

11. Assuming there are no longer any transients present, determine the current labeled i_L in the circuit of Fig. 10.47. Express your answer as a single sinusoid.

↳ only steady-state = particular solution

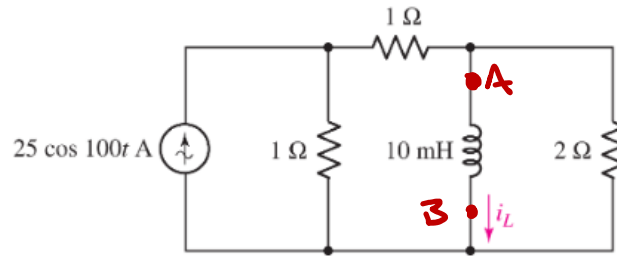
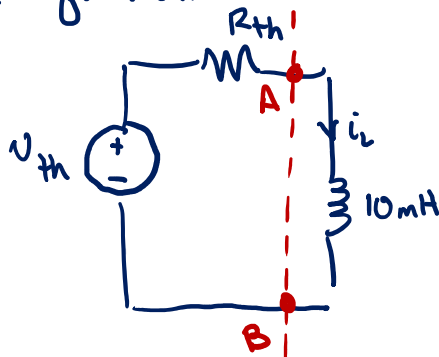


FIGURE 10.47

Redraw, focusing on terminals A & B:



Thevenin equivalent:



$$R_{th} = (1\Omega + 1\Omega) \parallel 2\Omega = 1\Omega$$

$$V_{th} = \frac{25(1\Omega)}{(1\Omega + 1\Omega)} (1\Omega) = 12.5V$$

$$V_{th} = 12.5 \cos 100t$$

$$V_{th} = i_L R_{th} + L \frac{di_L}{dt}$$

Need only the particular solution so "guess"

$$i_L(t) = I_L \cos(100t + \phi)$$

$$\frac{di_L}{dt} = -100 I_L \sin(100t + \phi)$$

$$V_{th} = i_L R_{th} + L \frac{di_L}{dt}$$

$$12.5 \cos(100t) = R_{th} I_L \cos(100t + \varphi) - 100L I_L \sin(100t + \varphi)$$

$$= \sqrt{(R_{th} I_L)^2 + (100L I_L)^2} \cos(100t + \varphi - \tan^{-1}\left(\frac{-100L I_L}{R_{th} I_L}\right))$$

$$12.5 \cos(100t) = I_L \sqrt{R_{th}^2 + (100L)^2} \cos(100t + \varphi - \tan^{-1}\left(\frac{-100L}{R_{th}}\right))$$

