Limitations and Discussion

steady-state

$$X_{ss} = \boldsymbol{X_0} = \left(I - \prod_{i=k}^{0} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{k} \left(\prod_{j=k}^{i+1} e^{\boldsymbol{A}_j t_j}\right) \boldsymbol{A}_i^{-1}\left(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}\right)\boldsymbol{B}_i \boldsymbol{u}_i$$

Gives steady-state solution to any arbitrary periodically switched system where:

① Valid circuit which possesses a periodic steady-state solution

② $u(t) \simeq u_i$ within one subinterval

③ All A_i non-singular

④ Within any subinterval, we need to be able to model circuit w/ LTI equivalent

⑤ All $A_i, B_i,$ & t_i are known

Discuss

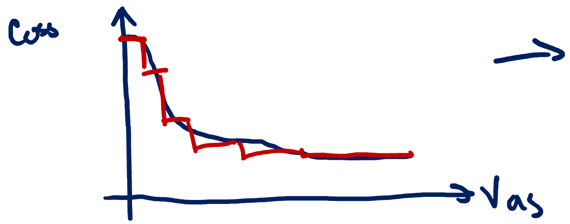
① Satisfied for circuits of interest

② Static or quasi-static inputs must be true for converter to respond
Alternative is to model the quickly-varying input as an internal waveform

③ We will develop methods next

④ → Nonlinear elements like $C_{oss}(V_{ds})$, $R_{ac}(f)$, P_{fe} cannot be included in A_i & B_i

can model with piecewise linear elements



→ can add additional A_i & B_i

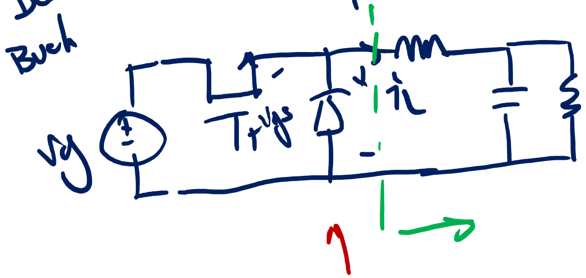
→ High-η approximation or otherwise just neglect & calculate afterwards

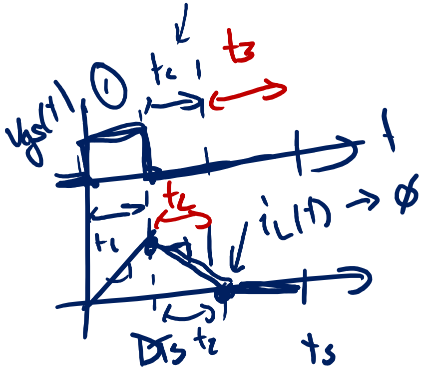
⑤ Can solve LTI equivalents for any A_i & B_i

for t_i → known for active time-dependent switching

"state-dependent" switching

e.g. DCM Buck





$$\dot{x}(t) = A(\underline{x}, t)\,x(t) + B(\underline{x}, t)\,u(t)$$

→ Both time-varying & nonlinear with state-dependent switching

Topological Time Invariance (HW2)

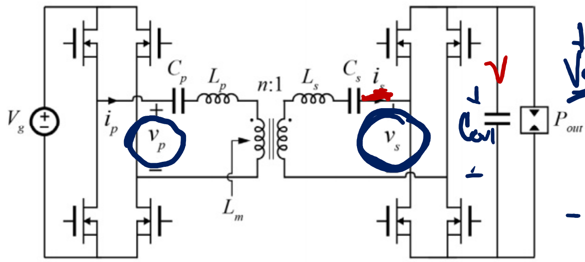


Fig. 1: Series Resonant Converter

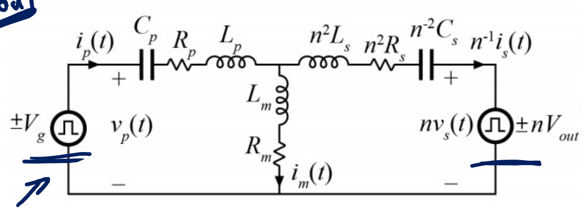


Fig. 2: Time-Invariant Circuit Model of SRC

if $u = [V_g \;\; V_{out}]^T$, All $\underline{A_i}$ will be the same & B_i differ only by +/- signs

Alternately, if we pick $u(t) = [v_p(t) \;\; v_s(t)]^T$

$\underline{A_i = A \;\; \& \;\; B_i = B}$

Not time-varying

Can model as a completely LTI system

$\dot{x} = Ax(t) + Bu(t)$

Augmented State Space Approach

u_i ✓✓

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}t}\boldsymbol{x}_0 + \underline{\int_0^t e^{\boldsymbol{A}(t-\tau)}\boldsymbol{B}\overset{\uparrow}{\boldsymbol{u}}(\tau)d\tau} \Longrightarrow \boldsymbol{x}(t) = e^{\boldsymbol{A}t}\boldsymbol{x}_0 + \underline{\boldsymbol{A}^{-1}(e^{\boldsymbol{A}t} - \boldsymbol{I})}\boldsymbol{B}\boldsymbol{u}$$

