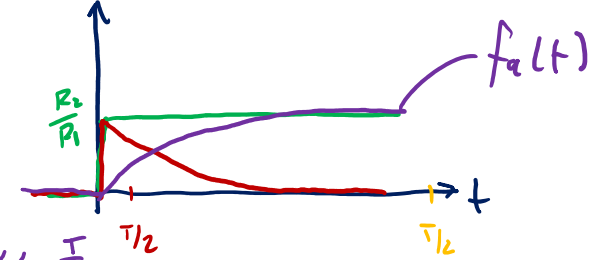
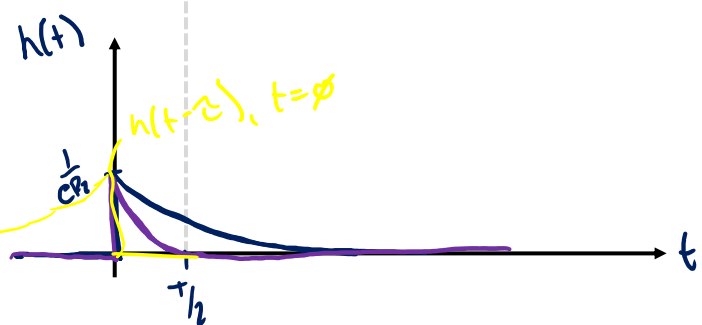


