47. For the circuit of **Fig. 13.66**, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$, and $R_L = 1\ \Omega$. Select a and b to achieve a peak voltage of 200 mV magnitude across R_L.

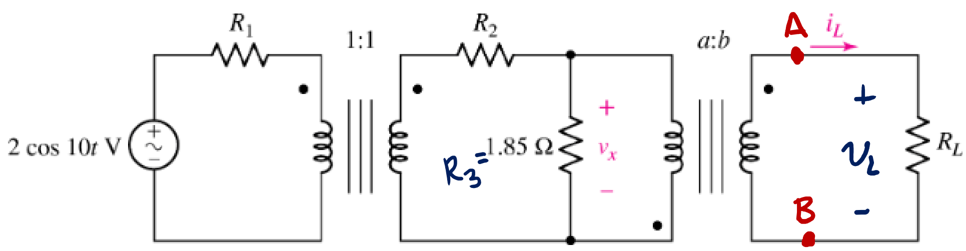
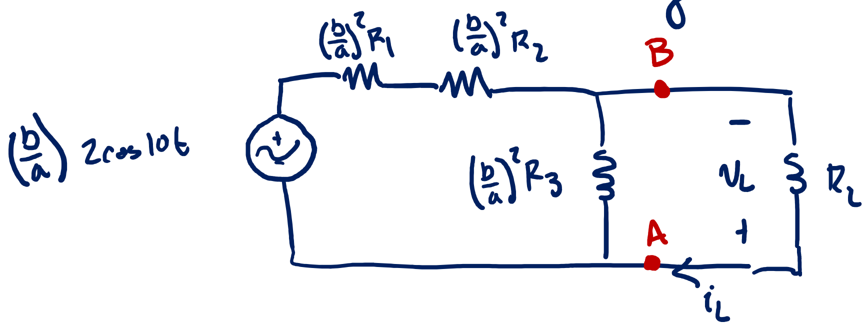


FIGURE 13.66

First, lets reflect everything to the right. Note that the a:b transformer is inverting



$$v_L = -\left(\frac{b}{a}\right) 2\cos 10t \; \frac{R_L \| \left(\frac{b}{a}\right)^2 R_3}{R_L \| \left(\frac{b}{a}\right)^2 R_3 + \left(\frac{b}{a}\right)^2 R_1 + \left(\frac{b}{a}\right)^2 R_2}$$

, by voltage divider with R_3 & R_L in parallel

Peak voltage will occur when $\cos 10t = -1$, so we can drop the negative sign & cosine term. Expanding the parallel resistance & dividing through by $\left(\frac{b}{a}\right)^2$:

$$|v_L| = 200\text{ mV} = \left(\frac{b}{a}\right) 2 \left[\frac{\dfrac{R_L R_3}{R_L + \left(\frac{b}{a}\right)^2 R_3}}{\dfrac{R_L R_3}{R_L + \left(\frac{b}{a}\right)^2 R_3} + R_1 + R_2} \right] \quad \frac{\left(R_L + \left(\frac{b}{a}\right)^2 R_3\right)}{\left(R_L + \left(\frac{b}{a}\right)^2 R_3\right)}$$

$$200\text{ mV} = 2\frac{b}{a} \; \frac{R_L R_3}{R_L R_3 + R_L(R_1 + R_2) + \left(\frac{b}{a}\right)^2 R_3 (R_1 + R_2)}$$

collecting terms:

$$0 = (200mV)(R_1+R_2)R_3\left(\frac{b}{a}\right)^2 - 2R_L R_3\left(\frac{b}{a}\right) + (200mV)\left[R_L R_3 + (R_1+R_2)R_L\right]$$

Plugging in

$$\phi = 1.85\left(\frac{b}{a}\right)^2 - 3.7\left(\frac{b}{a}\right) + 1.37$$

use quadratic formula:

$$\left(\frac{b}{a}\right) = \frac{3.7 \pm \sqrt{3.7^2 - 4(1.85)(1.37)}}{2(1.85)}$$

$$\boxed{\frac{b}{a} = 0.5 \text{ or } 1.5}$$

both are valid solutions. Further, we can choose <u>any</u> combination of b & a that meets this ratio.

<u>MATLAB to find roots:</u>

```
R1 = 1;
R2 = 4;
RL = 1;
R3 = 1.85;

a = 1/5*(R1+R2)*R3;
b = -2*RL*R3;
c = 1/5*(RL*R3) + 1/5*(R1+R2)*RL;

roots([a b c])
```