Must match **amplitude** and **phase** in this equation

$$I_L = \frac{12.5}{\sqrt{1^2 + (100 \cdot 10\text{mH})^2}}$$

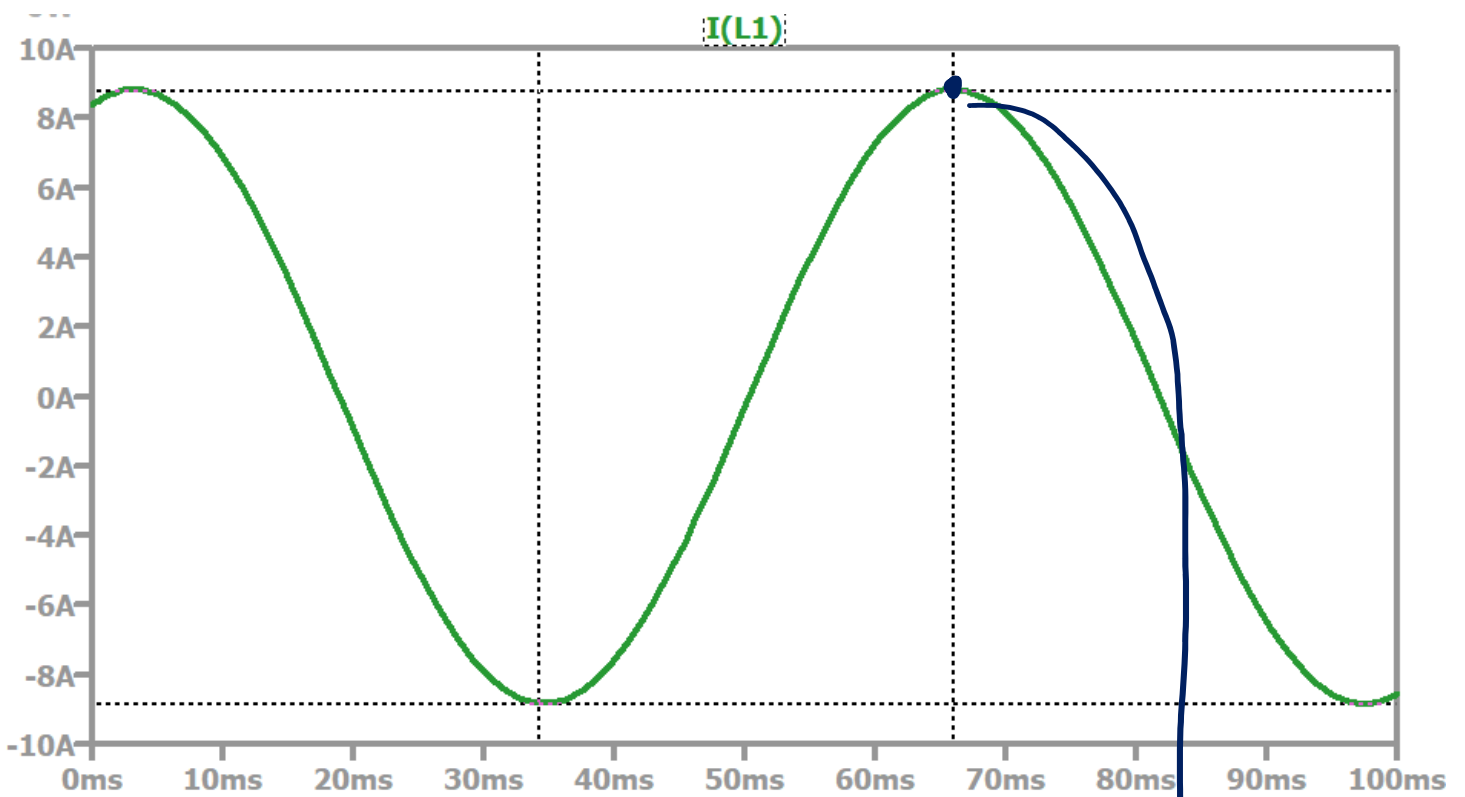
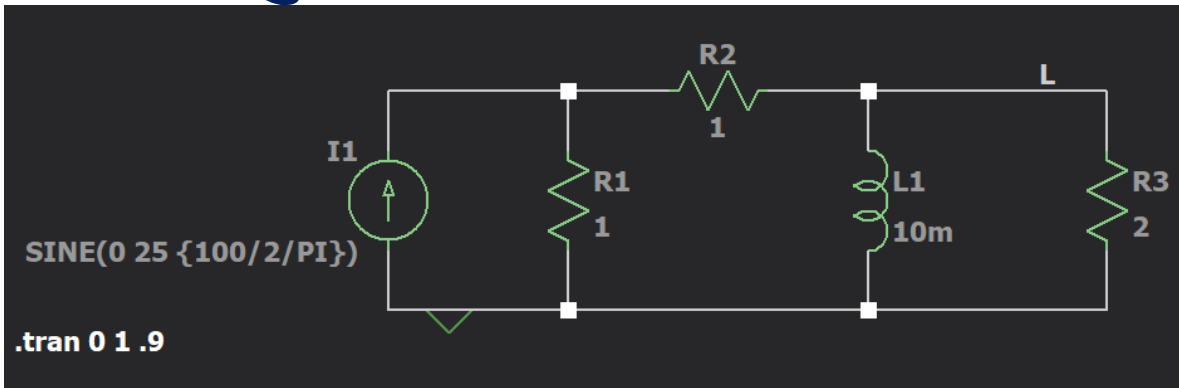
$$I_L = \frac{12.5}{\sqrt{2}} = 8.84$$

$$\varphi = \tan^{-1}\left(\frac{-100 \cdot 10\text{mH}}{1 \Omega}\right)$$

$$\varphi = \tan^{-1}(-1) = -45^\circ = -\frac{\pi}{4}$$

$$i_L(t) = 8.84 \cos(100t - 45^\circ)$$

check using LTSpice



Cursor 1	
I(L1)	
Horz: 65.882353ms	Vert: 8.8292119A
Cursor 2	
I(L1)	
Horz: 34.117647ms	Vert: -8.8093347A
Diff (Cursor2 - Cursor1)	
Horz: -31.764706ms	Vert: -17.638547A
Freq: 31.481481Hz	Slope: 555.288

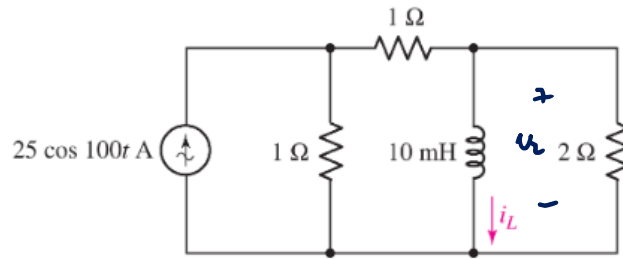


FIGURE 10.47

12. Calculate the power dissipated in the $2\ \Omega$ resistor of Fig. 10.47 assuming there are no transients present. Express your answer in terms of a single sinusoidal function.