$$f_a(t) = \frac{R_2}{R_1} [1 - e^{-\frac{1}{R_2 C} t}] u(t)$$



$T = R_2 C \ll \frac{T}{2}$
 $T = R_2 C \gg \frac{T}{2}$



$$H(s) = \frac{R_2}{R_1} \frac{\frac{1}{CR_2}}{s + \frac{1}{CR_2}}$$

$$h(t) = \frac{1}{CR_1} e^{-\frac{1}{CR_2} t} u(t)$$

$$v_o(t) = \int_0^{\infty} h(t-\tau) v_i(\tau) d\tau$$



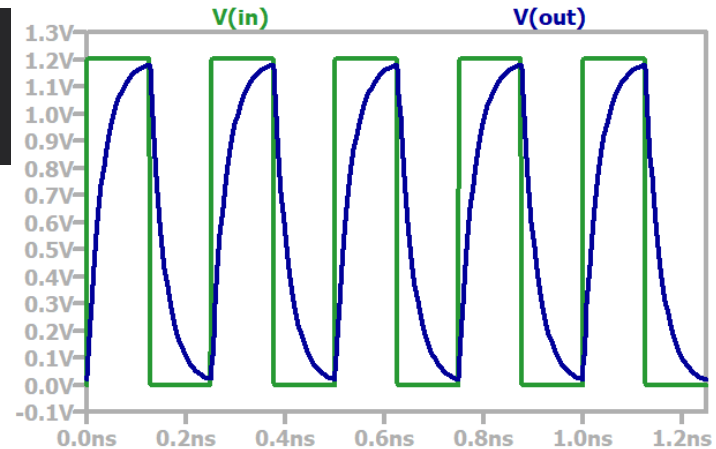
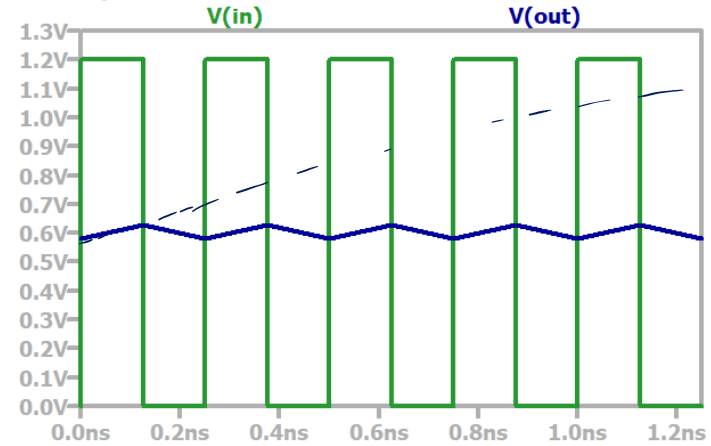
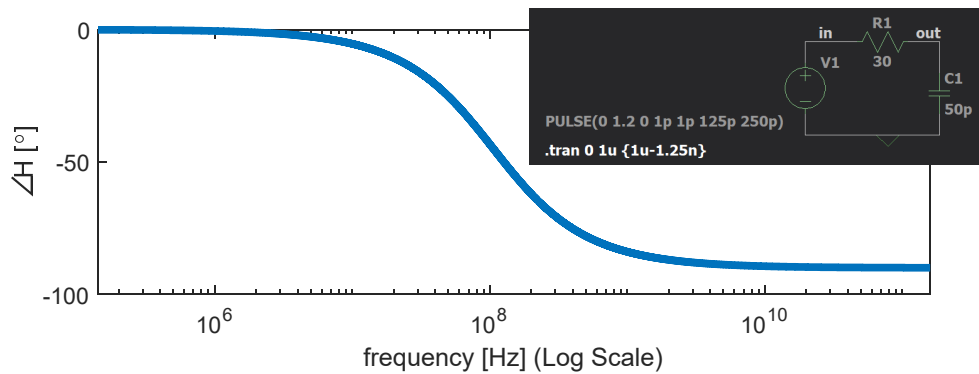
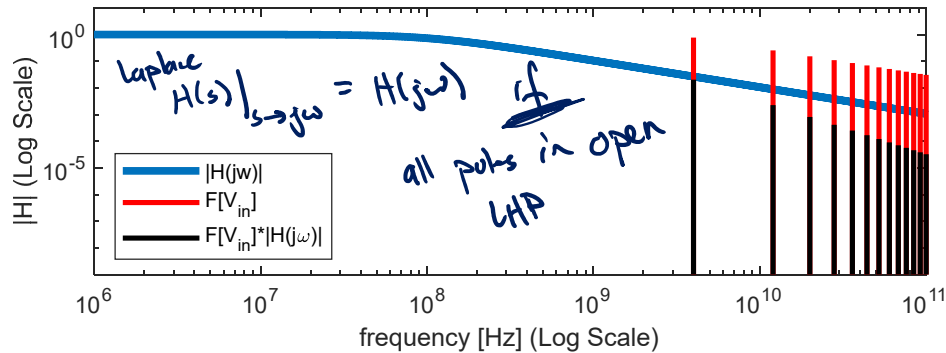
End of Material for Midterm 2

- Midterm Exam #2 Friday Apr 26th
 - Lectures 20 - 33
 - Homeworks 6 - 9
 - Quiz 3 - 4
 - Chapter 17 (17.1 -17.5) and Chapter 14 (all)
 - Experiment 3

Chapter 15

FREQUENCY RESPONSE

L22 - Frequency Response Interpretation



Laplace Explanation

$$F(s) = \int_0^- e^{-st} f(t) dt = \int_0^+ e^{-\sigma t} e^{-j\omega t} f(t) dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} e^{j\omega t} F(s) ds$$

L21

Fourier Series

Assume we have some function $f(t)$ which is periodic with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(k\omega_0 t) + b_n \sin(k\omega_0 t)$$

- $f(t)$ can be expressed this way if
- $f(t)$ is single-valued
- $f(t)$ has finite discontinuities and maxima/min in any closed interval

Need to find a_0, a_n, b_n for some function $f(t)$
for a_0 : $a_0 = \frac{1}{T} \int_0^T f(t) dt$ a_0 is average / DC value of $f(t)$

For a_n : $a_n \rightarrow$ but not $\frac{1}{T} \int_0^T f(t) \cos(n\omega_0 t) dt$
plug in Fourier Series for $f(t)$:

$$\int_0^T f(t) \cos(n\omega_0 t) dt = \int_0^T \left[a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right] \cos(n\omega_0 t) dt$$

$$= \int_0^T a_0 \cos(n\omega_0 t) dt + \int_0^T \left[\sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) \cos(n\omega_0 t) + b_k \sin(k\omega_0 t) \cos(n\omega_0 t) \right] dt$$

