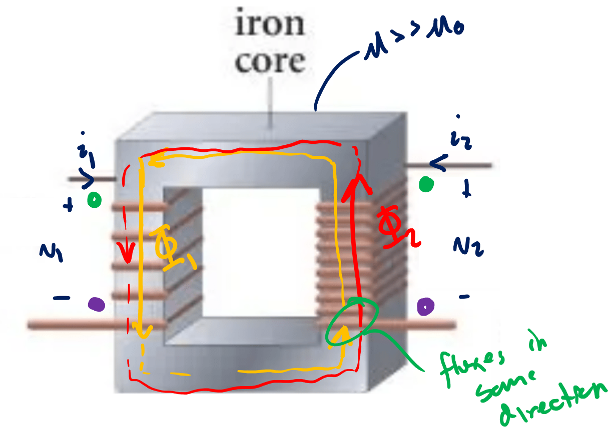
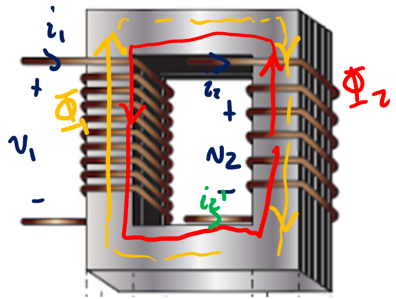
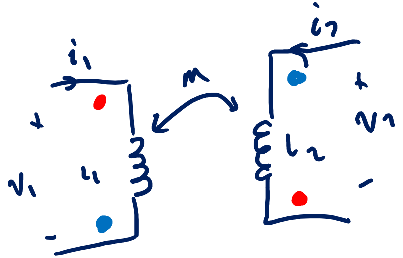
Dot Notation Example



Find which terminals should be dotted on each winding (2 possibilities)



Sketch circuit symbol with same reference polarities



https://energycentral.com/c/iu/transformer-polarity
https://stardelta.com.au/step-up-vs-step-down-transformers/

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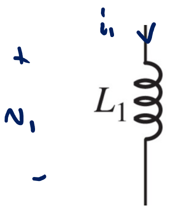
Energy Storage

@ $t=0$, $i_1=0$, @ $t=t_0$, $i_1=I_0$

Review

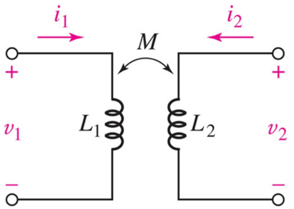


$$E_L = \int_0^{t_0} v_1 \cdot i_1 \, dt = \int_0^{t_0} L_1 \left[i_1 \cdot \frac{di_1}{dt} \right] dt$$

$$i_1 \frac{di_1}{dt} = \frac{1}{2}\left[\frac{d}{dt} i_1(t)^2\right] = \frac{1}{2}\left[2 i_1(t) \frac{di_1(t)}{dt}\right]$$

$$= \frac{1}{2} L_1 \int_0^{t_0} \frac{d}{dt} i_1(t)^2 dt = \frac{1}{2} L_1 \left[i_1(t=t_0)^2 - i_1(t=0)^2 \right]$$

$$\boxed{E_L = \frac{1}{2} L_1 I_0^2}$$



At $t=0$ $i_1=0$ & $i_2=0$, at $t=t_0$ $i_1=I_1$ $i_2=I_2$

$$E = \int_0^{t_0} (v_1 i_1 + v_2 i_2) \, dt$$

$$= \int_0^{t_0} \left(L_1 i_1 \frac{di_1}{dt} \pm M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} \pm M i_2 \frac{di_1}{dt} \right) dt$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm \int_0^{t_0} \underbrace{\left(M i_1 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt} \right)}_{= M \frac{d}{dt}(i_1 \cdot i_2)} dt$$

$$\boxed{E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2}$$

starting from zero currents & ramping up to $i_1 = I_1$ & $i_2 = I_2$, must be true that

$$E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2 > \emptyset$$

$$M < \frac{\frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2}{I_1 I_2} = \frac{1}{2} L_1 \frac{I_1}{I_2} + \frac{1}{2} L_2 \frac{I_2}{I_1}$$

find minimum of $\frac{1}{2} L_1 x + \frac{1}{2} L_2 \frac{1}{x}$, $\quad x = \frac{I_1}{I_2}$

$$\frac{\partial M}{\partial x} = \frac{1}{2} L_1 - \frac{1}{2} L_2 \frac{1}{x^2} = \emptyset \qquad x = \sqrt{\frac{L_2}{L_1}}$$

$$\frac{\partial^2 M}{\partial x^2} = L_2 \frac{1}{x^3} \quad \checkmark$$

so,

$$M < \frac{1}{2} L_1 \sqrt{\frac{L_2}{L_1}} + \frac{1}{2} L_2 \sqrt{\frac{L_1}{L_2}} = \frac{1}{2}\sqrt{L_1 L_2} + \frac{1}{2}\sqrt{L_1 L_2}$$

$$\boxed{M \leq \sqrt{L_1 L_2}}$$

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Coupling Coefficient

define coupling coefficient $k = \frac{M}{\sqrt{L_1 L_2}}$

$0 \leq k \leq 1$

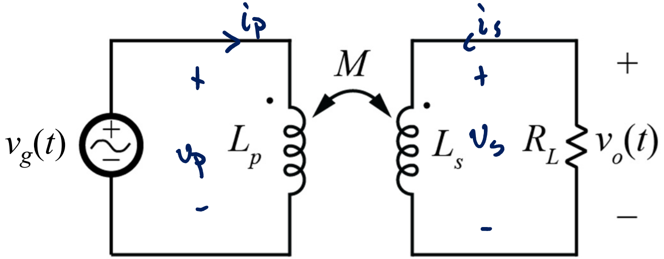
$k = 0 \rightarrow$ two separate inductors

$k = 1 \rightarrow$ perfect coupling between them

all of Φ_1 flows through L_2 & vica-versa

known as a "transformer"

Coupled Inductor Example



find $v_o(t)$

$$\begin{cases} v_p = L_p \frac{di_p}{dt} + M \frac{di_s}{dt} \\ v_s = M \frac{di_p}{dt} + L_s \frac{di_s}{dt} \end{cases}$$

$v_p = v_g$

$v_o = -i_s R_L$

$i_s = -\frac{v_o}{R_L}$

$$\begin{cases} v_g = L_p \frac{di_p}{dt} - M \frac{d}{dt} \frac{1}{R_L} v_o \\ v_o = M \frac{di_p}{dt} - L_s \frac{d}{dt} \frac{1}{R_L} v_o \end{cases}$$

$$\frac{di_p}{dt} = \frac{v_g + M \frac{1}{R_L} \frac{dv_o}{dt}}{L_p}$$

$$v_o = \frac{M}{L_p}\left(v_g + \frac{M}{R_L} \frac{dv_o}{dt} \right) - \frac{L_s}{R_L} \frac{dv_o}{dt} \rightarrow \boxed{-\frac{M}{L_p} v_g = -v_o + \left(\frac{M^2}{L_p R_L} - \frac{L_s}{R_L} \right) \frac{dv_o}{dt}}$$