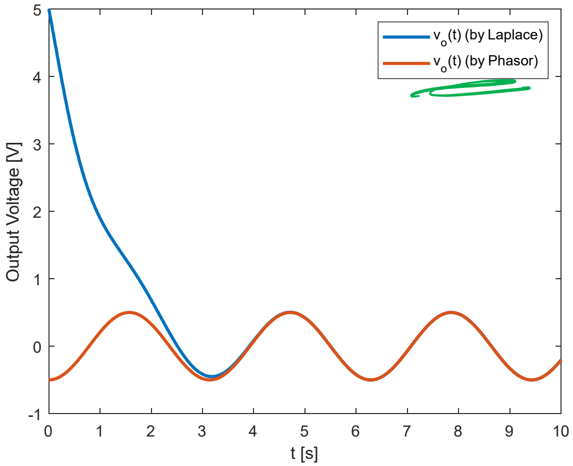
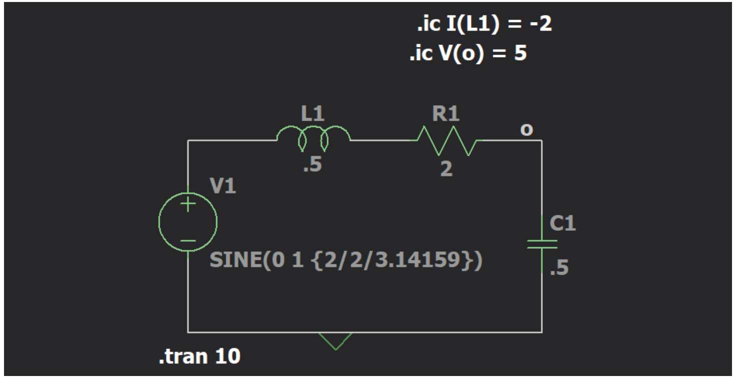
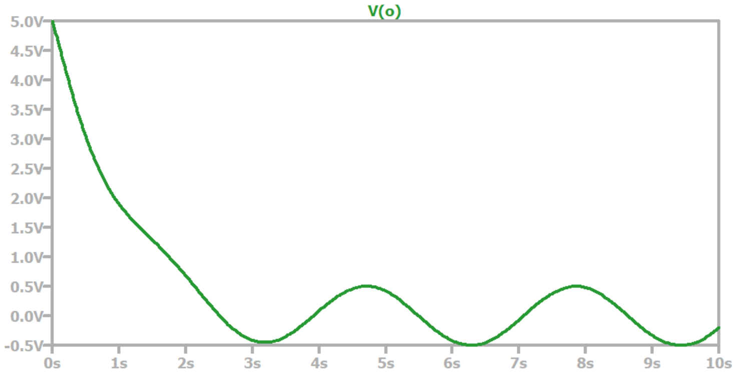
Comparison to Simulation

Phasor → $v_i(t) = \sin(2t)$ & neglect ICs } sinusoidal steady-state

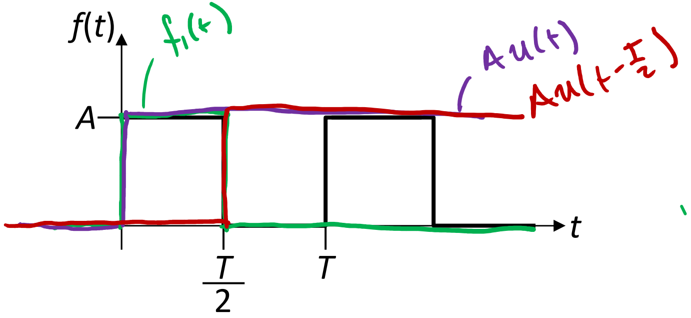
Laplace → $v_i(t) = \sin(2t)\,u(t)$ & include ICs } 'Any signal' transient & steady-state