<https://www.kitand.com/fourier/Fourier.html>



L24

Non-periodic Waveforms: Fourier Transform

Fourier Series \rightarrow work only for periodic waveforms
Fourier Transform \rightarrow for non-periodic signals
Idea: treat any non-periodic signal as if it was periodic with $T \rightarrow \infty$

Fourier Series: $C_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt$
Fourier Transform: $T C_n = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

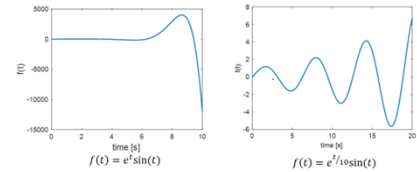
Fourier Series: Summation $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$
Smart Fourier Transform: $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$

- $f(t)$ can be expressed this way if
- $f(t)$ is single-valued
- $f(t)$ has finite discontinuities and maxima/min in any closed interval



L30

Unbounded Signals & Unstable Systems



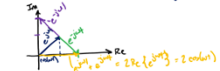
L23

Complex Form of Fourier Series

Euler: $e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

$$\sin(\omega t) = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$



Plug into Fourier Series:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$= a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2} (e^{jn\omega_0 t} + e^{-jn\omega_0 t}) + \frac{b_n}{2j} (e^{jn\omega_0 t} - e^{-jn\omega_0 t})$$

$$= a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n}{2} - \frac{b_n}{2j} \right) e^{jn\omega_0 t} + \left(\frac{a_n}{2} + \frac{b_n}{2j} \right) e^{-jn\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \rightarrow C_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt \quad C_n^* = C_{-n}$$



L25

Example Signal Laplace Transforms

$f(t) = u(t)$ $\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} u(t) dt = \int_0^{\infty} e^{-st} dt = \left[-\frac{1}{s} e^{-st} \right]_0^{\infty} = \left[0 - \left(-\frac{1}{s}\right) \right] = \frac{1}{s}$
Region of convergence for $\mathcal{L}\{u(t)\} \rightarrow \text{Re}\{s\} > 0$
 $s = \sigma + j\omega \rightarrow \sigma > 0$

$f(t) = e^{-at} u(t)$ $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} e^{-at} u(t) dt = \int_0^{\infty} e^{-(s+a)t} dt = \left[-\frac{1}{s+a} e^{-(s+a)t} \right]_0^{\infty} = \frac{1}{s+a}$
if $\text{Re}\{s+a\} > 0$

Generalize: $\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$
(where $F(s) = \mathcal{L}\{f(t)\}$)

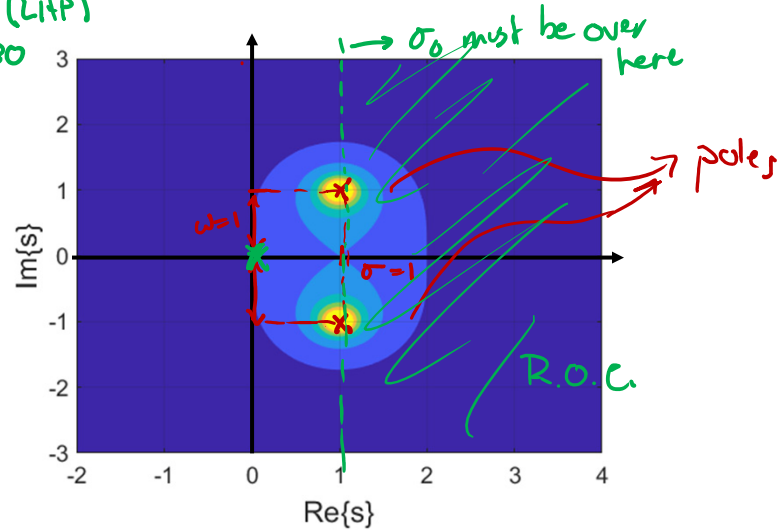
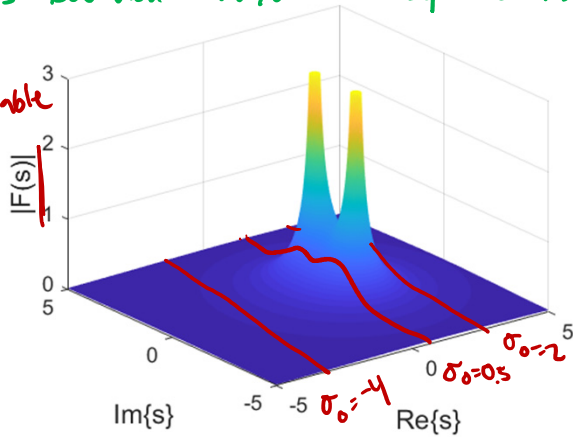


