Laplace and Fourier Revisited

Fourier Transform: $$F(j\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} f(t)\, dt$$

Laplace Transform (Bilateral): $$F(s) = \int_{-\infty}^{+\infty} e^{-st} f(t)\, dt$$

Inverse Fourier Transform: $$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega) e^{j\omega t} d\omega$$

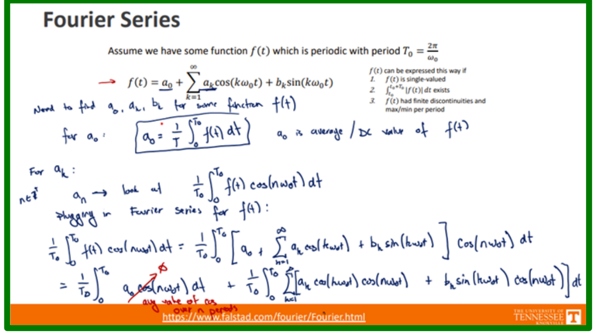
Inverse Laplace Transform (Bilateral): $$f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s) e^{st} ds$$

Laplace Explanation

$$F(s) = \int_{0^-}^{+\infty} e^{-st} f(t)\, dt = \int_{0^-}^{+\infty} e^{-\sigma t} e^{-j\omega t} f(t)\, dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} e^{j\omega t} F(s)\, ds$$

L21



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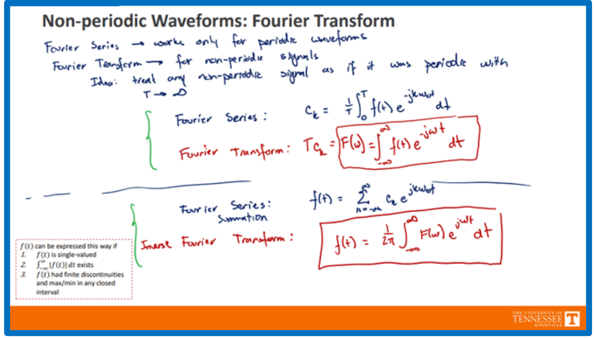

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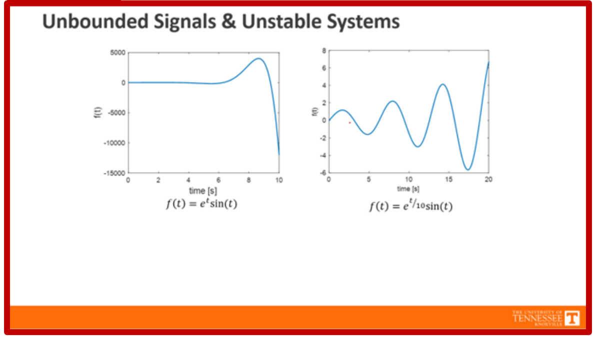

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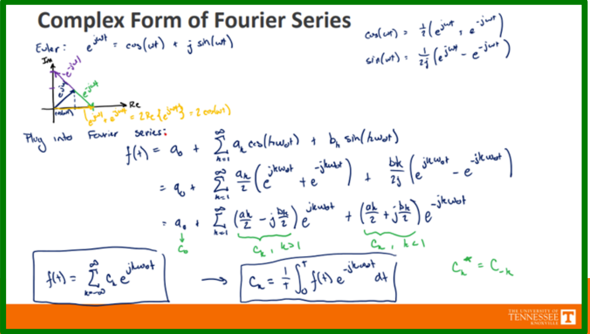

