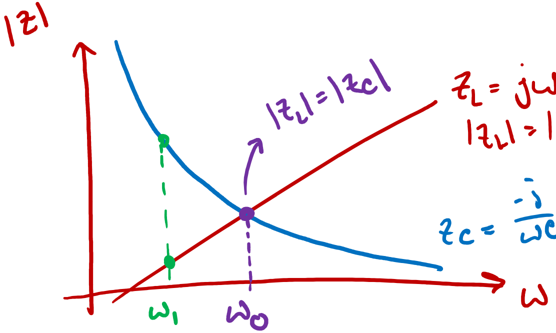
Reactance and Resonance



$Z_L = j\omega L$

$|Z_L| = |j\omega L| = |\omega L|$

$Z_C = \frac{-j}{\omega C} \rightarrow |Z_C| = \frac{1}{\omega C}$

@ $\omega_0 \rightarrow |Z_L| = |Z_C|$

$\omega_0 L = \frac{1}{\omega_0 C}$

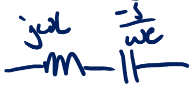
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

resonant frequency

At low frequency $\omega \rightarrow \phi$ ($\approx$ DC) $Z_C \rightarrow \infty$ (open), $Z_L \rightarrow \phi$ (short)

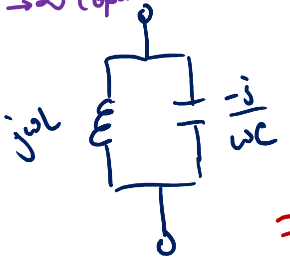
At high frequency $\omega \rightarrow \infty$ $Z_C \rightarrow \phi$ (short), $Z_L \rightarrow \infty$ (open)

At resonance



$\Rightarrow$ short @ resonance (ω_0)

$Z_{eq} = j\omega L + \frac{-j}{\omega C}$

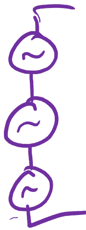


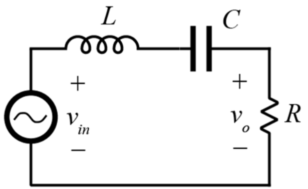
$Z_{eq} = \frac{1}{\frac{1}{j\omega L} + \frac{\omega C}{-j}} = \frac{j\omega L \cdot \left(\frac{-j}{\omega C}\right)}{j\omega L + \frac{-j}{\omega C}}$ for only 2 in parallel

$\Rightarrow$ open @ resonance

Phasor Superposition

3 frequencies → apply superposition in time domain





Find $v_o(t)$ for $v_{in}(t) = 10\cos(\omega t) + 10\cos(2\omega t) + 10\cos(0.5\omega t)$ and $\omega = 2\pi 100$ kHz, $R = 10\ \Omega$, $L = 10\ \mu$H, and $C = 253$ nF

Low frequency: $\omega_{LF} = \frac{1}{2}\omega = 2\pi 50\text{kHz}$

$V_{in} = 10\angle 0^\circ$

$Z_L = j\omega_{LF}L = j\pi$

$Z_C = \frac{-j}{\omega_{LF}C} = -j4\pi$

$Z_R = R = 10\Omega$

same ↓

$V_o = V_{in}\frac{Z_R}{Z_R + Z_L + Z_C} = 0.73\angle 43^\circ$

$v_{oLF}(t) = 0.73\cos(0.5\omega t + 43^\circ)$

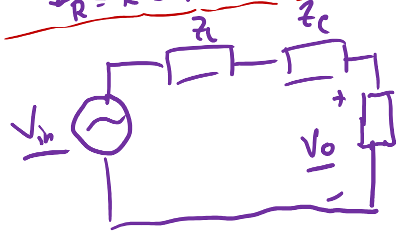
Fundamental: $\omega = 2\pi 100\text{kHz}$

$V_{in} = 10\angle 0^\circ$ → V_o

$Z_L = j\omega L = j2\pi$

$Z_C = \frac{-j}{\omega C} = -j2\pi$

$Z_R = R = 10$



same for all 3 frequencies

$V_o = V_{in}\frac{Z_R}{Z_R + Z_L + Z_C} = 10\angle 0^\circ$

$v_o(t) = 10\cos(\omega t)$

High frequency: $\omega_{HF} = 2\omega = 2\pi 200\text{kHz}$

$V_{in} = 10\angle 0^\circ$

$Z_L = j\omega_{HF}L = j4\pi$

$Z_C = \frac{-j}{\omega_{HF}C} = -j\pi$

↓

$V_o = 0.73\angle -43^\circ$

$v_{oHF}(t) = 0.73\cos(2\omega t - 43^\circ)$

$$v_o(t) = 10\cos(\omega t) + 0.73\cos(0.5\omega t + 43^\circ) + 0.73\cos(2\omega t - 43^\circ)$$

Example: WPT Problem

$v_{tx}(t) = 100 \sin(2\pi\, 6.78\,\text{MHz}\; t)$