The s-plane

The Region of convergence of a Laplace transform is the complex plane to the right of all poles

If all poles are in the open left half plane, (LHP) the signal is bounded and/or the system is BIBO stable

RHP poles \Leftrightarrow unstable



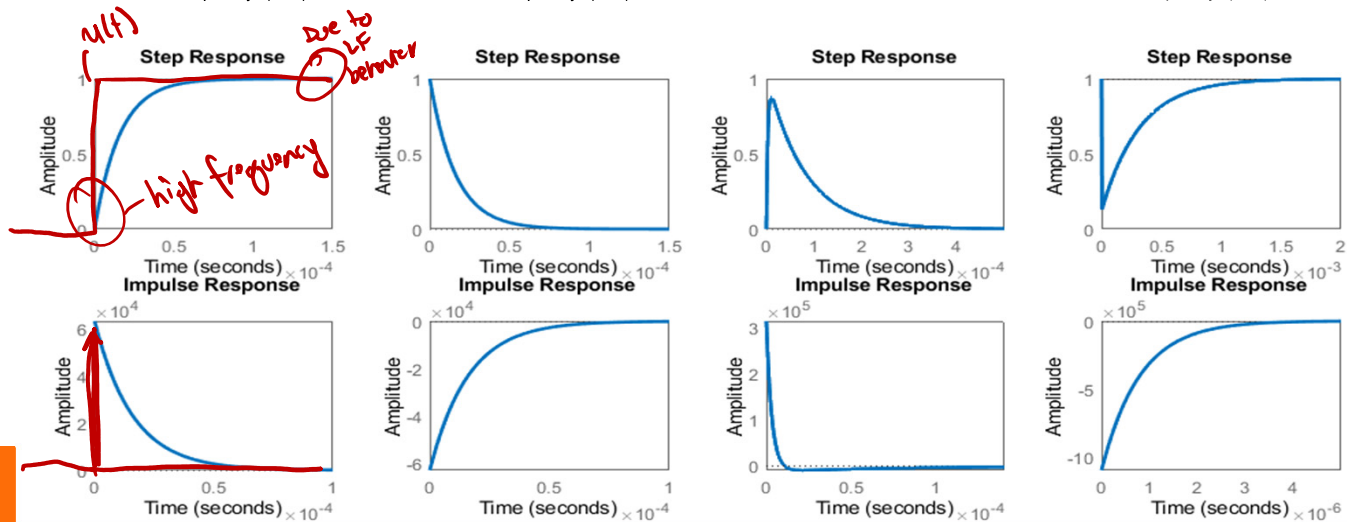
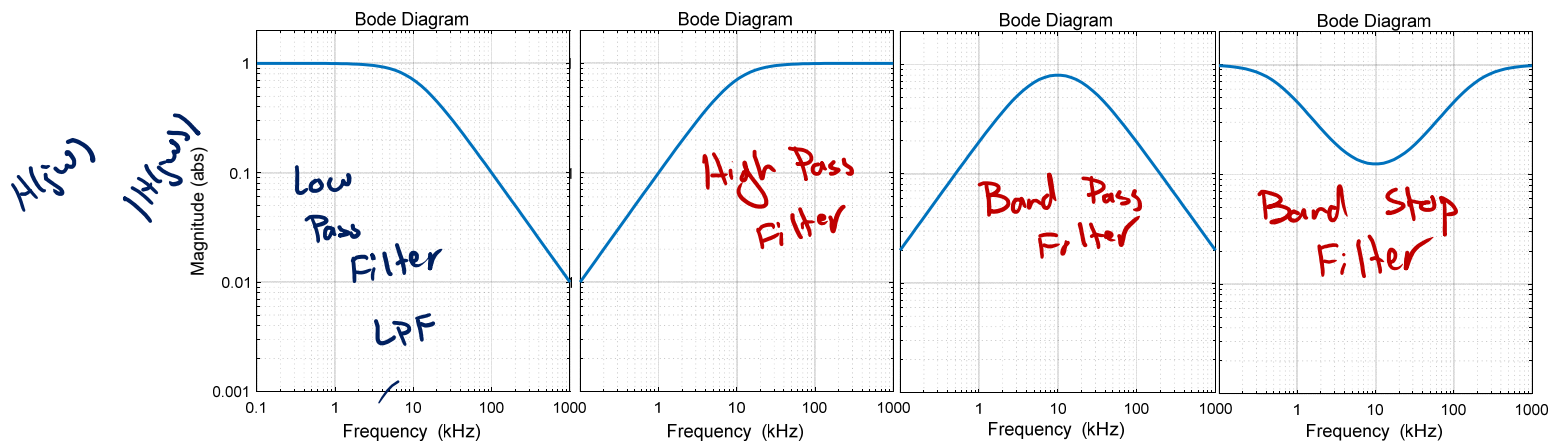
$$F(s) = \frac{1}{s^2 - 2s + 2} = \frac{1}{(s - (1 + j))(s - (1 - j))}$$