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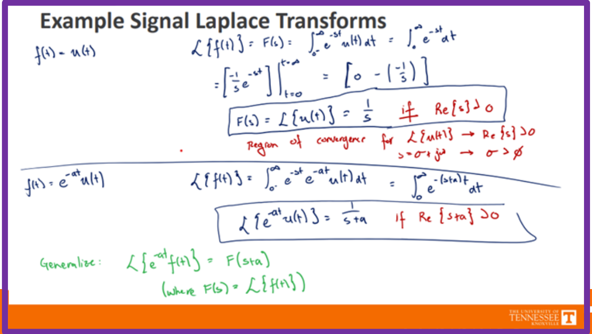
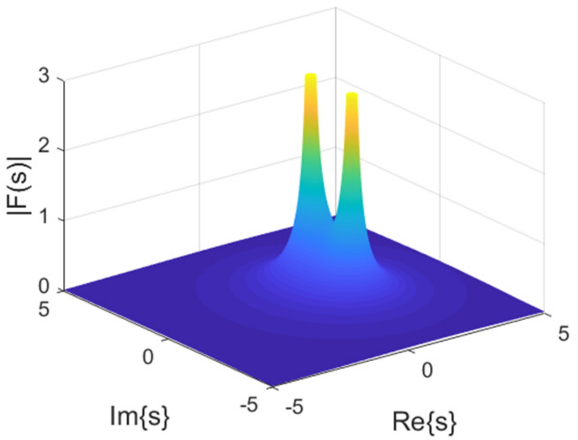
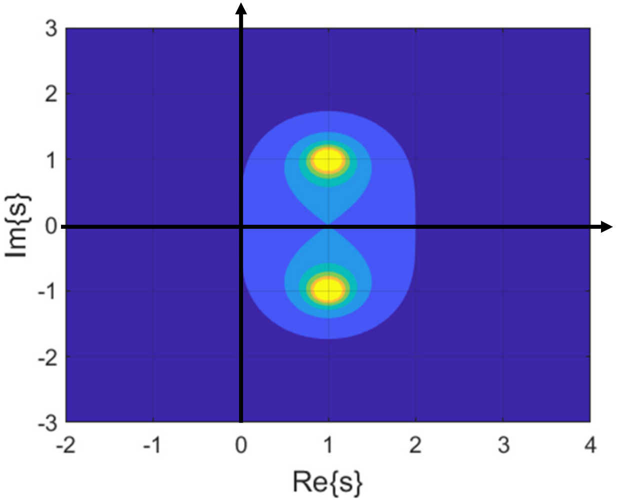


The s-plane



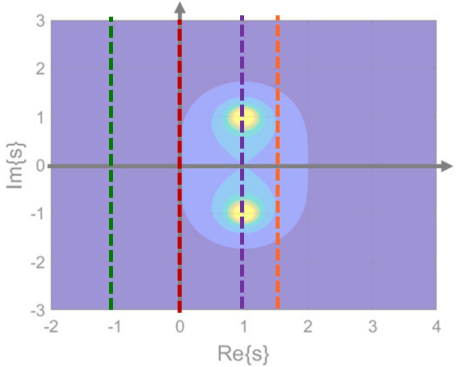


$$F(s) = \frac{1}{s^2 - 2s + 2} = \frac{1}{\big(s - (1 + j)\big)\big(s - (1 - j)\big)}$$

Example R.O.C.

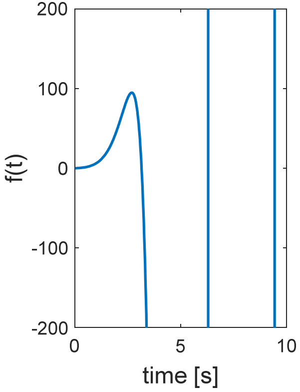
$$F(s) = \int_{0^-}^{+\infty} e^{-st} f(t)\, dt = \int_{0^-}^{+\infty} e^{-j\omega t} e^{-\sigma t} f(t) dt$$

