Announcements

- Return Analog Discovery Studio
- Office Hours
  - No (Dr. Costinett) Wednesday office hours due to EECS faculty meeting
  - Available via e-mail; Thursday office hours as normal
- TNvoice Open
  - Please fill out – Closes midnight May 8
  - +5 pts EC on final for 100% response rate

Final Exam

- Thursday May 9th, 3:30-6:00pm
- Roughly 2x midterm in length, w/ 3x time
- Covers all course material
  - Chapters 10-11, 13-15 & 17(partial)
  - All homeworks, quizzes, exams, and experiments 1-3
  - All lectures

Final Exam Problems (Tentative)

- Power and impedance matching with sinusoidal input
- Bode plot (plot $\longrightarrow H(s)$ and/or $H(s) \longrightarrow$ plot)
- Identify bounded/stable systems
- Solve Laplace with complex and repeated poles and zeroes
- Evaluate Fourier Series and frequency response
- Solve and compare different descriptions of circuit (e.g. $H(s)$, $h(t)$, Phasor circuit, etc.)

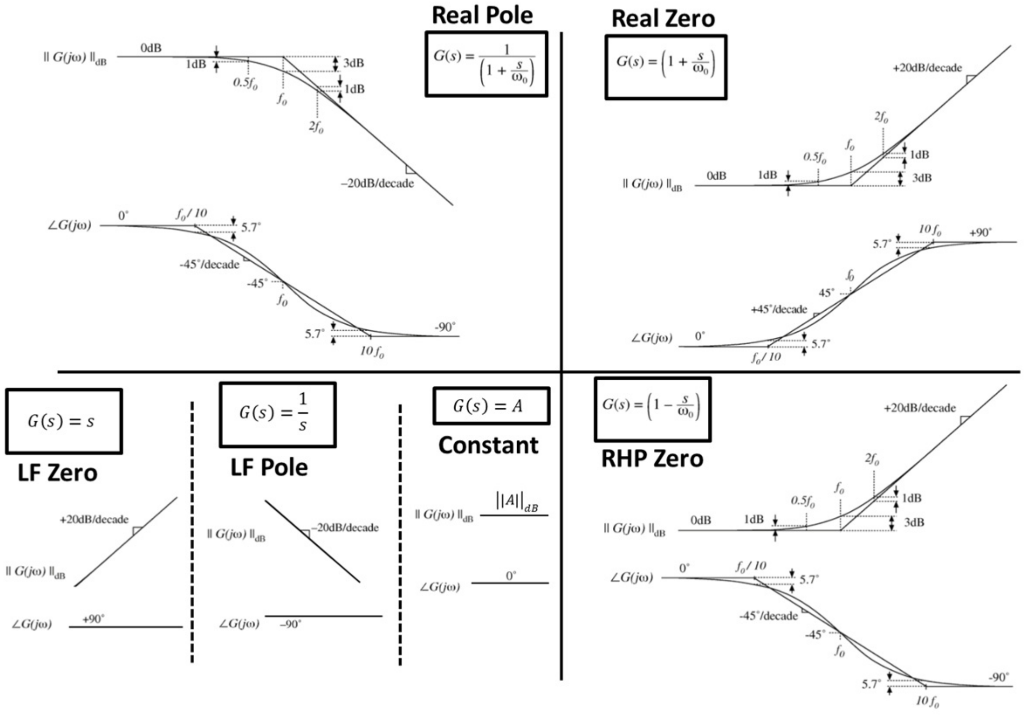
- Additional Notes:
  - Circuits will contain
    - coupled inductors and/or transformers
    - op-amps

Exam Tables

TABLE 14.1 Laplace Transform Pairs

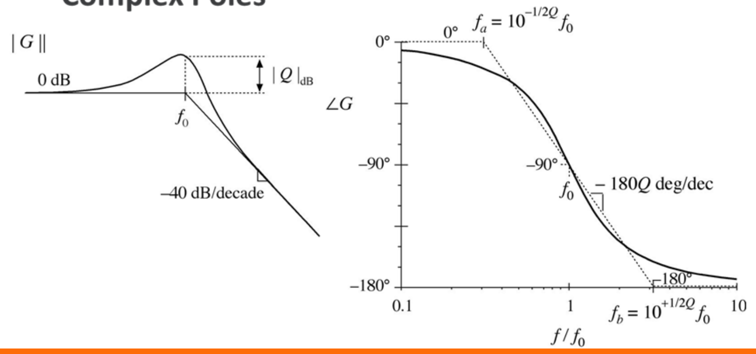
$f(t) = \mathscr{L}^{-1}\{\mathbf{F}(\mathbf{s})\}$	$\mathbf{F}(\mathbf{s}) = \mathscr{L}\{f(t)\}$	$f(t) = \mathscr{L}^{-1}\{\mathbf{F}(\mathbf{s})\}$	$\mathbf{F}(\mathbf{s}) = \mathscr{L}\{f(t)\}$
$\delta(t)$	1	$\frac{1}{\beta-\alpha}(e^{-\alpha t} - e^{-\beta t})u(t)$	$\frac{1}{(\mathbf{s}+\alpha)(\mathbf{s}+\beta)}$
$u(t)$	$\frac{1}{\mathbf{s}}$	$\sin \omega t\, u(t)$	$\frac{\omega}{\mathbf{s}^2+\omega^2}$
$tu(t)$	$\frac{1}{\mathbf{s}^2}$	$\cos \omega t\, u(t)$	$\frac{\mathbf{s}}{\mathbf{s}^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t), n = 1, 2, \ldots$	$\frac{1}{\mathbf{s}^n}$	$\sin(\omega t+\theta)\, u(t)$	$\frac{\mathbf{s}\sin\theta + \omega\cos\theta}{\mathbf{s}^2+\omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{\mathbf{s}+\alpha}$	$\cos(\omega t+\theta)\, u(t)$	$\frac{\mathbf{s}\cos\theta - \omega\sin\theta}{\mathbf{s}^2+\omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(\mathbf{s}+\alpha)^2}$	$e^{-\alpha t}\sin \omega t\, u(t)$	$\frac{\omega}{(\mathbf{s}+\alpha)^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t), n = 1,2,\ldots$	$\frac{1}{(\mathbf{s}+\alpha)^n}$	$e^{-\alpha t}\cos \omega t\, u(t)$	$\frac{\mathbf{s}+\alpha}{(\mathbf{s}+\alpha)^2+\omega^2}$
		$2\lvert k\rvert e^{\sigma t}\cos(\omega t - \angle k)\, u(t)$	$\frac{k}{s-(\sigma+j\omega)} + \frac{k^*}{s-(\sigma-j\omega)}$

