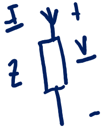
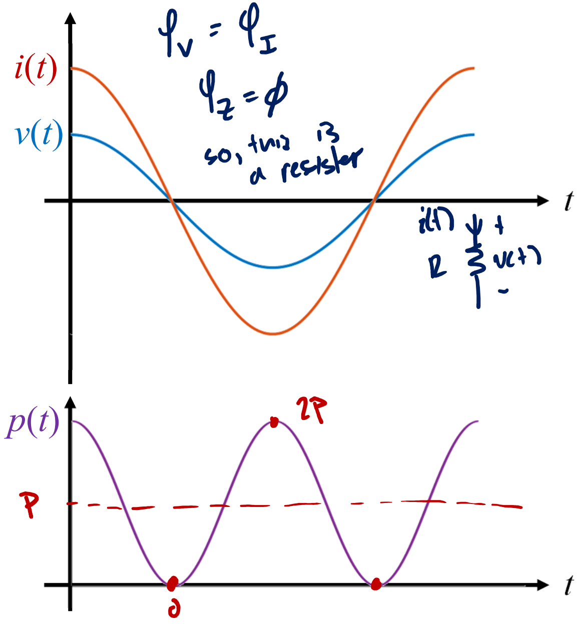
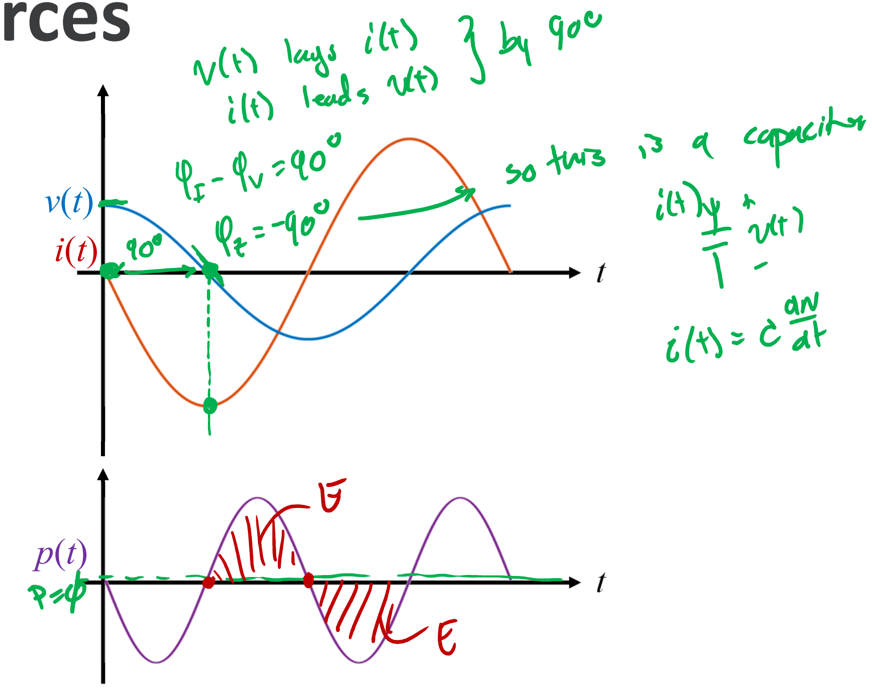
Power with Sinusoidal Sources







for a resistor

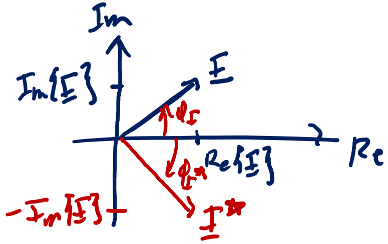
$$P = \frac{V_A I_A}{2}\cos(\varphi_V - \varphi_I) = \frac{V_A I_A}{2}\cos(\varphi_Z) = \frac{V_A I_A}{2}\cos(\phi) = \frac{V_A I_A}{2}$$

for a capacitor

$$P = \frac{V_A I_A}{2}\cos(-90°) = \phi$$

Capacitors & inductors must have $P=\phi$ in steady-state

Complex Power



$$P = \frac{V_A I_A}{2}\cos(\varphi_V - \varphi_I) = \boxed{\frac{1}{2}\mathrm{Re}\{\underline{V}\,\underline{I}^*\}}$$

$$= \frac{1}{2}\mathrm{Re}\left\{V_A e^{j\varphi_V} \cdot I_A e^{j(-\varphi_I)}\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{V_A I_A e^{j(\varphi_V - \varphi_I)}\right\}$$

$$= \frac{1}{2} V_A I_A \cos(\varphi_V - \varphi_I)$$

what about the imaginary part of $\underline{V}\,\underline{I}^*$

$$S = \frac{1}{2}\underline{V}\,\underline{I}^* = P + jQ$$

S: complex power [VA] Volt-amps

P: Average power "Real power" [W] watts

Q: Reactive power [VAR] "volt-amp reactive"