$f(t) = e^{t}\sin(t)$



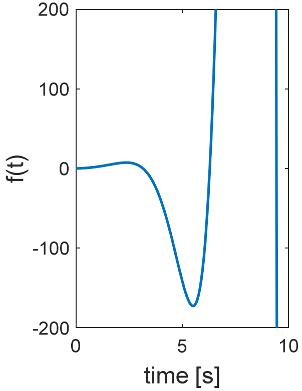
$\sigma = -1$

$e^{-(-1)t} f(t)$



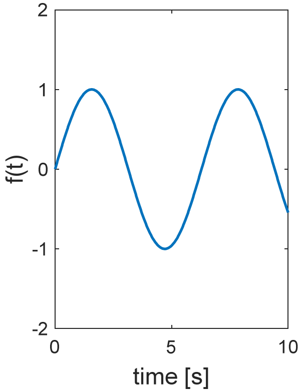
$\sigma = 0$

$f(t)$



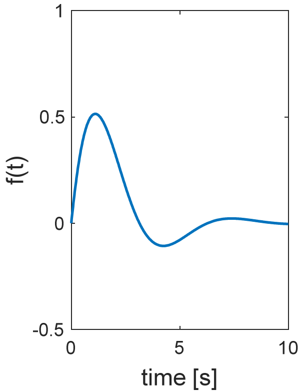
$\sigma = 1$

$e^{-(1)t} f(t)$



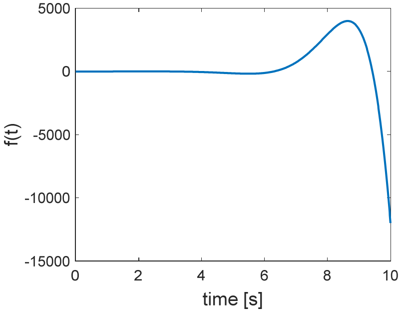
$\sigma = 1.5$

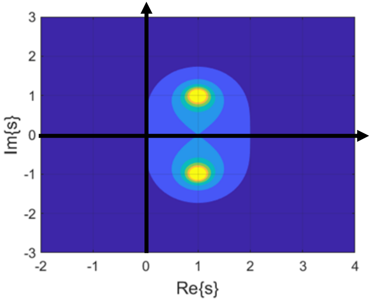
$e^{-(1.5)t} f(t)$



Example Functions

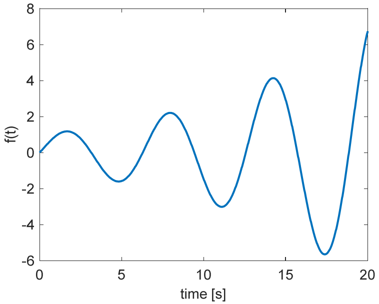
$$f(t) = e^{t}\sin(t)$$

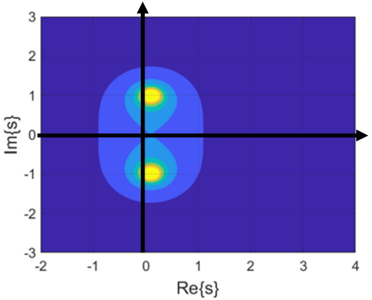




$$F(s) = \frac{1}{\left(s-(1+j)\right)\left(s-(1-j)\right)}$$

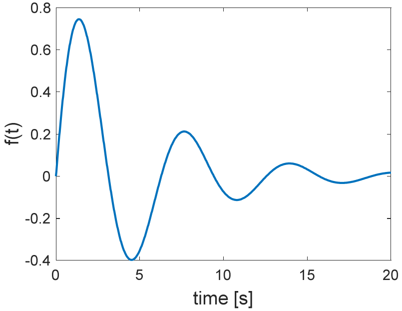
$$f(t) = e^{t/10}\sin(t)$$

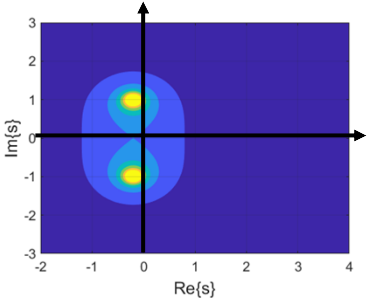




$$F(s) = \frac{1}{\left(s-\left(\frac{1}{10}+j\right)\right)\left(s-\left(\frac{1}{10}-j\right)\right)}$$

$$f(t) = e^{-t/5}\sin(t)$$

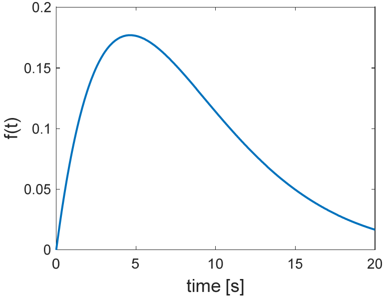


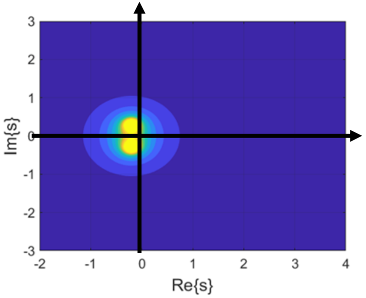


$$F(s) = \frac{1}{\left(s+\left(\frac{1}{5}+j\right)\right)\left(s+\left(\frac{1}{5}-j\right)\right)}$$