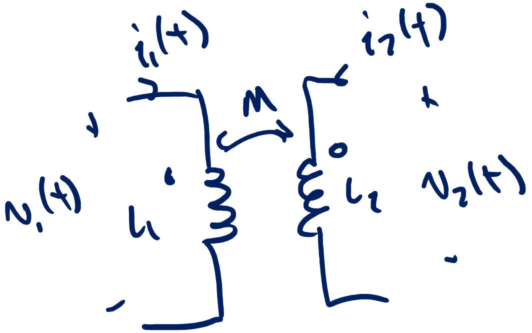
Operation	$f(t)$	$\mathbf{F}(\mathbf{s})$
Addition	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(\mathbf{s}) \pm \mathbf{F}_2(\mathbf{s})$
Scalar multiplication	$kf(t)$	$k\mathbf{F}(\mathbf{s})$
Time differentiation	$\frac{df}{dt}$	$\mathbf{s}\mathbf{F}(\mathbf{s}) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$\mathbf{s}^2\mathbf{F}(\mathbf{s}) - \mathbf{s}f(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$\mathbf{s}^3\mathbf{F}(\mathbf{s}) - \mathbf{s}^2f(0^-) - \mathbf{s}f'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^{t} f(t)\,dt$	$\frac{1}{\mathbf{s}}\mathbf{F}(\mathbf{s})$
	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{\mathbf{s}}F(\mathbf{s}) + \frac{1}{\mathbf{s}}\int_{-\infty}^{0^-} f(t)\,dt$
Convolution	$f_1(t) * f_2(t)$	$\mathbf{F}_1(\mathbf{s})\mathbf{F}_2(\mathbf{s})$
Time shift	$f(t-a)u(t-a), a \geq 0$	$e^{-a\mathbf{s}}\mathbf{F}(\mathbf{s})$
Frequency shift	$f(t)e^{-at}$	$\mathbf{F}(\mathbf{s}+a)$
Frequency differentiation	$tf(t)$	$-\frac{d\mathbf{F}(\mathbf{s})}{d\mathbf{s}}$
Frequency integration	$\frac{f(t)}{t}$	$\int_{\mathbf{s}}^{\infty}\mathbf{F}(\mathbf{s})\,d\mathbf{s}$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}\mathbf{F}\left(\frac{\mathbf{s}}{a}\right)$
Initial value	$f(0^+)$	$\lim_{\mathbf{s}\to\infty} \mathbf{s}\mathbf{F}(\mathbf{s})$
Final value	$f(\infty)$	$\lim_{\mathbf{s}\to 0} \mathbf{s}\mathbf{F}(\mathbf{s})$, all poles of $\mathbf{s}\mathbf{F}(\mathbf{s})$ in LHP
Time periodicity	$f(t) = f(t+nT)$, $n = 1, 2, \ldots$	$\frac{1}{1-e^{-T\mathbf{s}}}\mathbf{F}_1(\mathbf{s})$, where $\mathbf{F}_1(\mathbf{s}) = \int_{0^-}^{T} f(t)e^{-\mathbf{s}t}\,dt$



Complex Poles

$$H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$





$$\begin{cases} v_1(t) = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ v_2(t) = L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt} \end{cases}$$

Time

$$\underline{V_1} = L_1 (j\omega) \underline{I_1} + M (j\omega) \underline{I_2}$$

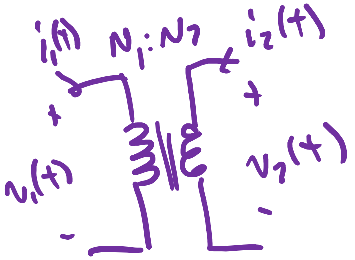
$$\underline{V_2} = L_2 (j\omega) \underline{I_2} + M (j\omega) \underline{I_1}$$

Phasor

Laplace:

$$V_1 = L_1 (sI_1 - i_1(0^-)) + M (sI_2 - i_2(0^-))$$

$$V_2 = L_2 (sI_2 - i_2(0^-)) + M (sI_1 - i_1(0^-))$$



$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$

$$N_1 i_1(t) + N_2 i_2(t) = \phi$$

Time

Phasor

$$\frac{\underline{V_1}}{N_1} = \frac{\underline{V_2}}{N_2} \quad \$ \quad N_1 \underline{I_1} + N_2 \underline{I_2} = \phi$$

Laplace

$$\frac{V_1(s)}{N_1} = \frac{V_2(s)}{N_2} \quad \$ \quad N_1 I_1(s) + N_2 I_2(s) = \phi$$

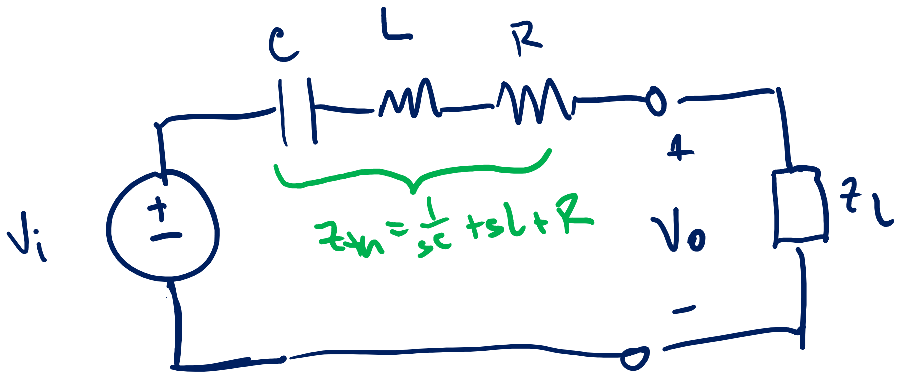
$$F(s) = \frac{7s^2 + 3s + 1}{1s^2 + 2s + 5}$$

