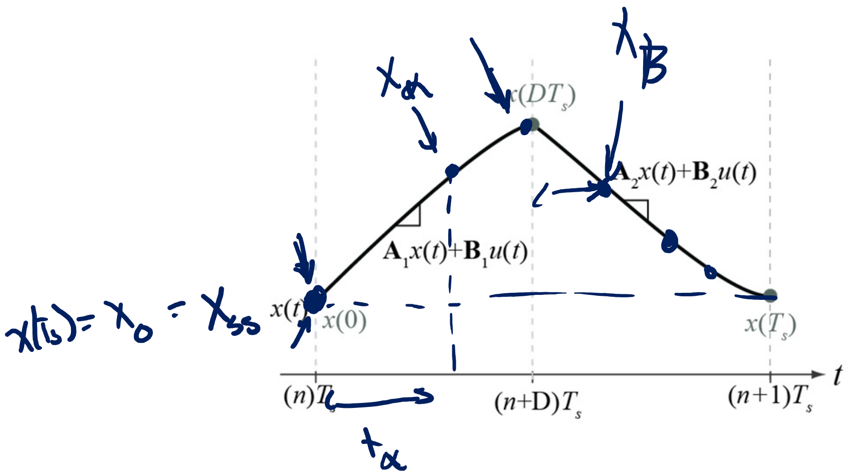
$$\boxed{x(T_s) = \prod_{i=k}^{1}\left(e^{A_i t_i}\right) x_0 + \sum_{i=1}^{k}\left[\prod_{j=k}^{i+1} e^{A_j t_j}\right] A_i^{-1}\left(e^{A_i t_i} - I\right) B_i u_i}$$

Steady-State Solution

in steady-state $x(T_s) = x_0 = X_{ss}$

$$\boxed{\underline{X_{ss}} = \left(I - \prod_{i=n}^{1} e^{A_i t_i}\right)^{-1} \sum_{i=1}^{n} \left[\prod_{j=n}^{i+1} e^{A_j t_j}\right] A_i^{-1}\left(e^{A_i t_i} - I\right) B_i u_i}$$

Waveform Reconstruction



$$\boxed{x_\alpha = e^{A_1 t_\alpha} x_{ss} + A_1^{-1}\left(e^{A_1 t_\alpha} - I\right) B_1 u}$$

can reconstruct states at any time point by re-applying solution

Comparison to Averaging

$$x_{ss} = \left(I - \prod_{i=k}^{1} e^{A_i t_i}\right)^{-1} \sum_{i=1}^{k}\left(\prod_{j=k}^{i+1} e^{A_j t_j}\right) A_i^{-1}\left(e^{A_i t_i} - I\right) B_i u$$

If we let $e^{A_i t_i} \approx I + A_i t_i$

$$x_{ss} = \left(\cancel{I} - \left(\cancel{I} + A_1 t_1 + A_2 t_2 + \ldots + \cancel{A_1 t_1 A_2 t_2 + A_1 t_1 A_3 t_3 + \ldots}\right)\right)^{-1} \cdots$$

$$\cdot \sum_{i=1}^{k}\left(\prod_{j=k}^{i+1} e^{A_j t_j}\right) A_i^{-1} \underbrace{\left(\cancel{I} + A_i t_i - \cancel{I}\right) B_i u}_{B_i t_i u}$$

$$\sum_{i=1}^{k}\left(\cancel{\prod_{j=k}^{i+1} e^{A_j t_j}}\right) \cdot B_i t_i u$$

Also neglect any terms with two $A_k t_k$ factors
& eliminate the product term in the forced response

$$x_{ss} = -\left(\sum_{i=1}^{k} A_i t_i\right)^{-1}\left(\sum_{i=1}^{k} B_i t_i\right) u = \langle x_{av} \rangle \rightarrow \text{same as state space averaging}$$