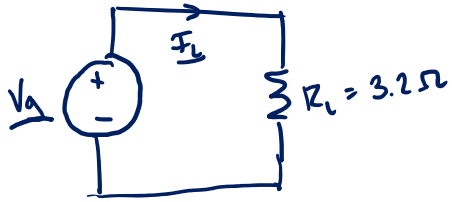
Matching Example

$\omega = 2\pi(60\text{Hz})$

$\underline{V_g} = 170\angle 0^\circ\ \text{V}$

residential wall outlet is 120 V rms = 170 Vpk @ 60 Hz



with $R_L = 3.2\Omega$ ≃ equivalent model of plugged-in load

$$\underline{I_L} = \frac{\underline{V_g}}{R_L} = \frac{170\angle 0^\circ}{3.2} = 53\angle 0^\circ\ \text{A}$$

$$S_g = \frac{1}{2}\underline{V_g}\,\underline{I_L} = \frac{1}{2}(170\angle 0^\circ)(53\angle 0^\circ) = 4.5\text{kW} + j0\ \text{VAR} = 4.5\angle 0^\circ\ \text{kVA}$$

$j\omega L \rightarrow L = 5\text{mH}$

$Z_L = j(2\pi 60)(5e\text{-}3) = j1.89\Omega$

$$\underline{I_L} = \frac{\underline{V_g}}{j\omega L + R_L} = \frac{170\angle 0^\circ}{3.2 + j1.89} = 45\angle -30^\circ\ \text{A}$$

$$\underline{V_L} = R_L\,\underline{I_L} = (3.2\Omega)(45\angle -30^\circ\ \text{A}) = 146.5\angle -30^\circ\ \text{V}$$

$$P_L = \frac{1}{2}R_e\{\underline{V_L}\,\underline{I_L}^*\} = \frac{|V_L||I_L|}{2}\cos(\theta_{V_L} - \theta_{I_L}) = \frac{1}{2}(146.5)(45) = 3.3\text{kW}$$

$$S_g = \frac{1}{2}\underline{V_g}\,\underline{I_L}^* = \frac{1}{2}(170\angle 0^\circ)(45\angle +30^\circ) = 3.8\angle 30^\circ\ \text{kVA} = \underbrace{3.3\text{kW}}_{\text{to } R_L} + j\underbrace{1.9\text{kVAR}}_{\text{to } L}$$

$$PF_g = \frac{P_g}{|S_g|} = \frac{3.3\text{kW}}{3.8\text{kVA}} = 0.87\ \text{lagging}$$

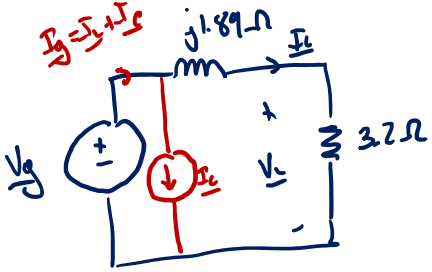
↳ current w.r.t. voltage

Generally, "lagging" corresponds to inductive loads

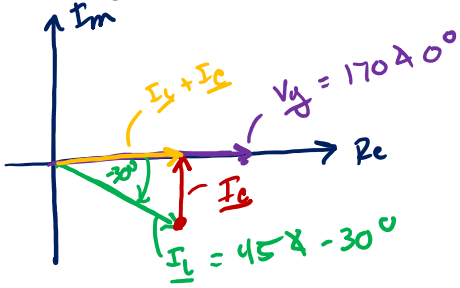
"leading" to capacitive

$PF = \cos(\theta_V - \theta_I) = \cos(\theta_Z)$

$\theta_Z = \angle(j\omega L + R_L)$



Can we return grid PF to unity PF = 1, S_g is all real

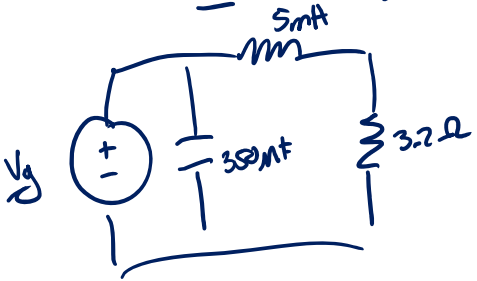


PF = cos($\varphi_V - \varphi_I$)

$$\underline{I_c} = j I_m\{\underline{I_L}^*\} = j45\sin(30°)$$
$$= j\,22.5\,A = 22.5\angle +90°$$

Can this be a passive impedance?

$$\underline{z_c} = \frac{\underline{V_g}}{\underline{I_c}} = \frac{170\angle 0°}{22.5\angle +90°} = 7.56\angle -90° = -j7.56\,\Omega = \frac{-j}{\omega C} \rightarrow C = 350\,\mu F$$



$$\rightarrow \quad S_g = \frac{1}{2}(\underline{V_g})(\underline{I_L} + \underline{I_c})^* = 3.3kW + j0\,VAR$$