$M = N$, order of numerator = order of denominator

$$
\begin{array}{r|l}
 & 7 \\
\hline
s^2 + 2s + 5 & 7s^2 + 3s + 1 \\
 & -7s^2 - 14s - 35 \\
\hline
 & 0 - 11s - 34
\end{array}
$$

$$F(s) = 7 + \frac{-11s - 34}{s^2 + 2s + 5}$$

Find Z_L that maximizes P_{out} (Real power to Z_L)



For max power transfer

$$Z_L = Z_{th}^*$$

$$Z_{th} = R + j\left(\omega L - \frac{1}{\omega C}\right),$$

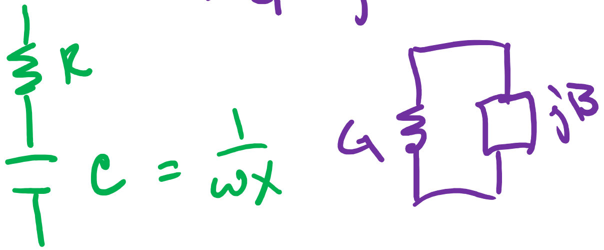
$$Z_L = Z_{th}^* = R - j\left(\omega L - \frac{1}{\omega C}\right), \quad Y_L = \frac{1}{Z_L} = \frac{R + j\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= G + jB$$

if $\omega L > \omega C$

$Z_L = R - jX$, $\quad X > \phi$

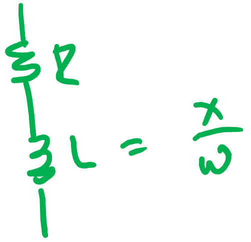
$\frac{-j}{\omega C} = -jX$ $\longrightarrow$



if $\omega L < \omega C$

$Z_L = R + jX$, $\quad X > \phi$ $\longleftrightarrow$

$j\omega L = jX$



COURSE REVIEW

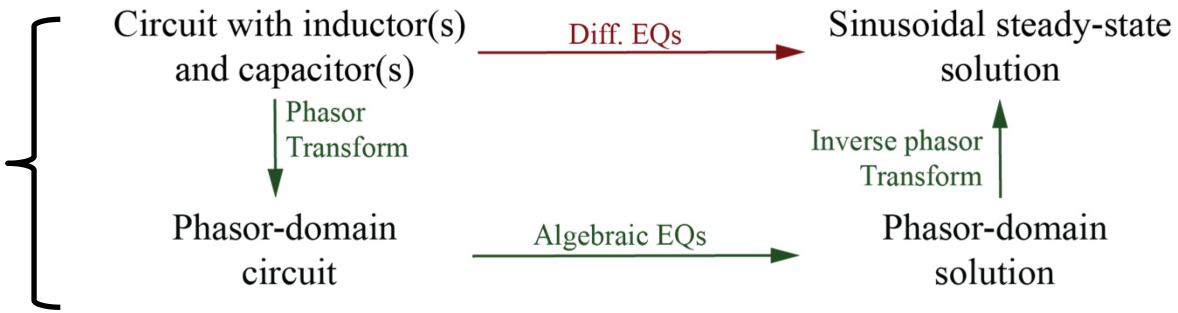
Course Content

- Magnetically Coupled Circuits (Ch 13)
- Sinusoidal Steady-State Analysis (Ch 10)
- AC Circuit Power Analysis (Ch 11)
- Circuit An Analysis in the s-Domain (Ch 14)
- Frequency Response (Ch 15)
- Fourier Circuit Analysis (Ch 17)
- Polyphase Circuits (Ch 12)
- Two-Port Networks (Ch 16)