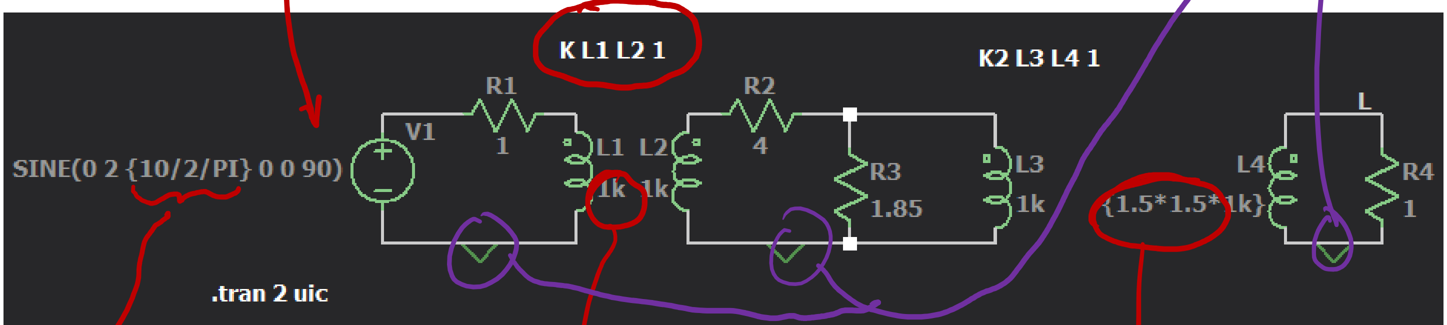
```
ans =

   1.5094
   0.4906
```

Though not required, lets check answer using LTspice:

90° phase because its a cosine

k-statement
k = name
L_1 = 1st inductor
L_2 = 2nd inductor
1 = coupling coefficient

Need ground ref for each isolated subcircuit



$\omega = 10 \frac{rad}{sec}$

$f = \frac{\omega}{2\pi}$ Hz

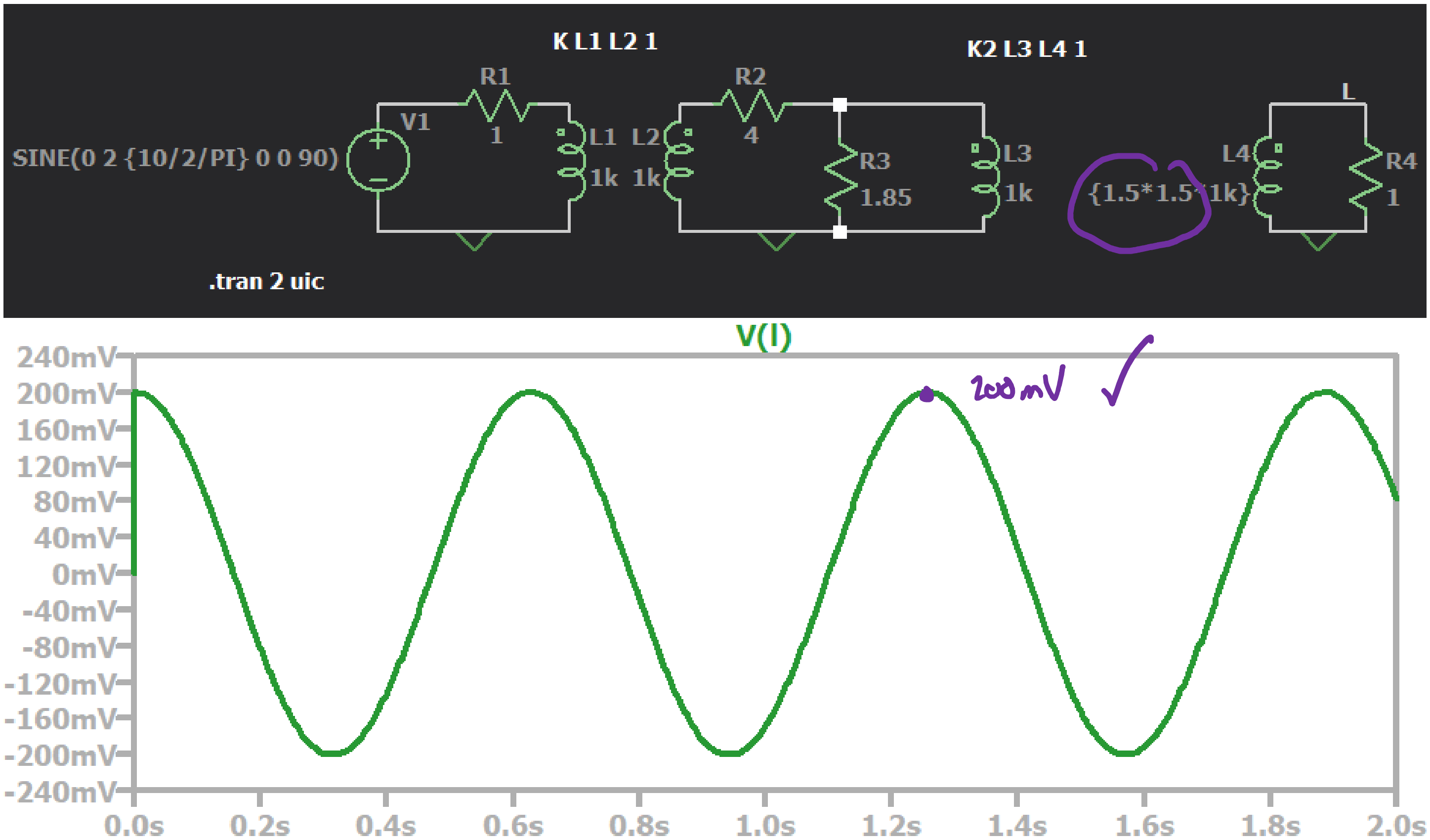
for ideal transformer, inductance should be "large" → try 1k Henries