Need a new way to deal with the convolution integral

H. R. Visser and P. P. J. van den Bosch, "Modelling of periodically switching networks," in PESC '91

Non-invertible A Matrices

$$C_i = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix} \longrightarrow e^{C_i t} = \begin{bmatrix} e^{A_i t} & \int_0^t e^{A_i(t-\tau)} B_i \\ 0 & I \end{bmatrix}$$

$$C = \left[\begin{array}{cccc|l} A_1 & B_1 & C_1 & D_1 & \}\, n_1 \\ 0 & A_2 & B_2 & C_2 & \}\, n_2 \\ 0 & 0 & A_3 & B_3 & \}\, n_3 \\ 0 & 0 & 0 & A_4 & \}\, n_4 \\ \hline \underbrace{}_{n_1} & \underbrace{}_{n_2} & \underbrace{}_{n_3} & \underbrace{}_{n_4} & \end{array}\right]$$

$$e^{Ct} = \begin{bmatrix} F_1(t) & G_1(t) & H_1(t) & K_1(t) \\ 0 & F_2(t) & G_2(t) & H_2(t) \\ 0 & 0 & F_3(t) & G_3(t) \\ 0 & 0 & 0 & F_4(t) \end{bmatrix}$$

$$F_j(t) = e^{A_j t}, \qquad j = 1,2,3,4$$

$$G_j(t) = \int_0^t e^{A_j(t-s)} B_j e^{A_{j+1} s}\, ds, \qquad j = 1,2,3$$

$$H_j(t) = \int_0^t e^{A_j(t-s)} C_j e^{A_{j+2} s}\, ds$$

$$+ \int_0^t \int_0^s e^{A_j(t-s)} B_j e^{A_{j+1}(s-r)}$$

$$\cdot B_{j+1} e^{A_{j+2} r}\, dr\, ds, \qquad j = 1,2$$

$$K_1(t) = \int_0^t e^{A_1(t-s)} D_1 e^{A_4 s}\, ds + \int_0^t \int_0^s e^{A_1(t-s)} [\, C_1 e^{A_3(s-r)} B_3$$

$$+ B_1 e^{A_2(s-r)} C_2] e^{A_4 r}\, dr\, ds$$

$$+ \int_0^t \int_0^s \int_0^r e^{A_1(t-s)} B_1 e^{A_2(s-r)} B_2 e^{A_3(r-w)} B_3 e^{A_4 w}\, dw\, dr\, ds.$$

→ C. Van Loan, "Computing Intergrals Invovling the Matrix Exponential", *IEEE Trans on Automatic Control*, 1978

Augmented circuit description:

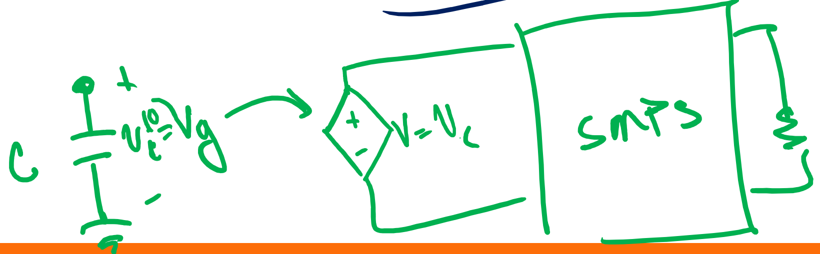
$$\tilde{A} = \begin{array}{c|cc|c} & n_s & p_i & \\ n_s & A_i & B_i & n_s \\ p_i & \emptyset & \emptyset & p_i \end{array} \rightarrow \text{Augmented state matrix}$$

$$\rightarrow \boxed{\dot{\tilde{x}} = \tilde{A}\tilde{x}} \rightarrow \text{homogeneous system}$$

$$\tilde{x} = \begin{bmatrix} x \\ u \end{bmatrix}, \quad \dot{\tilde{x}} = \begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A_i x + B_i u \\ \emptyset \end{bmatrix}$$

solution $\boxed{\tilde{x}(t) = e^{\tilde{A}t}\tilde{x}_0}$

$u = V_g$, $x \rightarrow$ actual circuit states



$$\rightarrow u = [\ ] \quad \hat{x} = \begin{bmatrix} x \\ v_c \end{bmatrix} \quad A \rightarrow \hat{A} = \begin{bmatrix} A_i & B_i \\ \emptyset & \emptyset \end{bmatrix}$$