Transform Domains

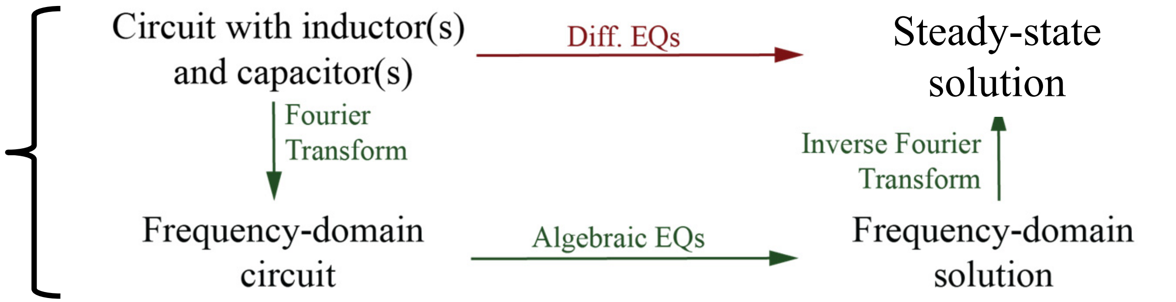
Phasor Transform

$$f(t) = A\cos(\omega_0 t + \varphi)$$



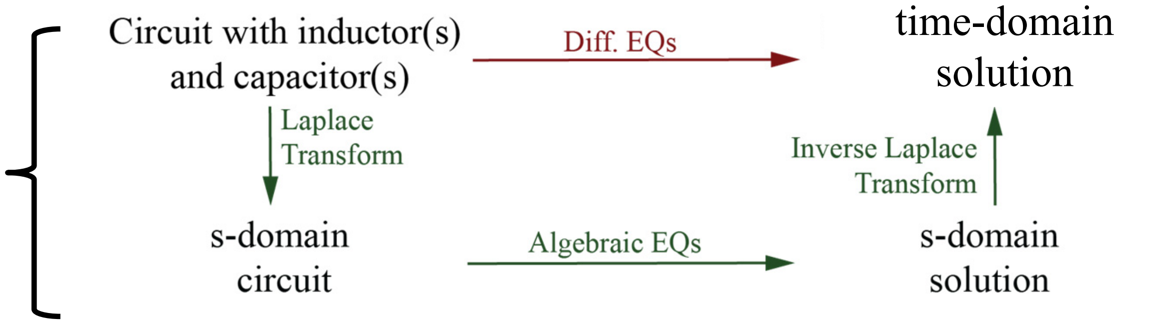
Fourier Transform

$$f(t) = \sum C_n \cos(n\omega_0 t + \varphi_n)$$



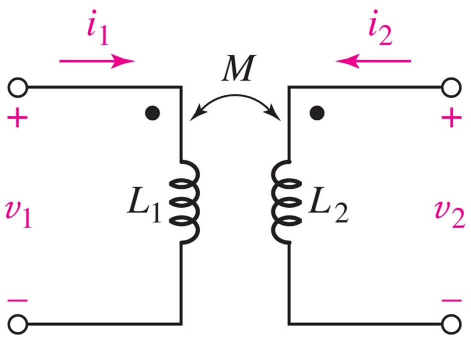
Laplace Transform

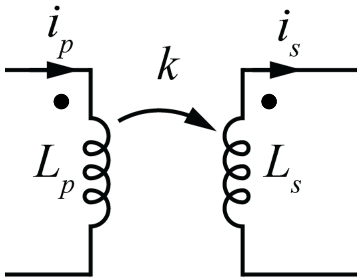
$$f(t) = \int e^{st} F(s) ds$$



Ch 13 – Magnetically Coupled Circuits

Coupled Inductors





Defining Equations

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Dot convention

- Current into the dot on one terminal produces a positive open circuit voltage w.r.t. the dot on the other

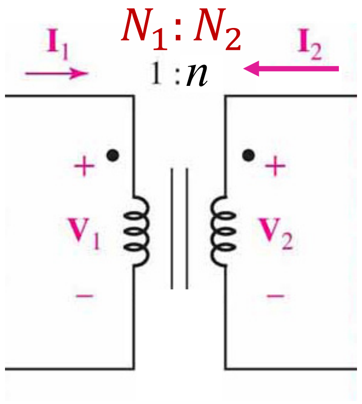
Coupling Coefficient

$$k = \frac{M}{\sqrt{L_p L_s}}$$

Recall

- Equivalent circuits

Ideal Transformer



Defining Equations

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = \cdots$$

$$0 = N_1 I_1 + N_2 I_2 + \cdots$$

Coupled inductors with:

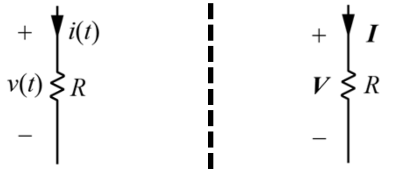
- no energy storage (L→ ∞)
- Perfect coupling (*k=1*)

Recall

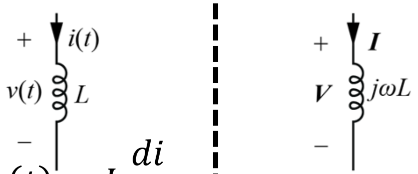
- Z/V/I reflection

Ch 10 – Sinusoidal Steady State

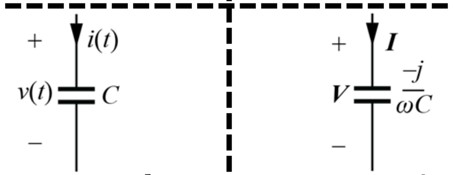
Time Domain	Phasor Domain



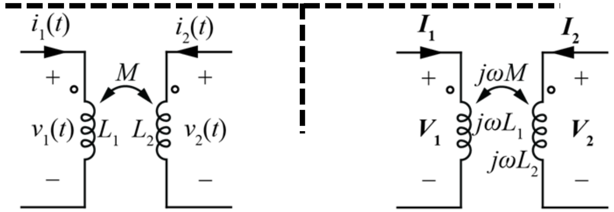
$v(t) = i(t)R$ | $\boldsymbol{V} = \boldsymbol{I}R$



$v(t) = L\frac{di}{dt}$ | $\boldsymbol{V} = j\omega L\boldsymbol{I}$



$i(t) = C\frac{dv}{dt}$ | $\boldsymbol{V} = \frac{-j}{\omega C}\boldsymbol{I}$



$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$v_2(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

$$\boldsymbol{V}_1 = j\omega L_1\boldsymbol{I}_1 + j\omega M\boldsymbol{I}_2$$
$$\boldsymbol{V}_2 = j\omega M\boldsymbol{I}_1 + j\omega L_2\boldsymbol{I}_2$$

Phasor Notation

$$A\cos(\omega t + \varphi) \Leftrightarrow A\measuredangle\varphi$$
$$= Re\{Ae^{j(\omega t+\varphi)}\}$$
$$= Re\{A\cos(\omega t + \varphi) + jA\sin(\omega t + \varphi)\}$$

Impedance and Admittance

$$Z = R + jX$$

Impedance → Z, Resistance → R, Reactance → X

$$Y = \frac{1}{Z} = G + jB$$

Admittance → Y, Conductance → G, Susceptance → B

Circuit Analysis

- Real circuits always have all real signals in the time domain
- All 201 analysis techniques apply
- Gives only forced/steady-state/particular response, for single sinusoidal source
- Phasor superposition

Ch 11 – AC Power Analysis

Average (DC) Power: $P = \int_{-\infty}^{\infty} p(t)dt$ For periodic signals: $P = \int_{t_0}^{t_0+T} p(t)dt$

Average power in a resistor: $P_R = \left[\underbrace{\sqrt{\int i(t)^2 dt}}_{I_{rms} = I_{eff}}\right]^2 R$ For sinusoids: $I_{rms} = \frac{I_A}{\sqrt{2}}$