Apparent Power & Power Factor

"Apparent Power" = $|S| = \frac{V_A I_A}{2}$

- maximum possible real power for V_A & I_A stresses on circuit

"Power Factor" $PF = \frac{P}{|S|} = \frac{\cancel{\frac{V_A I_A}{2}} \cos(\varphi_Z)}{\cancel{\frac{V_A I_A}{2}}} = \cos(\varphi_Z) = \cos(\varphi_V - \varphi_I)$

either leading or lagging current with respect to voltage

$0 \leq PF \leq 1$

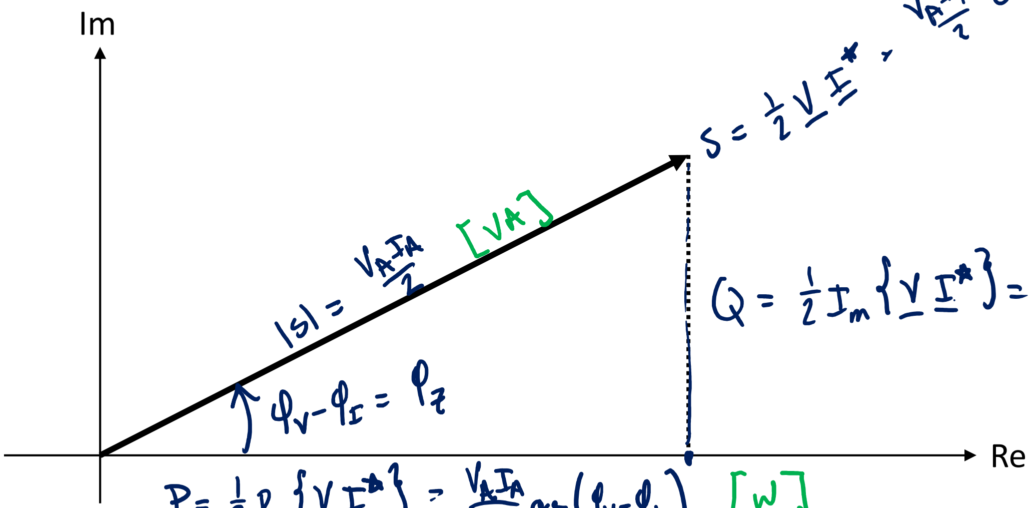
Power Triangle

$V_A = \sqrt{2}\, V_{rms}$

$I_A = \sqrt{2}\, I_{rms}$

$\varphi_V - \varphi_I = \varphi_Z$

$\underline{V} = \underline{I}\,\underline{Z}$



$S = \frac{1}{2}\underline{V}\,\underline{I}^* = \frac{V_A I_A}{2} e^{j(\varphi_V - \varphi_I)}$ [VA]

$|S| = \frac{V_A I_A}{2}$ [VA]

$\varphi_V - \varphi_I = \varphi_Z$

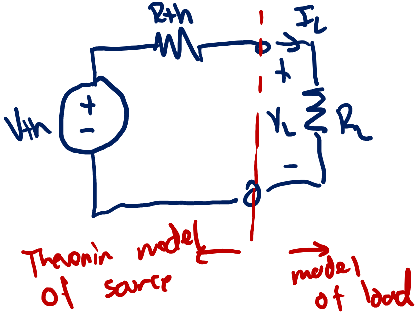
$Q = \frac{1}{2} I_m\{\underline{V}\,\underline{I}^*\} = \frac{V_A I_A}{2} \sin(\varphi_V - \varphi_I)$ [VAR]

$P = \frac{1}{2} R_e\{\underline{V}\,\underline{I}^*\} = \frac{V_A I_A}{2} \cos(\varphi_V - \varphi_I)$ [W]

$= \frac{1}{2} Re\left\{\underline{V} \left(\frac{\underline{V}}{\underline{Z}}\right)^*\right\}$

$PF = \frac{P}{|S|} = \cos(\varphi_V - \varphi_I)$ leading or lagging

Maximum Power Transfer



what value of R_L will yield maximum power $\quad P_L = V_L I_L$

Answer: $\underline{R_L = R_{th}}$

$$P_L = V_L I_L = \left(V_{th} \frac{R_L}{R_L + R_{th}}\right)\left(\frac{V_{th}}{R_L + R_{th}}\right) = V_{th}^2 \frac{R_L}{(R_L + R_{th})^2}$$

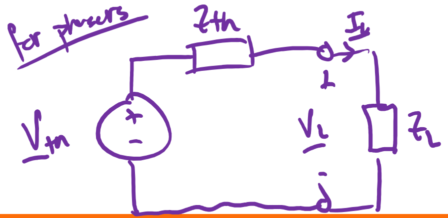
to find extrema

$$\frac{\partial P_L}{\partial R_L} = \cancel{V_{th}^2}\left[\frac{1 \cdot (R_L + R_{th})^{\cancel{2}} - R_L(2)\cancel{(R_L + R_{th})}}{(R_L + R_{th})^{\cancel{4}\,3}}\right] = \emptyset$$

$$0 = \frac{R_L + R_{th} - 2R_L}{(R_L + R_{th})^3} \rightarrow \text{numerator} \quad \underline{R_{th} - R_L = \emptyset}$$

what value of R_{th} will give maximum power to a fixed R_L?

$$\boxed{R_{th} = \emptyset}$$



what Z_L will maximize power transfer to the load?

$$\boxed{Z_L = Z_{th}^*}$$