60. The circuit in **Fig. 11.51** uses a Pi network to match the impedance between source and load for maximum power transfer. A 60 Hz source has a series impedance of $50 + j5\ \Omega$. What load impedance would a Pi network with $C_S = 70.1\ \mu F$, $C_L = 39.8\ \mu F$, and $L = 0.225$ H correspond to for maximum power transfer?

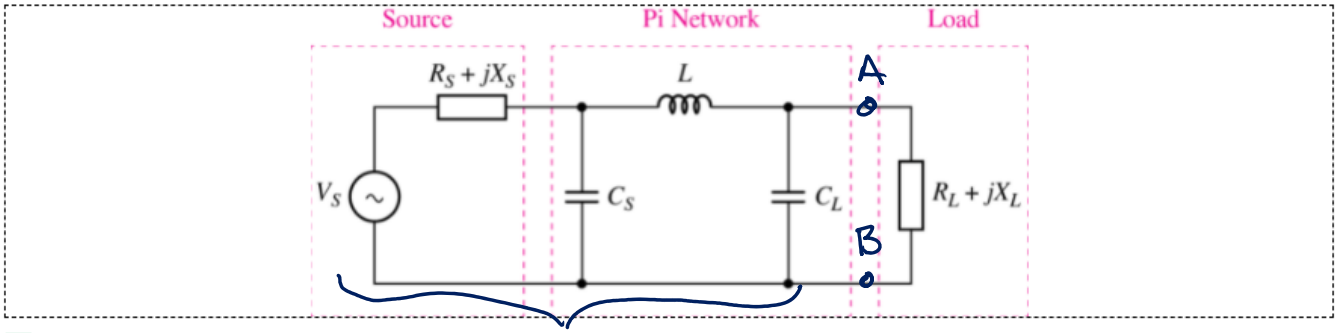
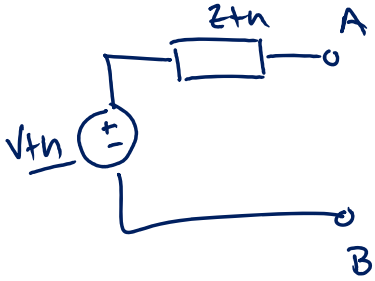


FIGURE 11.51



$\omega = 2\pi 60$

$$Z_{th} = \left(\left[(R_S + jX_S) \parallel \frac{-j}{\omega C_S} \right] + j\omega L \right) \parallel \frac{-j}{\omega C_L}$$

MATLAB Calculation:

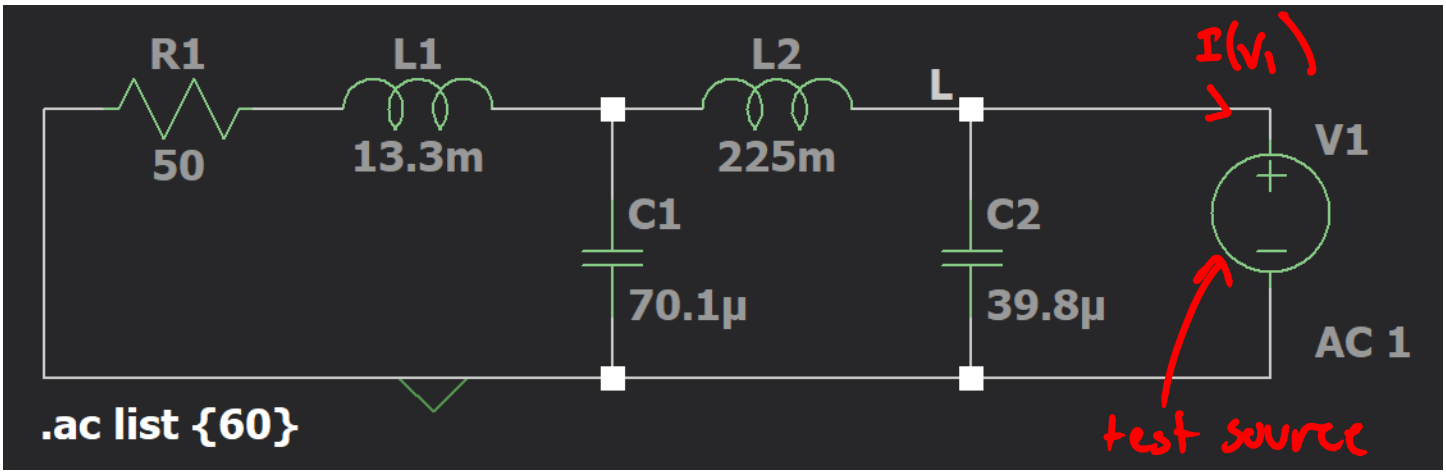
```
w = 2*pi*60
Zs = 50 +1j*5;
ZCs = -1j/w/70.1e-6;
ZCL = -1j/w/39.8e-6;
ZL = 1j*w*225e-3;

Zeq1= 1/(1/Zs + 1/ZCs);
Zth = 1/(1/(Zeq1 + ZL) + 1/ZCL)

>> Zth = 2.0068e+02 - 1.2038e+00i
```

$$Z_{th} = 200 - j1.2$$

for maximum power $Z_L = Z_{th}^*$

so $\boxed{Z_L = Z_{th}^* = R_L + jX_L = 200 + j1.2}$

Validate with LTSpice



* H:\Courses\ECE 202\Spring 2022\Solutions\Draft1.asc

```
        --- AC Analysis ---

frequency:      60              Hz
V(n001):        mag:    0.499204 phase:    -128.304°          voltage
V(n002):        mag:    0.501718 phase:    -122.577°          voltage
V(1):           mag:           1 phase: -2.42657e-020°        voltage
I(C2):          mag:   0.0150042 phase:          90°          device_current
I(C1):          mag:   0.0132589 phase:    -32.5772°          device_current
I(L2):          mag:   0.0157818 phase:     108.411°          device_current
I(L1):          mag:  0.00998409 phase:     51.6964°          device_current
I(R1):          mag:  0.00998409 phase:    -128.304°          device_current
I(V1):          mag: 0.00498454 phase:    -179.652°          device_current
```

$$Z_{th} = \frac{V(V_1)}{-I(V_1)} = \frac{1 \angle 0^\circ \text{ V}}{-4.98 \angle -179.6^\circ \text{ mA}}$$

$$= -200 \angle 179.6^\circ \ \Omega$$

$$= 200 - j1.2 \ \Omega \quad \checkmark$$