Sinusoidial Power

$$p(t) = [V_A \cos(\omega t + \varphi_V)][I_A \cos(\omega t + \varphi_I)]$$

$$= \underbrace{\frac{V_A I_A}{2}\cos(2\omega t + \varphi_V + \varphi_I)}_{\text{Double-frequency}} + \underbrace{\frac{V_A I_A}{2}\cos(\varphi_V - \varphi_I)}_{\text{DC = average}}$$

$$\frac{V_A I_A}{2} = V_{rms} I_{rms}$$

Complex Power

$$S = \frac{\boldsymbol{V}\boldsymbol{I}^*}{2} = \boldsymbol{V}_{rms}\boldsymbol{I}_{rms}$$

$S = P + jQ$

Complex [VA] Real/Avg [W] Reactive/Quad. [VAR]

Apparent Power: $|S| = \frac{V_A I_A}{2} = V_{rms} I_{rms}$ Power Factor: $PF = \frac{P}{|S|}$ (leading/lagging)

Impedance match for max power transfer: $Z_L = {Z_{th}}^*$

Ch 14 – Laplace Transform

Unilateral Laplace Transform: $F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t) dt$

Inverse Laplace Transform: $f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j}$, σ_0 in ROC

(to the right of all poles in complex plane)

Complex frequency = Laplace variable = $s = \sigma + j\omega$

Laplace transform is a **linear transformation**. Other properties and transforms in tables

Inverse Transforms: *Long Division* → *Factor* → *PFE* → *Tables*

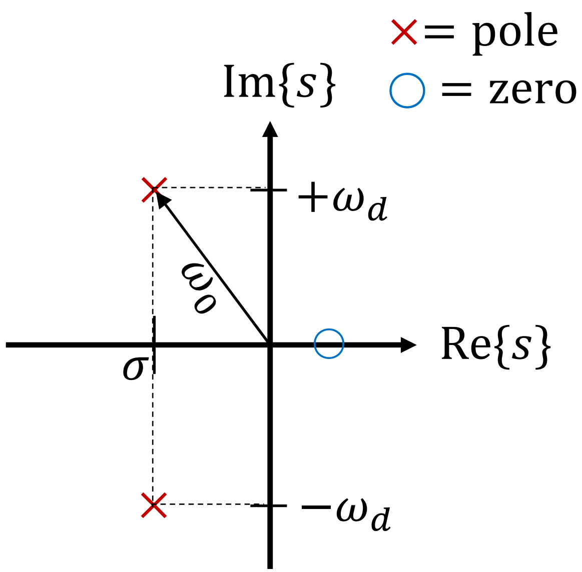
PFE Special cases:

Repeated: $\frac{N(s)}{(s+5)^2} = \frac{k_1}{s+5} + \frac{k_2}{(s+5)^2}$ (Differentiation or coefficient matching)

Complex: $\frac{N(s)}{s^2+4} = \frac{k_1 s + k_2}{s^2+4} = \frac{k_1}{s-j2} + \frac{{k_1}^*}{s+j2}$

Other properties from tables including delay and time periodicity

Ch 14 – Laplace Circuit Analysis



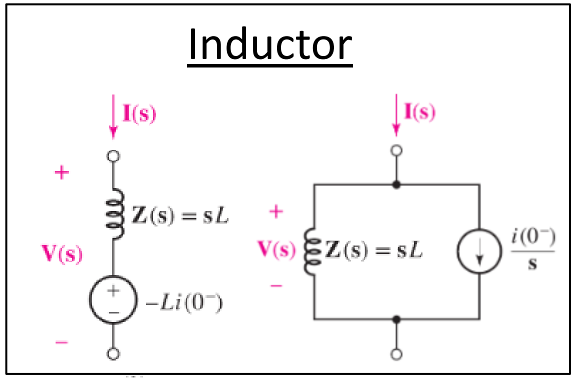
Poles and Zeros:

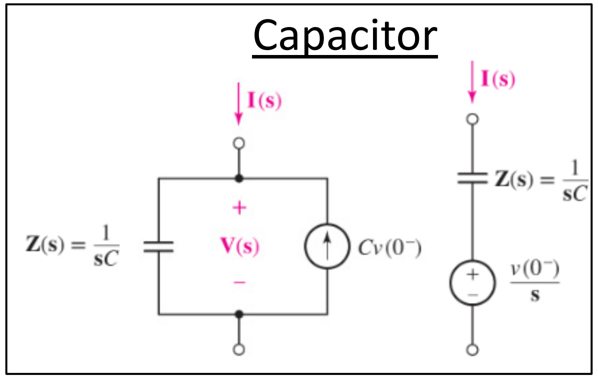
$$F(s) = \frac{N(s)}{D(s)}$$

Zeros: roots of $N(s)$

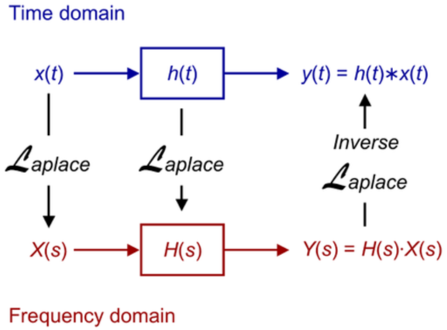
Poles: roots of $D(s)$

Circuit Transformation:

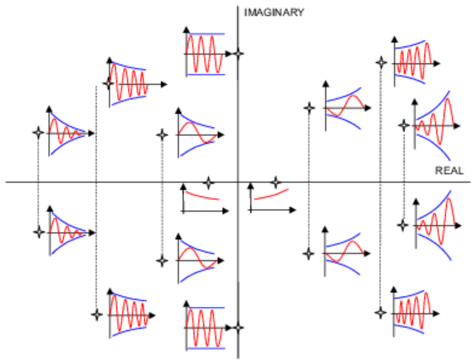




Convolution:



$$v_o(t) = v_i(t) * h(t) = \int_{-\infty}^{\infty} v_i(t-\tau)h(\tau)d\tau$$



Transfer Functions:

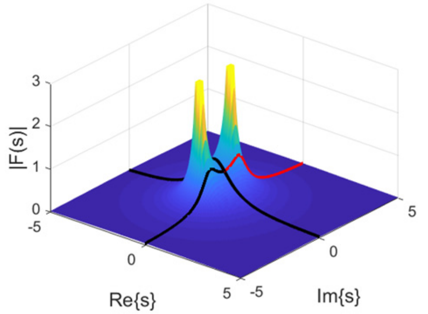
$$H(s) = \frac{V_o(s)}{V_i(s)}$$

Poles of $H(s)$ define "form" of terms in natural response of the circuit

Poles of $V_i(s)$ define "form" of terms in forced response of the circuit

$$V_o(s) = V_I(s)H(s) = \left(\frac{\sum_{i=0}^{M_I} c_i s^i}{\sum_{i=0}^{N_I} d_i s^i}\right)\left(\frac{\sum_{i=0}^{M_H} a_i s^i}{\sum_{i=0}^{N_H} b_i s^i}\right) = \frac{(s-z_1)(s-z_2)\cdots(s-z_{M_H+M_I})}{(s-p_1)(s-p_2)\cdots(s-p_{N_H+N_I})} = \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \cdots + \frac{k_x}{(s-p_x)}$$

Ch 15 – Frequency Response



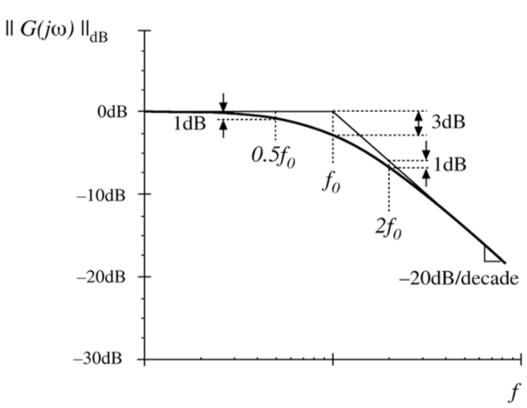
Frequency Response: $H(s \to j\omega)$

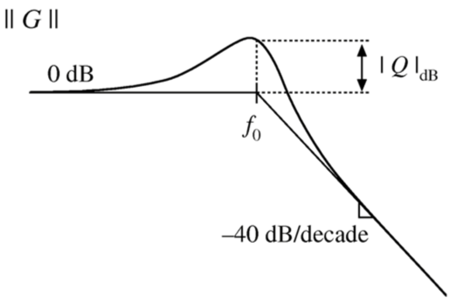
- Circuit response to sinusoidal inputs
- Valid if all poles in LHP

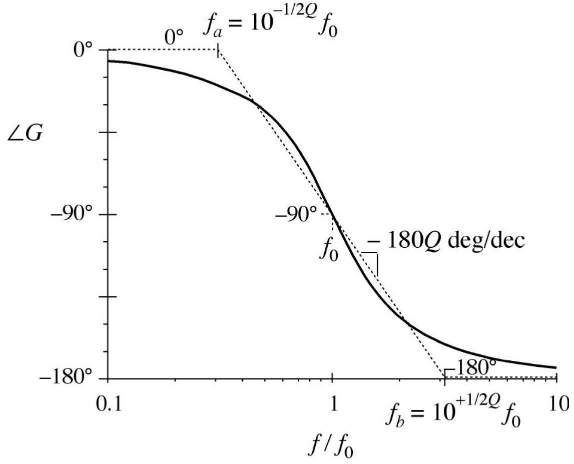
Bode plot: mag-phase plots on log-log axes

- $\|H(j\omega)\|_{dB} = 20\log(|H(j\omega)|)$
- $\measuredangle H(j\omega)$

Templates and Approximations:







Filter Design

- Graphical analysis, Chebyshev and Butterworth, Sallen-Key Amplifier
- Resonant circuits

Ch 17 – Fourier Series and Transform

https://en.wikipedia.org/wiki/Fourier_transform

Fourier Series:

For periodic $f(t)$ with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t)\cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t)\sin(k\omega_0 t)\, dt$$

Fourier Transform:

For periodic or non-periodic $f(t)$

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t} f(t) dt$$

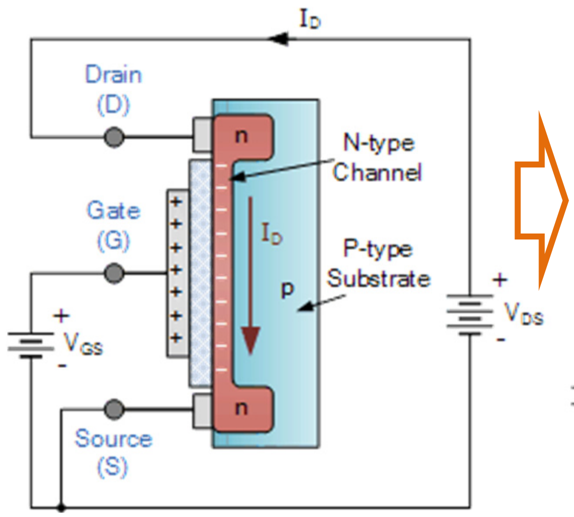
$$f(t) = \mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} F(\omega) d\omega$$

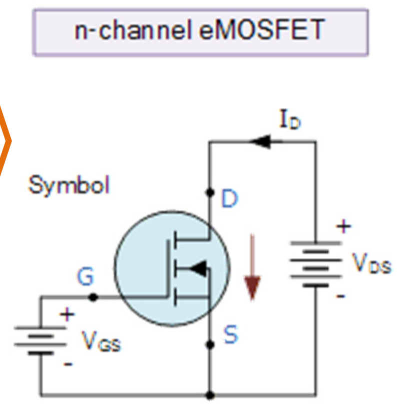
- Frequency content of a signal
- Gives mag/phase of sinusoids that add up to original signal
- Symmetry can speed up calculations

