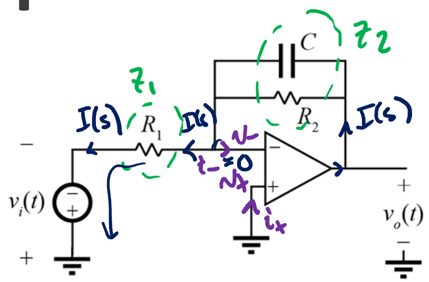
Example Problem

$Z_1 = R_1$

$Z_2 = \frac{1}{sC} \| R_2$



Ideal op-amp assumptions:
if there is negative feedback

(1) Virtual short : $v_+ = v_-$ $\rightarrow$ $V_+(s) = V_-(s)$

(2) $i_+ = i_- = \emptyset$ $\longrightarrow$ $I_+(s) = I_-(s) = \emptyset$

Inverting op-amp configuration $V_o(s) = \frac{-Z_2}{Z_1}\left(-V_I(s)\right)$

$I_-(s) = \emptyset$, $V_-(s) = \emptyset$

$$I(s) = \frac{0 - (-V_I(s))}{Z_1} = \frac{V_I(s)}{Z_1}$$

$$V_o(s) = V_-(s) + I(s) Z_2 = \emptyset + \frac{V_I(s)}{Z_1} Z_2 \longrightarrow \boxed{V_o = \frac{Z_2}{Z_1} V_I(s)}$$

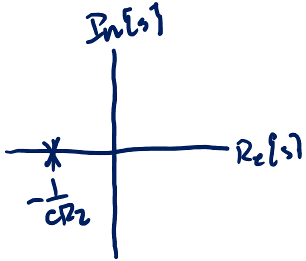
$$Z_2 = \frac{\frac{1}{sC} R_2}{R_2 + \frac{1}{sC}} = \frac{\frac{1}{C}}{s + \frac{1}{CR_2}}$$

$Z_1 = R_1$

$$V_o(s) = \underbrace{\frac{R_2}{R_1} \frac{\frac{1}{CR_2}}{s + \frac{1}{CR_2}}}_{H(s) \,=\, \text{Transfer function}} V_I(s)$$

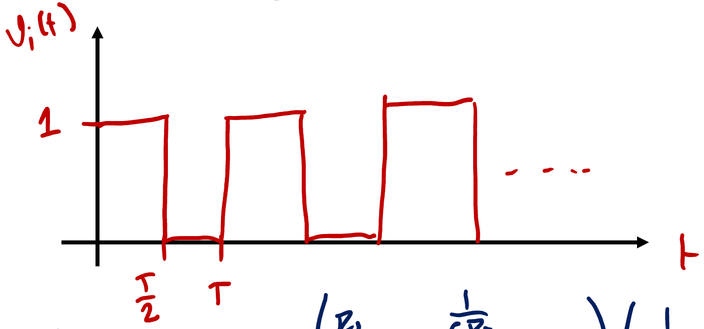
single pole @ $s = \frac{-1}{CR_2}$

pole-zero plot



Poles in open LHP
BIBO stable

Example Problem



$$v_i(t) = \sum_{n=0}^{\infty} u(t-kT) - u\left(t-kT-\frac{T}{2}\right)$$

$$V_I(s) = \mathcal{L}\{v_i(t)\} = \frac{1}{s}\,\frac{\left(1-e^{-sT/2}\right)}{1-e^{-sT}}$$

$$V_o(s) = V_I(s)\,H(s) = \underbrace{\left(\frac{R_2}{R_1}\,\frac{\frac{1}{CR_2}}{s+\frac{1}{CR_2}}\right)\left(\frac{1}{s}\right.}_{F_a(s)}\;\underbrace{\left.\frac{\left(1-e^{-sT/2}\right)}{1-e^{-sT}}\right)}$$

set aside knowing delay + periodic function

$$F_a(s) = \frac{R_2}{R_1}\,\frac{\frac{1}{CR_2}}{s+\frac{1}{CR_2}}\cdot\frac{1}{s} = \frac{k_1}{s} + \frac{k_2}{s+\frac{1}{CR_2}}$$

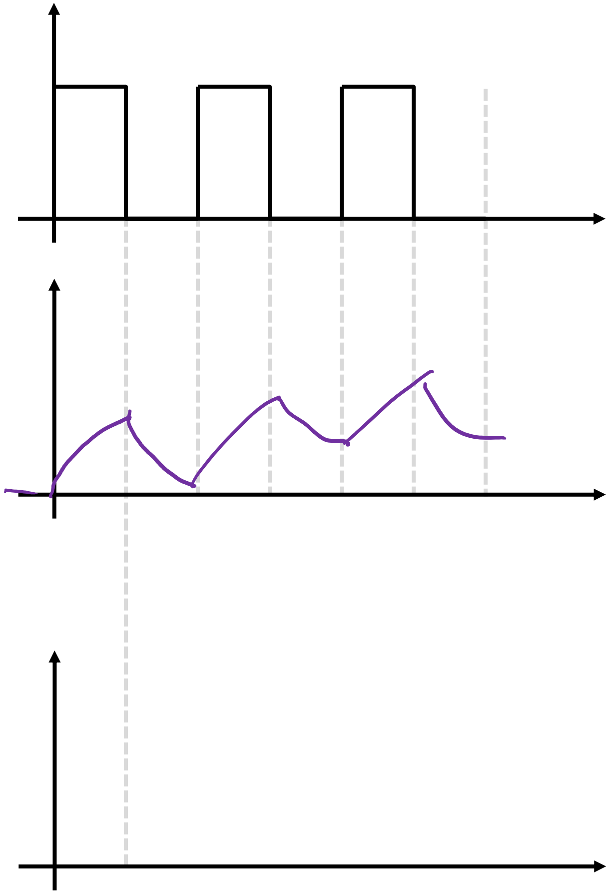
$$k_1 = \frac{R_2}{R_1}$$

$$k_2 = -\frac{R_2}{R_1}$$

$$f_a(t) = \frac{R_2}{R_1}\left[1 - e^{-\frac{1}{CR_2}t}\right]u(t)$$

$$V_o(t) = \sum_{k=0}^{\infty} f_a(t-kT) - f_a\left(t-\frac{T}{2}-kT\right)$$

$$v_o(t) = \sum_{k=0}^{\infty} \frac{R_2}{R_1}\left[1-e^{-\frac{1}{CR_2}(t-kT)}\right]u(t-kT) - \frac{R_2}{R_1}\left[1-e^{-\frac{1}{CR_2}\left(t-\frac{T}{2}-kT\right)}\right]u(t-T/2-kT)$$



$$f_a(t) = \frac{R_2}{R_1}\left[1 - e^{-\frac{1}{R_2 C}t}\right]u(t)$$