

TABLE 14.2 Laplace Transform Operations

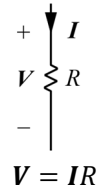
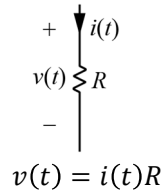
Operation	$f(t)$	$\mathbf{F}(s)$
Addition	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
Scalar multiplication	$kf(t)$	$k\mathbf{F}(s)$
Time differentiation	$\frac{df}{dt}$	$s\mathbf{F}(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2\mathbf{F}(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3\mathbf{F}(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^t f(t) dt$	$\frac{1}{s}\mathbf{F}(s)$
	$\int_{-\infty}^t f(t) dt$	$\frac{1}{s}\mathbf{F}(s) + \frac{1}{s} \int_{-\infty}^{0^-} f(t) dt$
Convolution	$f_1(t) * f_2(t)$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$
Time shift	$f(t-a)u(t-a), a \geq 0$	$e^{-as}\mathbf{F}(s)$
Frequency shift	$f(t)e^{-at}$	$\mathbf{F}(s+a)$
Frequency differentiation	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty \mathbf{F}(s) ds$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right)$
Initial value	$f(0^+)$	$\lim_{s \rightarrow \infty} s\mathbf{F}(s)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} s\mathbf{F}(s)$, all poles of $s\mathbf{F}(s)$ in LHP
Time periodicity	$f(t) = f(t+nT),$ $n = 1, 2, \dots$	$\frac{1}{1 - e^{-Ts}}\mathbf{F}_1(s),$ where $\mathbf{F}_1(s) = \int_{0^-}^T f(t) e^{-st} dt$

Circuit Laplace Transform

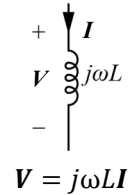
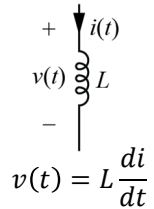
Time Domain

Phasor Domain

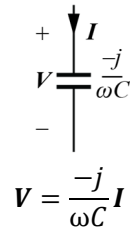
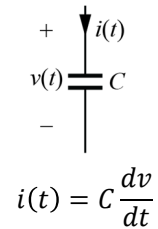
s-Domain



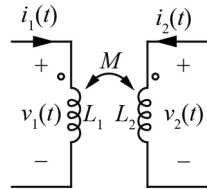
$$= \frac{V}{R}$$



$$= \frac{V}{j\omega L}$$



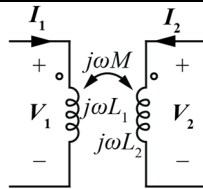
Time Domain



$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

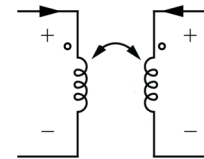
Phasor Domain



$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

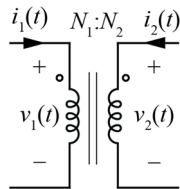
$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

s-Domain



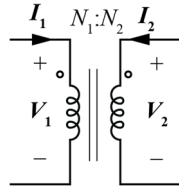
$$V_1(s) = sL_1 I_1(s) - L_1 i_1(0^-) + sM I_2(s) - M i_2(0^-)$$

$$V_2(s) = sM I_1(s) - M i_1(0^-) + sL_2 I_2(s) - L_2 i_2(0^-)$$



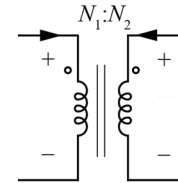
$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$

$$N_1 i_1(t) + N_2 i_2(t) = 0$$



$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$N_1 I_1 + N_2 I_2 = 0$$



$$\frac{V_1(s)}{N_1} = \frac{V_2(s)}{N_2}$$

$$N_1 I_1(s) + N_2 I_2(s) = 0$$

Laplace Transform of Diff EQs

N^{th} order circuit with sinusoidal input described by ($M \leq N$ for causality)

$$b_N \frac{d^N}{dt^N} v_o(t) + \dots + b_1 \frac{d}{dt} v_o(t) + b_0 v_o(t) = a_M \frac{d^M}{dt^M} v_i(t) + \dots + a_1 \frac{d}{dt} v_i(t) + a_0 v_i(t)$$

$$\sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t)$$

Then the Laplace transform of the circuit, neglecting initial conditions, is

$$\mathcal{L} \left\{ \sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) \right\} = \mathcal{L} \left\{ \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t) \right\}$$

$$\sum_{i=0}^N b_i s^i V_o(s) = \sum_{i=0}^M a_i s^i V_i(s)$$

Rearranging:

$$\boxed{\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}}$$

Laplace Circuit Solution Algorithm

1. Transform all sources, signals into Laplace Domain
2. Transform circuit components (including initial conditions) into Laplace Domain
3. Solve the circuit using 201 techniques

$$V_o(s) = H(s)V_i(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}$$

4. Inverse Laplace Transform to get back to time domain

Example Laplace Circuit Analysis

Inverse Transforms

Transfer Functions

$$H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_M)}$$