$$f(t) = e^t \sin(t)$$

$$F(s) = \int_0^{\infty} e^{-st} e^t \sin(t) dt \rightarrow \text{Need } \text{Re}\{s\} > 1 \text{ for convergence}$$

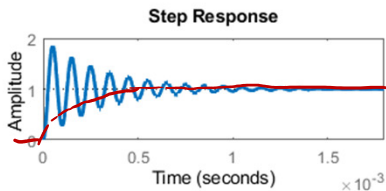
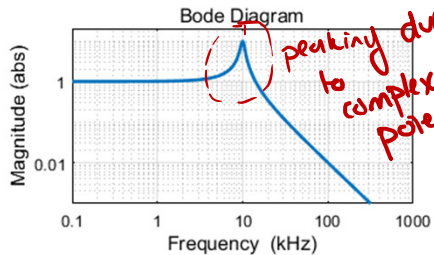
ROC is $\text{Re}\{s\} > 1$

Frequency Response and Circuit Behavior

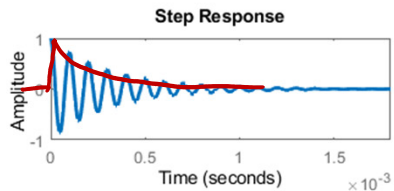
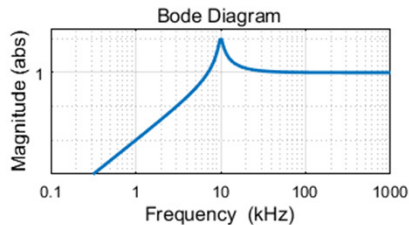


Complex Poles

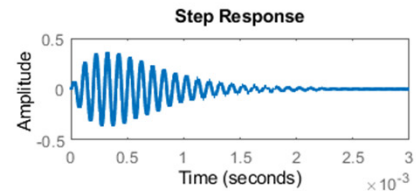
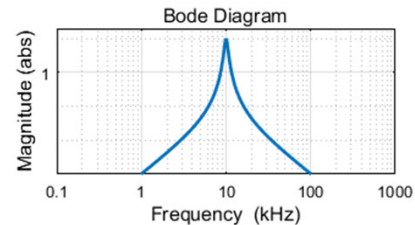
low pass