Example Functions

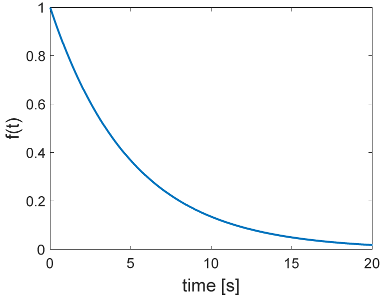
$$f(t) = e^{-t/5}\sin(t/10)$$

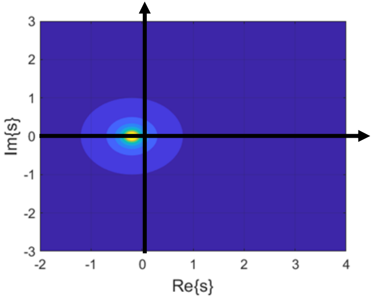




$$F(s) = \frac{1/10}{\left(s + \left(\frac{1}{5} + \frac{j}{10}\right)\right)\left(s + \left(\frac{1}{5} - \frac{j}{10}\right)\right)}$$

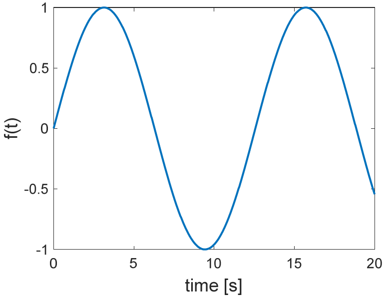
$$f(t) = e^{-t/5}$$

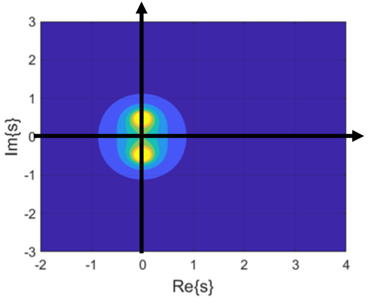




$$F(s) = \frac{1}{\left(s + \frac{1}{5}\right)}$$

$$f(t) = \sin(t/2)$$

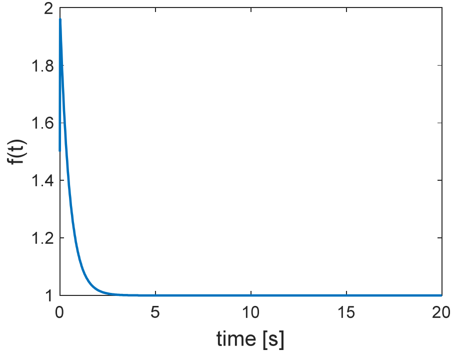


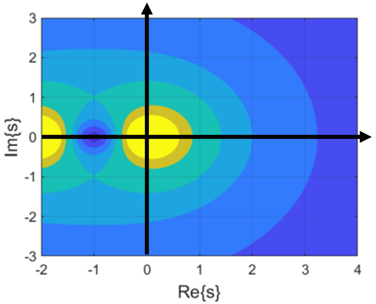


$$F(s) = \frac{1/2}{\left(s + \frac{j}{2}\right)\left(s - \frac{j}{2}\right)}$$

Example Functions

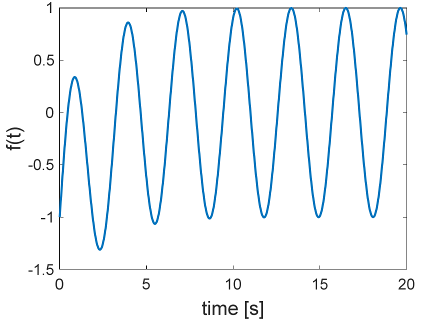
$$f(t) = -e^{-t/2} + u(t)$$