Laplace Transform of Periodic PWL Signals



$$f(t) = \begin{cases} A & kT+0 < t < \frac{T}{2} + kT \\ \emptyset & kT + \frac{T}{2} < t < T + kT \end{cases} \qquad k \in \mathbb{Z}^+$$

$$f_1(t) = Au(t) - Au(t-\tfrac{T}{2}) \rightarrow \text{first period only}$$

$$f(t) = \sum_{k=0}^{\infty} f_1(t-kT)$$

Laplace transform:

$$\mathcal{L}\{f_1(t)\} = F_1(s) = A\frac{1}{s} - Ae^{-s\frac{T}{2}}\frac{1}{s} = \frac{A}{s}\left(1-e^{-s\frac{T}{2}}\right)$$

$$\mathcal{L}\{f(t)\} = F(s) = \sum_{k=0}^{\infty} e^{-sTk} F_1(s) = \sum_{k=0}^{\infty} e^{-sTk}\left(\frac{A}{s}\left(1-e^{-s\frac{T}{2}}\right)\right) = \frac{A}{s}\left(1-e^{-s\frac{T}{2}}\right) \cdot \sum_{k=0}^{\infty}\left(e^{-sT}\right)^k$$

$$r = e^{-sT}$$

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad |r| < 1$$

$$\boxed{F(s) = \frac{\frac{A}{s}\left(1-e^{-s\frac{T}{2}}\right)}{\left(1-e^{sT}\right)} = \frac{F_1(s)}{1-e^{-sT}}}$$

TABLE 14.2 Laplace Transform Operations

Operation	f(t)	F(s)
Addition	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
Scalar multiplication	$kf(t)$	$k\mathbf{F}(s)$
Time differentiation	$\frac{df}{dt}$	$s\mathbf{F}(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2\mathbf{F}(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3\mathbf{F}(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^{t} f(t)\,dt$	$\frac{1}{s}\mathbf{F}(s)$
	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Convolution	$f_1(t) * f_2(t)$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$
Time shift	$f(t-a)u(t-a),\ a \geq 0$	$e^{-as}\mathbf{F}(s)$
Frequency shift	$f(t)e^{-at}$	$\mathbf{F}(s+a)$
Frequency differentiation	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} \mathbf{F}(s)\,ds$
Scaling	$f(at),\ a \geq 0$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right)$
Initial value	$f(0^+)$	$\lim_{s \to \infty} s\mathbf{F}(s)$
Final value	$f(\infty)$	$\lim_{s \to 0} s\mathbf{F}(s)$, all poles of $s\mathbf{F}(s)$ in LHP
Time periodicity	$f(t) = f(t+nT)$, $n = 1, 2, \ldots$	$\frac{1}{1-e^{-T_s}}\mathbf{F}_1(s)$, where $\mathbf{F}_1(s) = \int_{0^-}^{T} f(t)e^{-st}\,dt$

not subscript

TABLE 14.1 Laplace Transform Pairs

$f(t) = \mathscr{L}^{-1}\{\mathbf{F}(s)\}$	$\mathbf{F}(s) = \mathscr{L}\{f(t)\}$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$\frac{t^{n-1}}{(n-1)!}u(t),\ n = 1, 2, \ldots$	$\frac{1}{s^n}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t),\ n = 1,2,\ldots$	$\frac{1}{(s+\alpha)^n}$
$\frac{1}{\beta-\alpha}(e^{-\alpha t} - e^{-\beta t})u(t)$	$\frac{1}{(s+\alpha)(s+\beta)}$
$\sin \omega t\, u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t\, u(t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)\, u(t)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)\, u(t)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-\alpha t}\sin \omega t\, u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos \omega t\, u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
$2\lvert k\rvert e^{\sigma t}\cos(\omega t - \angle k)\, u(t)$	$\frac{k}{s-(\sigma+j\omega)} + \frac{k^*}{s-(\sigma-j\omega)}$