from previous problem, $i_L(t) = 8.84 \cos(100t - 45^\circ)$

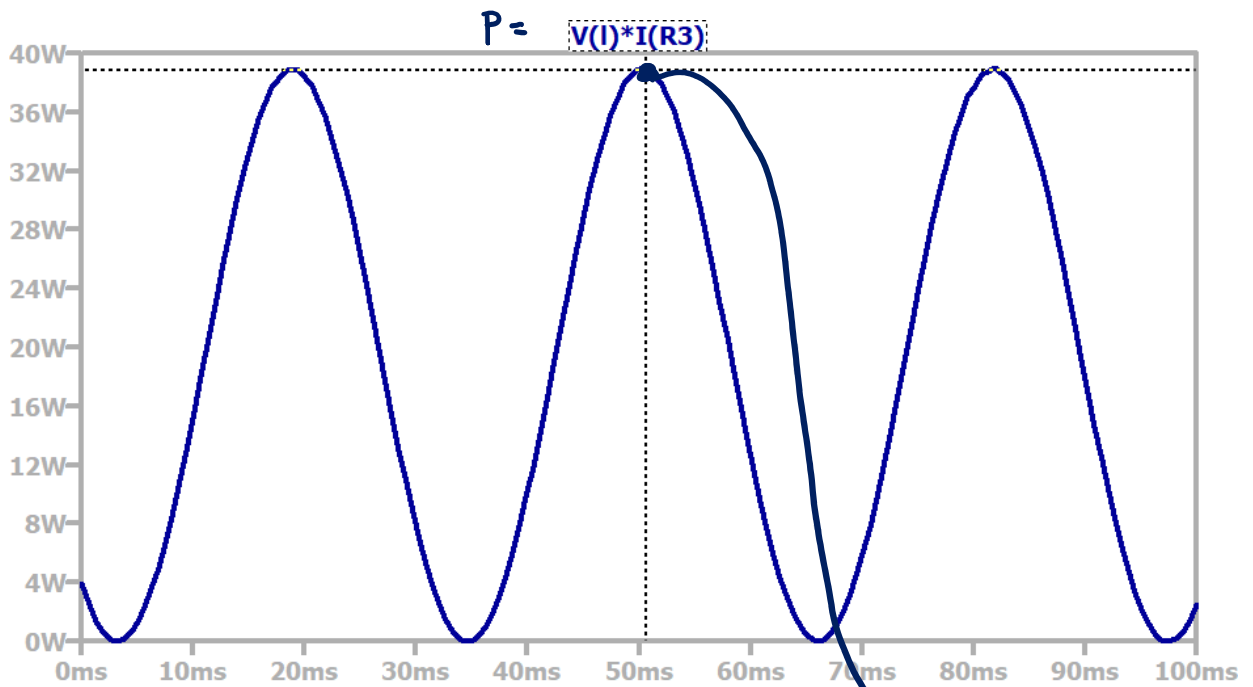
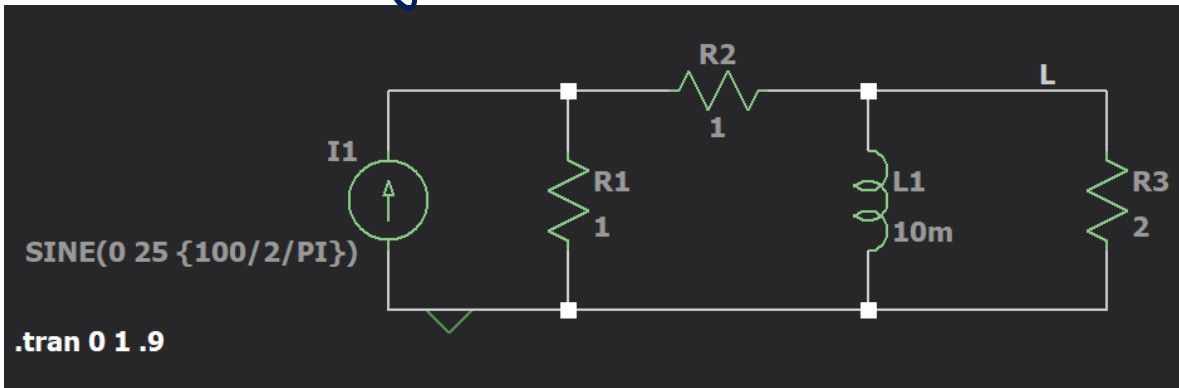
$$u_L = L \frac{di_L}{dt} = -(10\text{ mH})(100) 8.84 \sin(100t - 45^\circ)$$

$$= -8.84 \sin(100t - 45^\circ)$$

$$P_{2\Omega}(t) = \frac{u_L(t)^2}{2\Omega} = \left(\frac{8.84^2}{2} \right) \sin^2(100t - 45^\circ)$$

$$= 39.1 \sin^2(100t - 45^\circ) \text{ W}$$

Check using LTspice



Draft1.raw

Cursor 1	
V(I)*I(R3)	
Horz: 50.470588ms	Vert: 38.942076W
Cursor 2	
Horz: -- N/A --	Vert: -- N/A --
Diff (Cursor2 - Cursor1)	
Horz: -- N/A --	Vert: -- N/A --
Freq: -- N/A --	Slope: -- N/A --