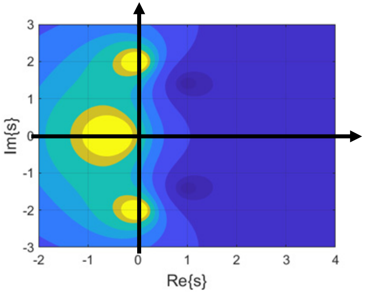




$$F(s) = 2\frac{s+1}{s(s+2)}$$

$$f(t) = -e^{-t/2} + \sin(2t)$$

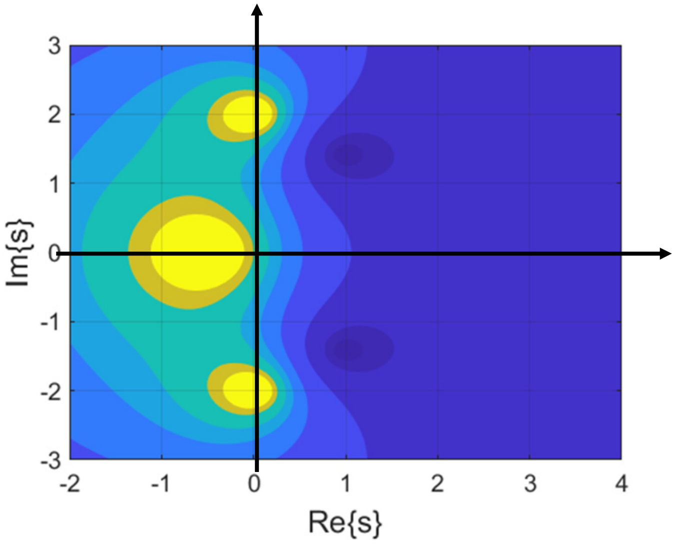


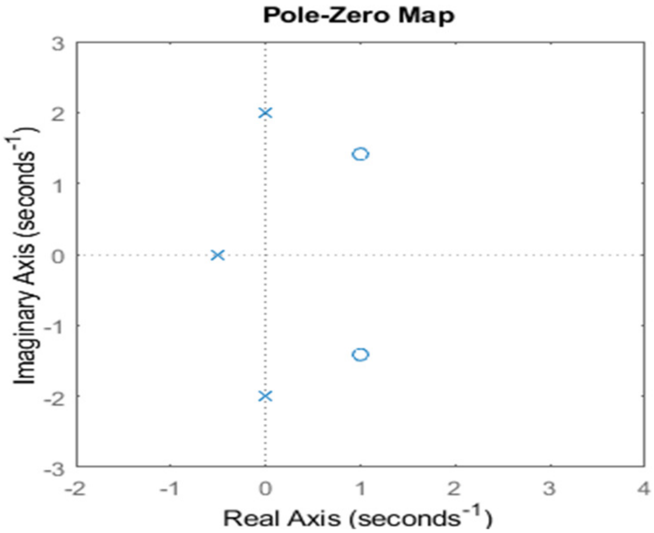


$$F(s) = -\frac{\left(s+\left(-1+j\sqrt{2}\right)\right)\left(s+\left(-1-j\sqrt{2}\right)\right)}{\left(s+{}^{1}\!/_{2}\right)(s+j2)(s-j2)}$$

Pole-Zero Map

$$F(s) = -\frac{\left(s + \left(-1 + j\sqrt{2}\right)\right)\left(s + \left(-1 - j\sqrt{2}\right)\right)}{\left(s + {}^{1}\!/_{2}\right)(s + j2)(s - j2)}$$





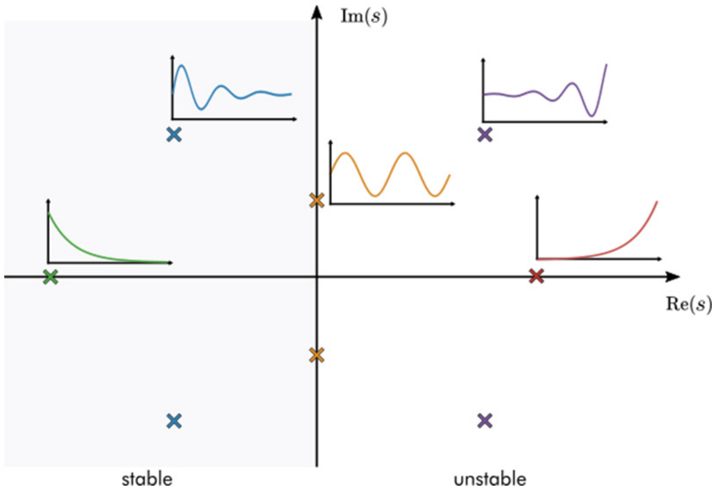
MATLAB:

```
s = tf('s’);
Fs = 2/(s^2 + 4) - 1/(s + 1/2);
pzmap(Fs);
```

MATLAB:

```
s = tf('s’);
Fs = 2/(s^2 + 4) - 1/(s + 1/2);
pzmap(Fs);
```

Poles-Zero Plot



System I/O Relationship