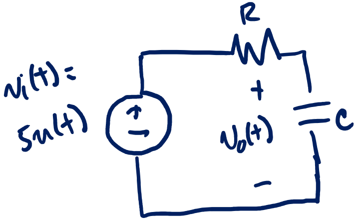
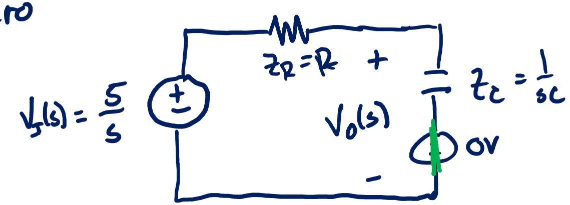
Example Laplace Circuit Analysis



$v_o(t)$ @ $t \le 0$ is zero



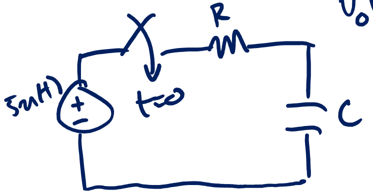
$v_o(t) = (5 - 5e^{-\frac{1}{RC}t})u(t)$

$$V_o(s) = V_I(s)\frac{Z_C}{Z_C + Z_R} = \frac{5}{s}\,\frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{5}{s}\,\frac{\frac{1}{RC}}{\frac{1}{RC} + s}$$

$$v_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = \mathcal{L}^{-1}\left\{\frac{A}{s} + \frac{B}{\frac{1}{RC} + s}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{s} + \frac{-5}{\frac{1}{RC} + s}\right\}$$

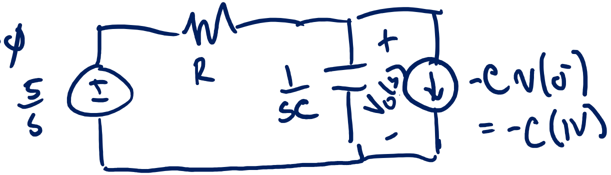
PFE (more review coming)

$$v(t) = \mathcal{L}^{-1}\left\{\frac{5}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{-5}{\frac{1}{RC} + s}\right\} = 5u(t) - 5e^{-\frac{1}{RC}t}u(t)$$



$V_o(t) = 1V$ for $t \le 0$

for $t > 0$



$$V_o(s) = V_I(s)\frac{Z_C}{Z_C + Z_R} + C(1V)\left[R \,\|\, \frac{1}{sC}\right]$$

Inverse Transforms

Transfer Functions

$$H(s) = \frac{\sum_{i=0}^{M} a_i s^i}{\sum_{i=0}^{N} b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}$$

Partial Fraction Expansion / Decomposition

$M < N$

$$\frac{\sum_{i=0}^{M} a_i s^i}{\sum_{i=0}^{N} b_i s^i} = \frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_M)} = \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \cdots \frac{k_N}{(s-p_N)}$$

$$\frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_M)} = \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \cdots \frac{k_N}{(s-p_N)}$$

PFE: Repeated Roots