Power Spectrum

\[ I_1 = I_2 \quad \text{By inspection} \quad I_R = \phi \]

\[ \therefore P_R = \phi \]

Correctly apply superposition:

\[ I_R = I_{R1} + I_{R2} = (-I_1) + I_2 = \phi \]

Incorrect to apply superposition to power (because it's nonlinear)

If I did it anyway:

\[ P_R = P_1 + P_2 = I_{1\text{rms}}^2 R + I_{2\text{rms}}^2 R \neq \phi \quad \text{(wrong)} \]

However, this will work if I have two sources at different frequencies:

\[ i_1(t) = I_{1\text{rms}} \cos(\omega_1 t) \quad i_2(t) = I_{2\text{rms}} \cos(\omega_2 t) \quad , \quad \omega_1 \neq \omega_2 \]

\[ P_R(t) = p_R^2 R = (i_1(t) - i_2(t))^2 R \]

\[ P_R = \frac{1}{T} \int_{t=0}^{T} p_R(t)^2 R \, dt = \frac{1}{T} \int_{t=0}^{T} (I_{1\text{rms}}^2 \cos^2(\omega_1 t) + I_{2\text{rms}}^2 \cos^2(\omega_2 t) + 2 I_{1\text{rms}} I_{2\text{rms}} \cos(\omega_1 t) \cos(\omega_2 t)) \, dt \]

\[ = I_{1\text{rms}}^2 R + I_{2\text{rms}}^2 R \quad \text{(only for } \omega_1 \neq \omega_2) \]
Limitations of Phasor Analysis

1. Single frequency
2. Sinusoids only
3. Only steady-state response
4. LTI systems only

Reminder of the course: develop techniques to address (1) - (3)

Approaches:
1. Use superposition in the time-domain
   → Ch 15 on Frequency Response
2. Express arbitrary signal as a sum of (infinite) sinusoids
   → Ch 17 Fourier Series/Transform
3. Include exponentials with our sinusoids
   → Ch 14 Laplace Transform
Limitations of Phasor Analysis
Frequency Response

Phasor Analysis:
\[ V_o = V_i \left( \frac{Z_c}{Z_c + Z_R} \right) = V_i \frac{-j\omega C}{j\omega C + R} = V_i \left( \frac{1}{1 - j\omega CR} \right) \]

\[ V_o = V_i \left( \frac{1}{1 - j\omega CR} \right) = V_i H(j\omega) \]

Frequency Response

tells us, at any \( \omega \), how does circuit alter input at the output

Any LTI circuit has \( V_o = V_i H(j\omega) \)

In polar form
\[ V_o = \left( |V_o|, \phi_o \right) = \left( |V_i| H(j\omega), \phi + \angle H(j\omega) \right) = \left( |V_i| H(j\omega), \phi \right) \]

Gain: \( |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \)

Phase: \( \angle H(j\omega) = -\tan^{-1}(\frac{\omega RC}{1}) \)

\[ \text{Re} \]
\[ \text{Im} \]

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Frequency Response

\[ R = 10 \]
\[ C = 10 \mu F \]
\[ \frac{1}{2\pi f_c} = 1.6 \text{kHz} \]

Gain

Phase

-100
-50
0
50
100

\[ \text{frequency [kHz]} \]

\[ 0 \]
\[ 0.5 \]
\[ 1 \]

\[ \text{frequency [kHz]} \]
Bode Plot – Frequency Response