Symmetry in Fourier Series

Even functions

\[ b_n = 0 \]

Odd functions

\[ a_n = 0 \]

Half-wave symmetric functions

\[ a_n, \ b_n = 0 \text{ for even } n \]
Application: Digital Communication

[Diagram of a circuit with a voltage source $v(t)$ and a resistor $R$, with $i(t)$ as the current through the circuit.]

[Graphs showing frequency response with $|H|$ (Log Scale) and $\Delta H$ (Log Scale) on the y-axis and frequency in Hz (Log Scale) on the x-axis.]
Application: Digital Communication
Applying Superposition
Calculated Output Voltage
Simulation Verification

V_{in} [V]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 \\
\hline
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
\end{array}
\]

time [ns]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 \\
\hline
0 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\
\end{array}
\]

V_{out} [V]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 \\
\hline
0.57 & 0.58 & 0.59 & 0.6 & 0.61 & 0.62 & 0.63 & 0.63 \\
\end{array}
\]

time [ns]

V_{in} (green)

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 \\
\hline
0 & 1.3 & 1.3 & 1.3 & 1.3 & 1.3 & 1.3 & 1.3 \\
\end{array}
\]

V_{out} (blue)

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 \\
\hline
0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 & 1.1 & 1.2 \\
\end{array}
\]

PULSE(0 1.2 0 1p 1p 125p 250p)

.in 0 1u {1u-125n}

R1

C1

V1

.out

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
in & V1 & 30 & R1 & out \\
\end{array}
\]

50p
Frequency Domain Interpretation

<table>
<thead>
<tr>
<th>Frequency [Hz] (Log Scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6$</td>
</tr>
<tr>
<td>$</td>
</tr>
<tr>
<td>$F[V_{in}]$</td>
</tr>
<tr>
<td>$F[V_{in}]*</td>
</tr>
</tbody>
</table>
Complex Form of Fourier Series
Fourier Series Representation

Assume we have some function $f(t)$ which is periodic with period $T_0 = \frac{2\pi}{\omega_0}$

\[ f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \]

\[ a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \cos(k\omega_0 t) \, dt \]
\[ b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \sin(k\omega_0 t) \, dt \]

Alternate forms

\[ f(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k) \]

\[ A_k = \sqrt{a_k^2 + b_k^2} \]
\[ \varphi_k = \tan^{-1} \left( \frac{b_k}{a_k} \right) \]

\[ f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \]
\[ c_k = \frac{1}{2} (a_k - jb_k) \]
\[ c_{-k} = \frac{1}{2} (a_k + jb_k) \]
\[ c_0 = a_0 \]

$f(t)$ can be expressed this way if

1. $f(t)$ is single-valued
2. $\int_{t_0}^{t_0+T_0} |f(t)| \, dt$ exists
3. $f(t)$ had finite discontinuities and max/min per period
Fourier Series & Frequency Domain

https://en.wikipedia.org/wiki/Fourier_transform
Input Spectrum