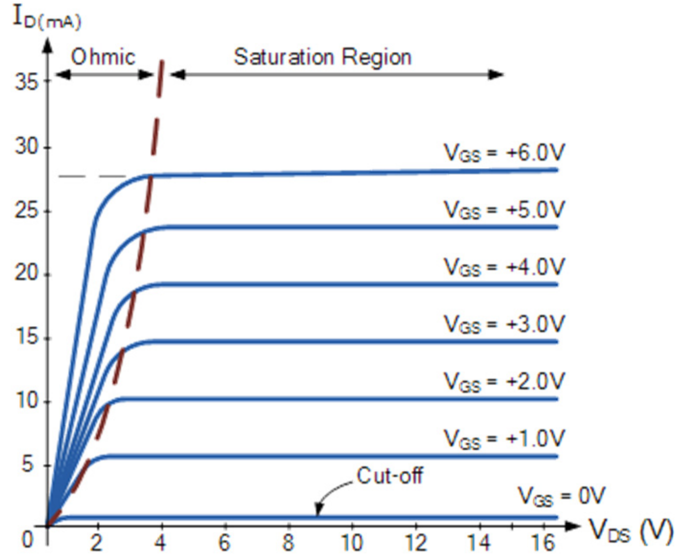
FUTURE TOPICS

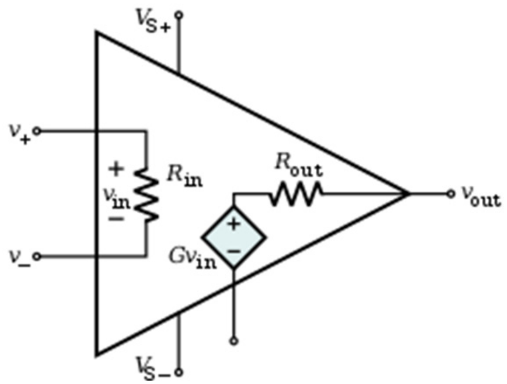
Nonlinear Circuits

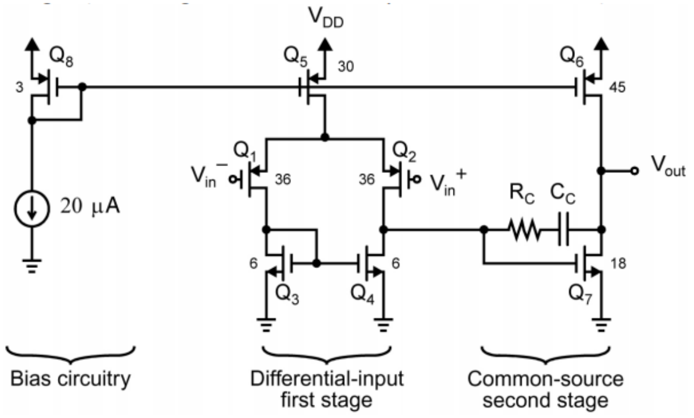
- ECE 335 & 336 – Electronic Devices









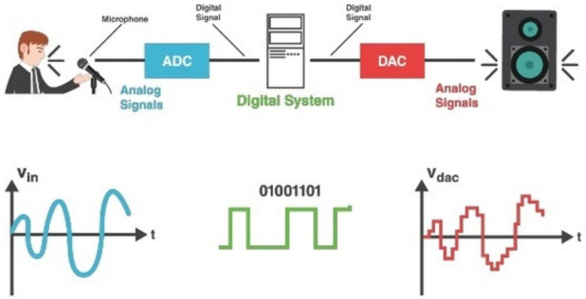


Digital / Discrete Time Signals

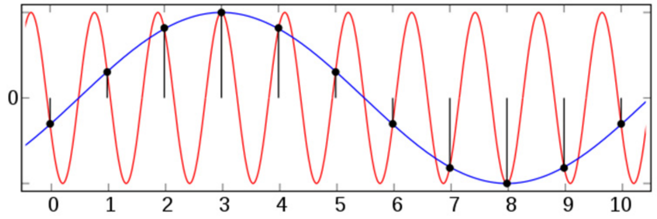
- ECE 315 – Signals and Systems

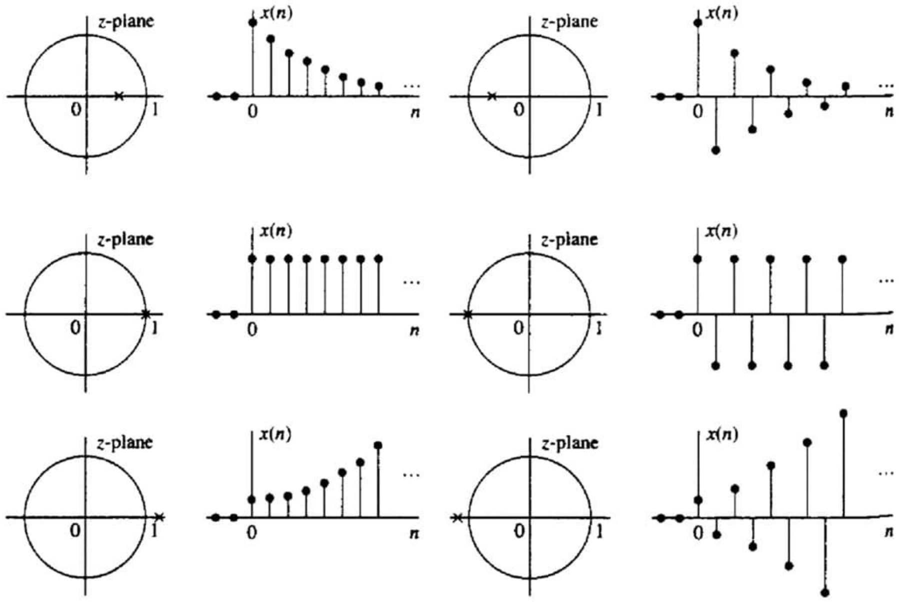
Cont. time $\sum_{n} A_n \frac{d^n}{dt^n} v_o(t) = \sum_{m} B_m \frac{d^m}{dt^m} v_i(t) \xrightarrow{\mathcal{L}} \sum_{n} A_n s^n V_o(s) = \sum_{m} B_m s^m V_i(s)$

