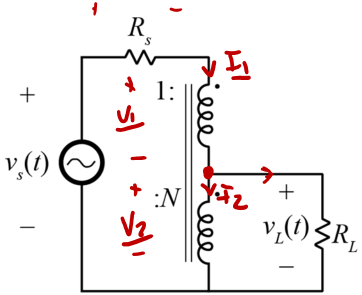
Example Problem



$\omega = 2\pi 60$

Find $v_L(t)$ for $v_s(t) = 170\cos(2\pi 60t)$ and for $R_s = 10\ \Omega$, $N = 0.1$, and $R_L = 50\ \Omega$

$v_L(t) \rightarrow \underline{V_L}$

$v_s(t) \rightarrow \underline{V_s} = 170\angle 0^\circ$ or $170e^{j0}$ or $170 + j\emptyset$

$R_s \rightarrow R_s$

$R_L \rightarrow R_L$

$1 + \frac{1}{N} = \frac{N+1}{N}$

$\frac{\underline{V_1}}{1} = \frac{\underline{V_2}}{N}$, $\underline{I_1} + N\underline{I_2} = \emptyset$

Node:

$\underline{I_1} = \underline{I_2} + \frac{\underline{V_L}}{R_L}$

$\left(1 + \frac{1}{N}\right)\underline{I_1} = \frac{\underline{V_L}}{R_L}$

Loop

$\underline{V_s} = \underline{V_1} + \underline{V_2} + \underline{I_1}R_s$

$\underline{V_s} = \left(\frac{1}{N} + 1\right)\underline{V_L} + \underline{I_1}R_s$

$\underline{V_s} = \left(\frac{N+1}{N}\right)\underline{V_L} + \frac{R_s}{R_L}\left(\frac{1}{N+1}\right)\underline{V_L} \longrightarrow$

$$\boxed{\underline{V_L} = \underline{V_s}\,\frac{1}{\frac{N+1}{N} + \frac{R_s}{R_L}\frac{1}{N+1}}} = 15.5\angle 0^\circ$$

compute & inverse transform

$v_L(t) = 15.5\cos(2\pi 60t + 0^\circ)$

Impedance

Phasor equivalent of ohm's law

$$\underline{V} = \underline{I}\, Z$$

$Z \rightarrow$ "impedance" , $Z = R + jX$

"resistence" $Re\{Z\}$

"reactance" $Im\{Z\}$

$$\begin{cases} Z_R = R & \text{for resistor} \\ Z_L = j\omega L & \text{for inductor} \\ Z_C = \dfrac{-j}{\omega C} & \text{for capacitor} \end{cases}$$

All Z have units of Ohms

Y = "Admittance" $= \dfrac{1}{Z} = G + jB$ $\longrightarrow$ units of siemens (or mhos)

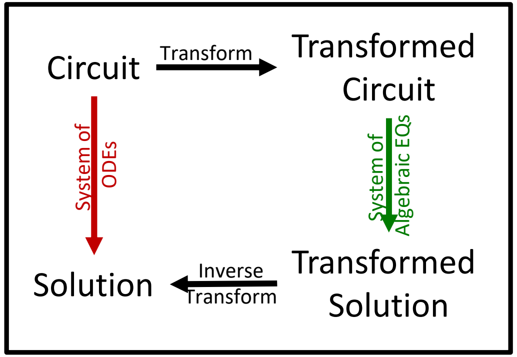
"conductance"

"susceptance"

$$\frac{1}{Z} \neq \frac{1}{R} + j\frac{1}{X}$$

$$\frac{1}{Z} = \frac{1}{R+jX}\,\frac{(R-jX)}{(R-jX)} = \frac{R-jX}{R^2+X^2} = \frac{R}{R^2+X^2} - j\,\frac{X}{R^2+X^2}$$

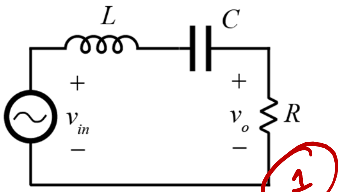
Phasor Circuit Analysis



Goal: Analyze a LTI circuit to find steady-state solution with only single-frequency sinusoidal source(s)

1. Transform all sources & signals into their phasor equivalents
2. Transform all passives into impedances
3. Solve the circuit
    - Use 201 techniques for DC resistor-only circuits
4. Transform solution back into the time domain

Resonance Example



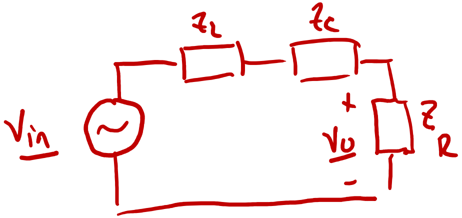
$10\cos(\omega t - 90^\circ)$

Find $v_o(t)$ for $v_{in}(t) = 10\sin(\omega t)$ and $\omega = 2\pi 100$ kHz, $R = 10\ \Omega$, $L = 10\ \mu\text{H}$, and $C = 253$ nF

1. $v_{in}(t) \rightarrow V_{in} = 10\angle -90^\circ = -j10 = 10e^{-j\frac{\pi}{2}}$

   $v_o(t) \rightarrow V_o$

2. $R \rightarrow Z_R = R = 10\,\Omega$ $\quad L \rightarrow Z_L = j\omega L = j2\pi$ $\quad C \rightarrow Z_C = \frac{-j}{\omega C} = -j2\pi$

3. 

$$\boxed{V_o = V_{in}\frac{Z_R}{Z_L + Z_C + Z_R}} = 10\angle 90^\circ$$

4. $v_o(t) = |V_o|\cos(\omega t + \angle V_o)$

$$\boxed{v_o(t) = 10\cos(\omega t - 90^\circ)}$$