Pole Locations

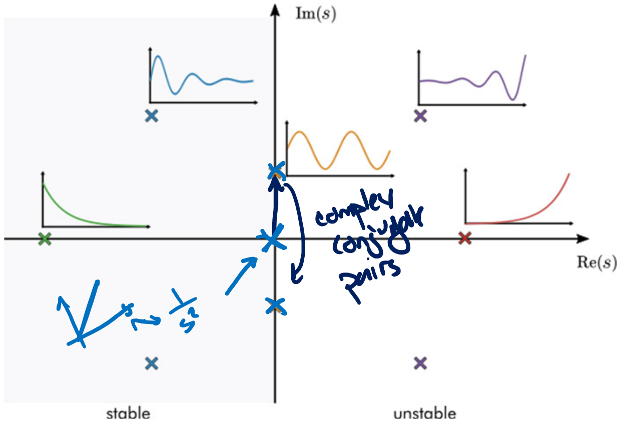
$$\boxed{x = N_H + N_I}$$

$$V_o(s) = V_I(s)H(s) = \left(\frac{\sum_{i=0}^{M_I} c_i s^i}{\sum_{i=0}^{N_I} d_i s^i}\right)\left(\frac{\sum_{i=0}^{M_H} a_i s^i}{\sum_{i=0}^{N_H} b_i s^i}\right) = \frac{(s-z_1)(s-z_2)\cdots(s-z_{M_H+M_I})}{(s-p_1)(s-p_2)\cdots(s-p_{N_H+N_I})}$$

$$V_o(s) = \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \cdots + \frac{k_x}{(s-p_x)}$$

poles of both $V_I(s)$ & $H(s)$ determine the "type" of signals in the output

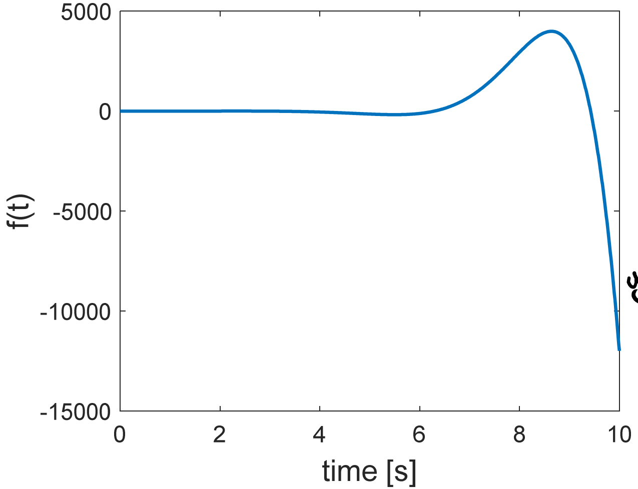
$$v_o(t) = k_1 e^{p_1 t} u(t) + k_2 e^{p_2 t} u(t) + \cdots$$



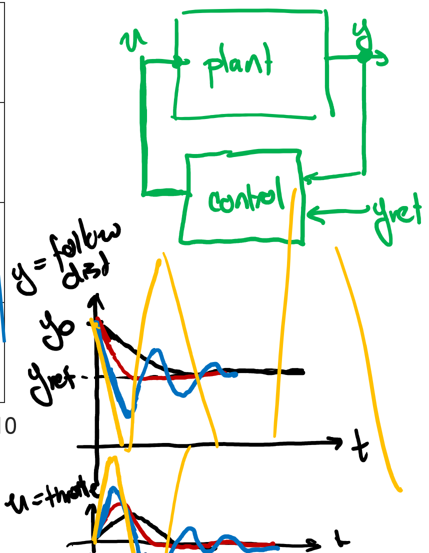
Output has poles/terms from both $H(s)$ & $V_I(s)$

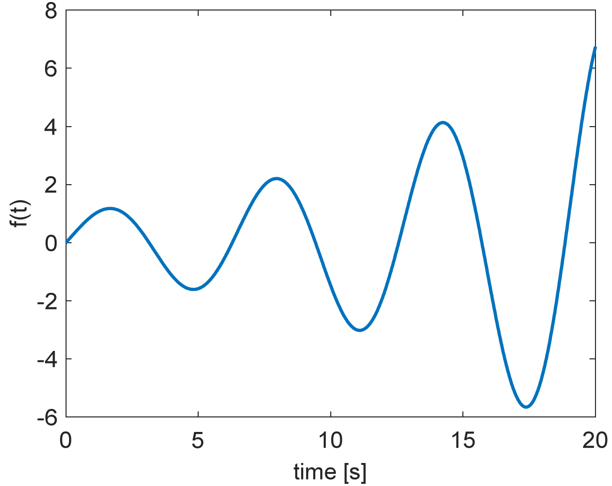
Unbounded Signals & Unstable Systems

feedback control



$f(t) = e^{t}\sin(t)$





$f(t) = e^{t/10}\sin(t)$

Bounded signals → mathematical definition $f(t)$ is bounded iff $\exists\, B$ s.t. $|f(t)| < B \;\forall t$

BIBO stability → "Bounded input, Bounded output" stability

Always want BIBO stable circuits / $H(s)$