Discrete time $\sum_{n} A_n y[k-n] = \sum_{m} B_m u[k-m] \xrightarrow{Z} \sum_{n} A_n z^{-n} Y(z) = \sum_{m} B_m z^{-m} U(z)$



Sampling and aliasing:

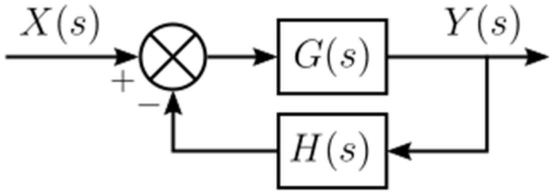




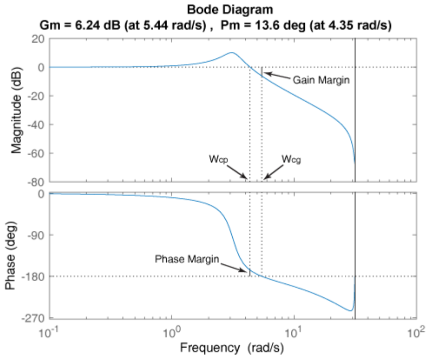
Closed-Loop Control

- ECE 316 – Signals and Systems

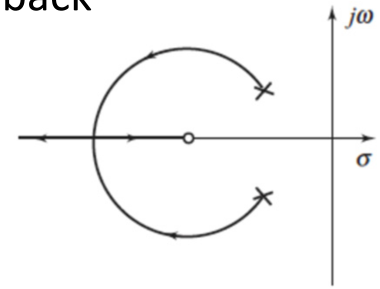
<u>Classic Control</u>



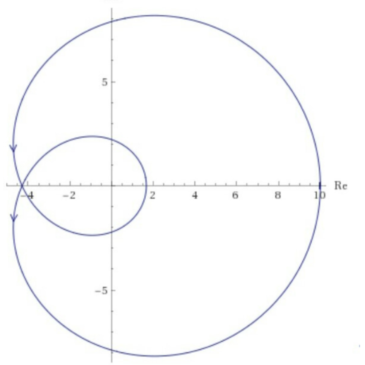
Margin Test: Examine closed-loop stability on bode plot



Root Locus: How poles and zeros move as you change feedback



Nyquist Plots: Frequency response in the complex plane



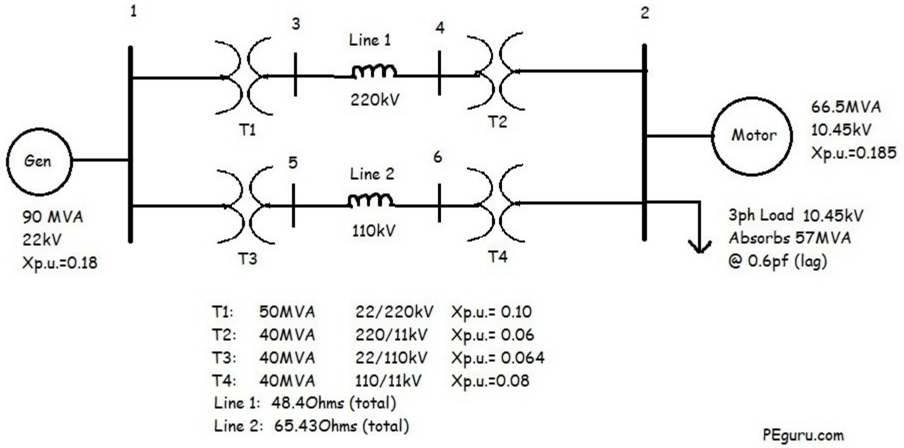
<u>Modern Control</u>

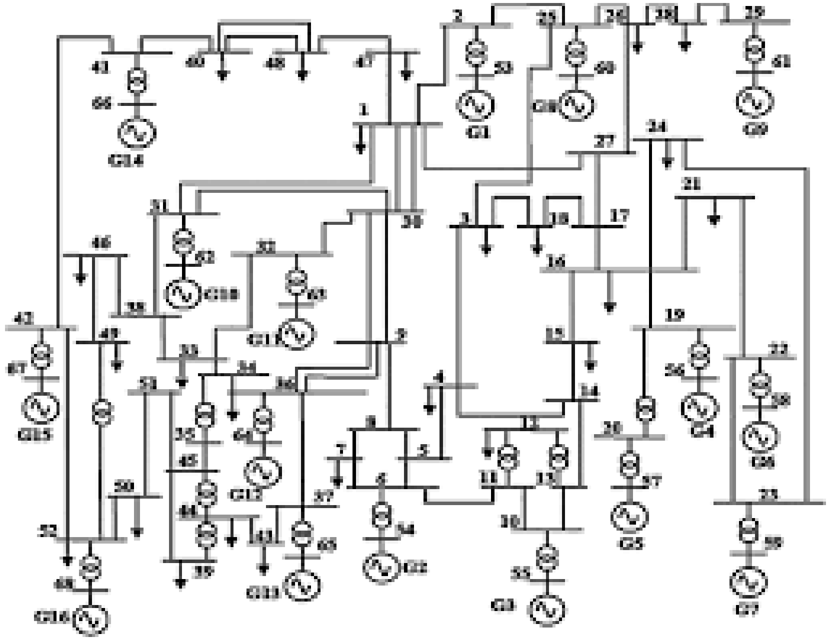
$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

$$H(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$$

Power Systems

- ECE 325 -- Electric Energy System Components





FINAL REMARKS

Thank you for all your hard work

Good luck with all your finals

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