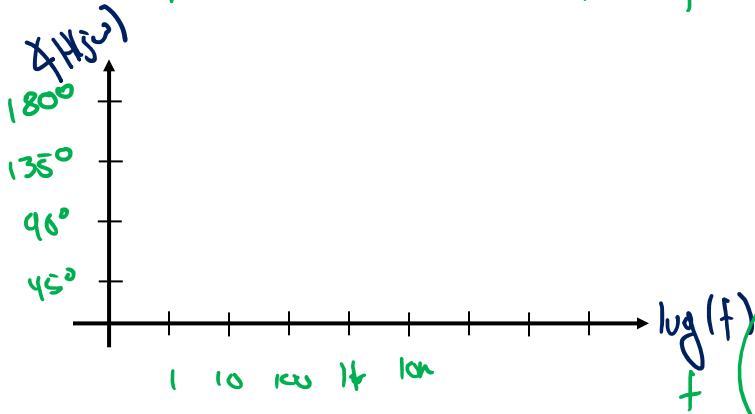
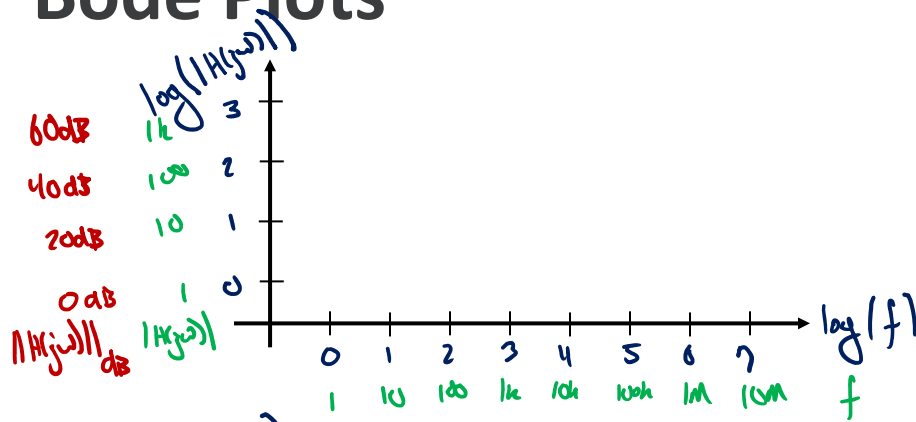
high pass



Band pass



Bode Plots



$$\|H(j\omega)\|_{\text{dB}} = 20 \log(|H(j\omega)|)$$

Frequency response:

@ ω_x

fourier component of $v_i(t)$ at ω_x is $A e^{j\phi}$

output component at ω_x is $A |H(j\omega_x)| e^{j(\phi + \angle H(j\omega))}$

- Magnitudes multiply

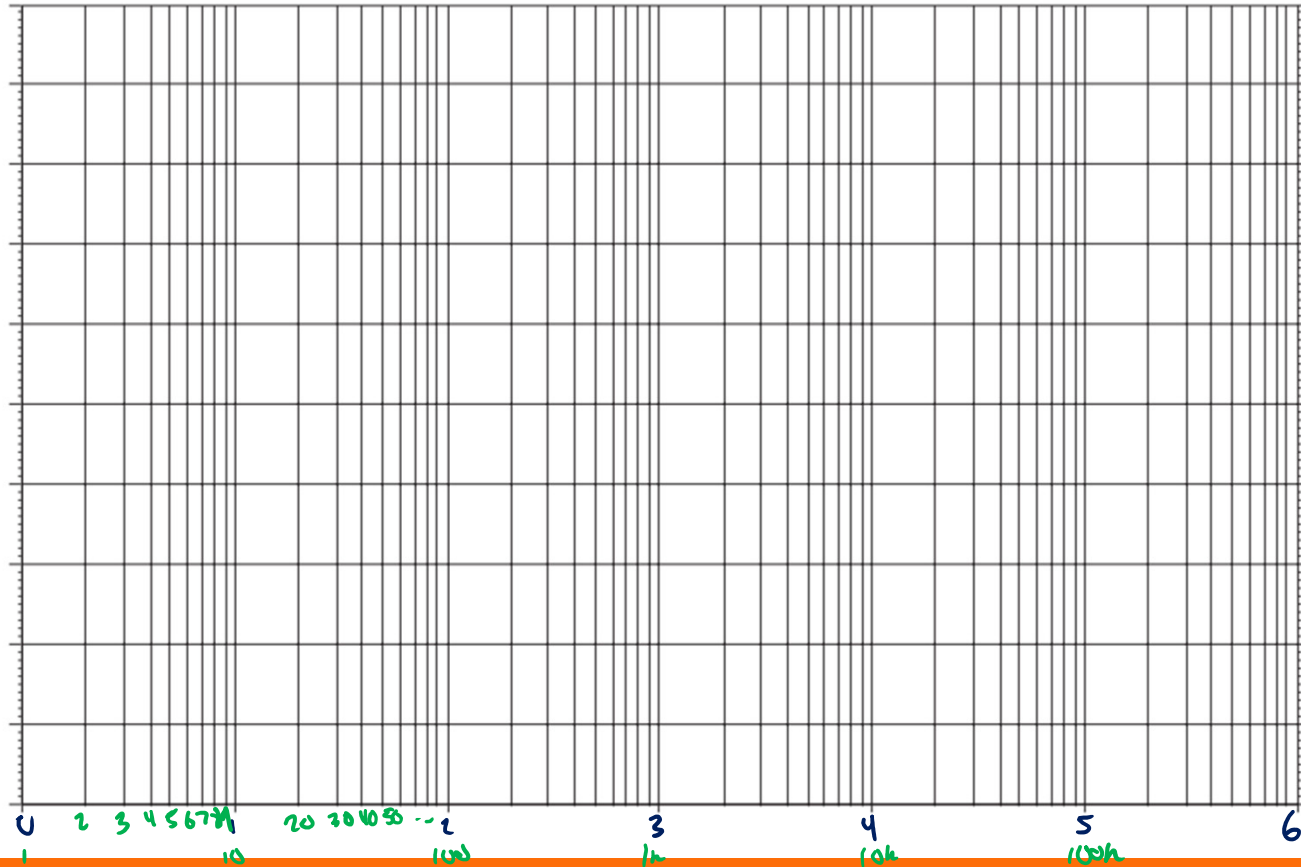
- Phases Add

$$\log(A |H(j\omega)|) = \log(|V_o @ \omega_x|)$$

$$= \log(A) + \log(|H(j\omega)|)$$

- log of Magnitudes add

Semilog Paper



0 1 2 3 4 5 6
1 10 100 1000 10000 100000 1000000

$\log(f)$
f



dB Scale

Decibels

$$\|G\|_{\text{dB}} = 20 \log_{10}(\|G\|)$$

Decibels of quantities having units (impedance example): normalize before taking log

$$\|Z\|_{\text{dB}} = 20 \log_{10}\left(\frac{\|Z\|}{R_{\text{base}}}\right)$$

Table 8.1. Expressing magnitudes in decibels

Actual magnitude	Magnitude in dB
1/2	- 6dB
1	0 dB
2	6 dB
5 = 10/2	20 dB - 6 dB = 14 dB
10	20dB
1000 = 10 ³	3 · 20dB = 60 dB

5Ω is equivalent to 14dB with respect to a base impedance of $R_{\text{base}} = 1\Omega$, also known as 14dBΩ.

60dBμA is a current 60dB greater than a base current of 1μA, or 1mA.