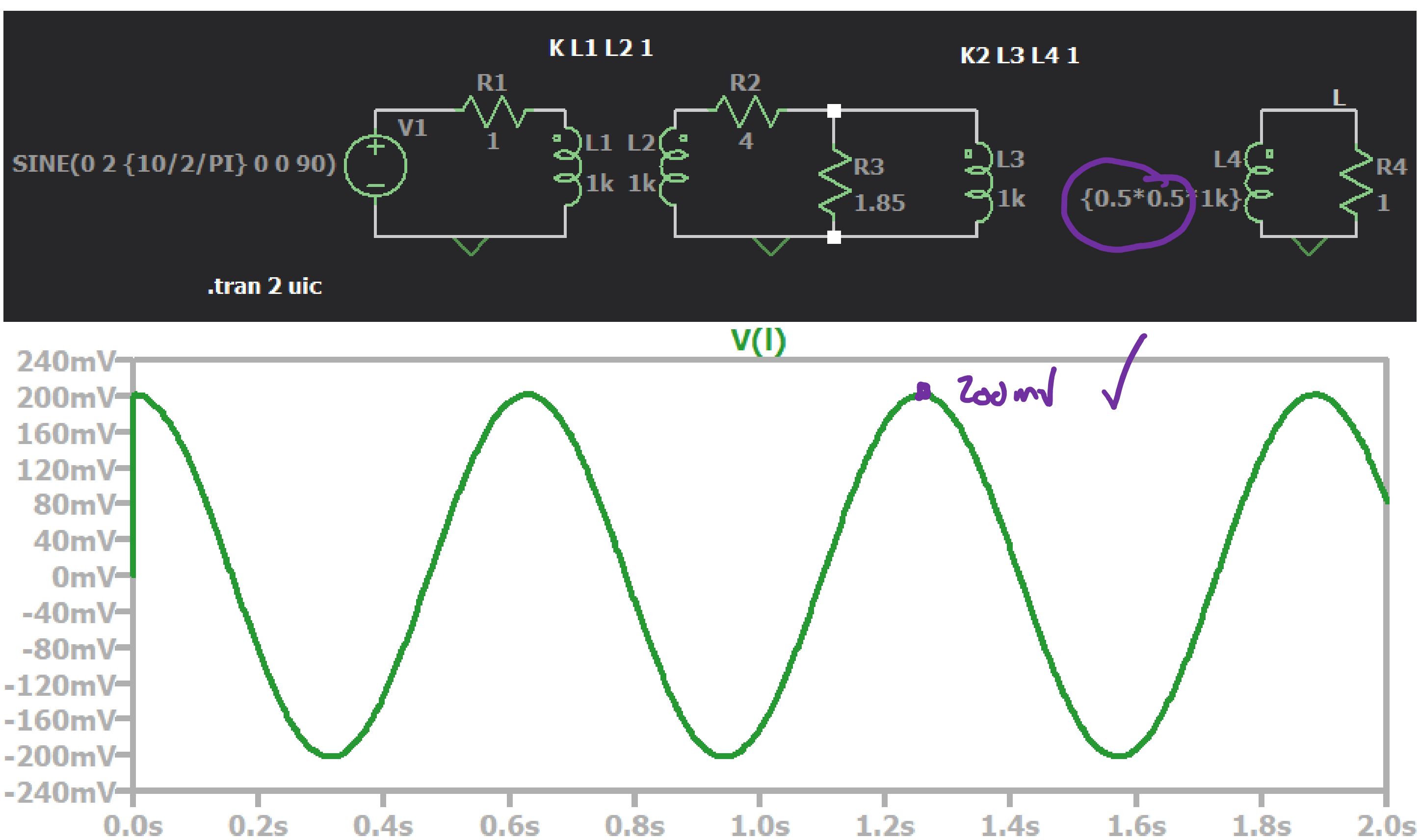
Recall from lecture when k=1

$N_1 = N_2\sqrt{\frac{L_1}{L_2}} = N_2\sqrt{\frac{N_1^2}{N_2^2}}$

so

$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} = \left(\frac{b}{a}\right)^2$


K L1 L2 1
K2 L3 L4 1
R1
1
V1
SINE(0 2 {10/2/PI} 0 0 90)
L1
1k
L2
1k
R2
4
R3
1.85
L3
1k
{1.5*1.5*1k}
L4
L
R4
1
.tran 2 uic
V(l)
200mV ✓


K L1 L2 1
K2 L3 L4 1
R1
1
V1
SINE(0 2 {10/2/PI} 0 0 90)
L1
1k
L2
1k
R2
4
R3
1.85
L3
1k
{0.5*0.5*1k}
L4
L
R4
1
.tran 2 uic
V(l)
200mV ✓