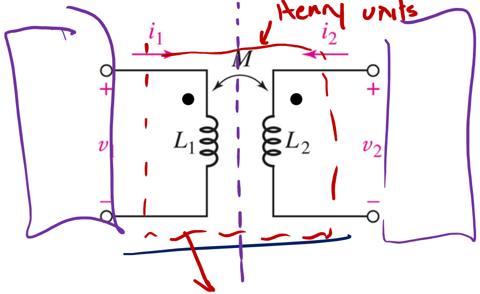
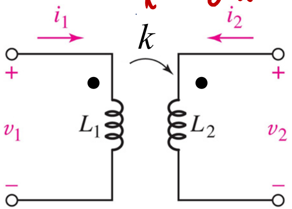
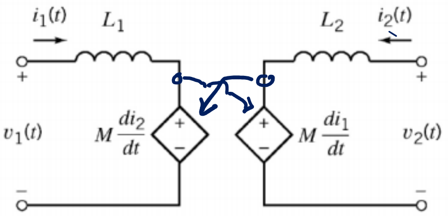
Equivalent Circuits

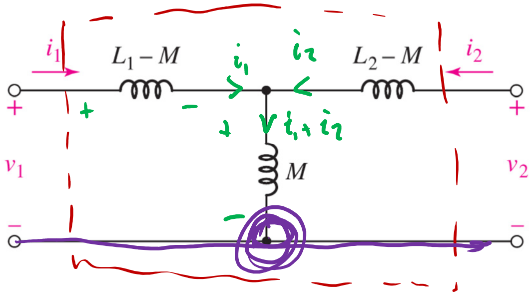






$$\begin{cases} v_1 = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt} \\ v_2 = \pm M\frac{di_1}{dt} + L_2\frac{di_2}{dt} \end{cases}$$

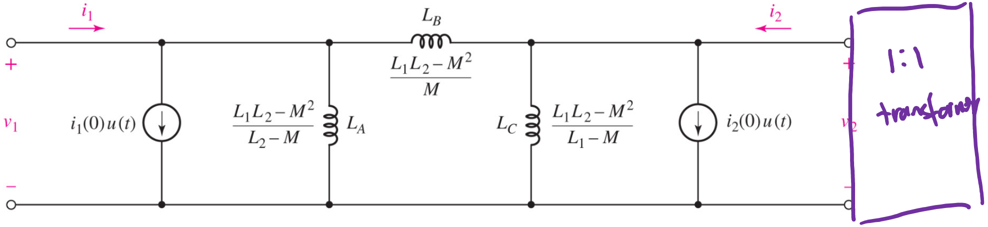
T-network



$$\begin{cases} v_1 = (L_1 - M)\frac{di_1}{dt} + M\frac{d}{dt}(i_1 + i_2) \\ v_2 = M\frac{d}{dt}(i_1 + i_2) + (L_2 - M)\frac{di_2}{dt} \end{cases}$$

for positive polarity

Π-network



Transformers

Special case of coupled inductors with $k = 1 \longleftrightarrow$ perfect coupling $\longleftrightarrow M = \sqrt{L_1 L_2}$

$$\begin{cases} v_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \\ v_2 = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{cases} \quad \xleftrightarrow{M = \sqrt{L_1 L_2}} \quad \begin{cases} v_1 = L_1 \frac{di_1}{dt} \pm \sqrt{L_1 L_2} \frac{di_2}{dt} \\ v_2 = \pm \sqrt{L_1 L_2} \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{cases}$$

$$v_1 = \sqrt{\frac{L_1}{L_2}}\, v_2$$

$$\downarrow$$

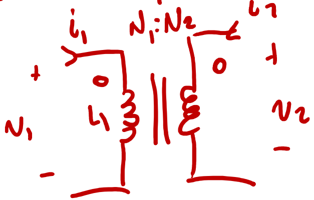
$$v_1 = \sqrt{\frac{\alpha_1 N_1^2}{\alpha_2 N_2^2}}\, v_2$$

if $k=1$, it must be true that $\alpha_1 = \alpha_2$

$$\boxed{v_1 = \frac{N_1}{N_2} v_2}$$

$\frac{N_1}{N_2}$ is the "turns ratio"

Circuit Symbol



$N_1 : N_2$ is the same as $1 : N$, $N = \frac{N_2}{N_1}$

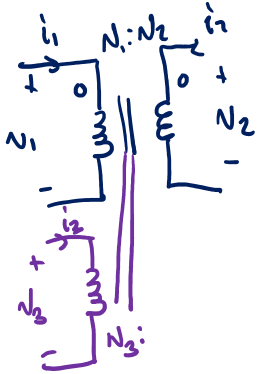
Ideal Transformer

"Ideal" transformer has very large L_1 & L_2

Recall: Inductors (& transformers) cannot have DC voltage applied to their windings

- $v_1 = L_1 \frac{di_1}{dt} \rightarrow i_1 = \frac{1}{L_1}\int_0^t v_1\, dt \rightarrow$ DC voltage causes current to go to ∞ & will saturate & stop
- Materials $\rightarrow$ core materials needed for $K \approx 1$ working at some finite current
- $v = N \frac{d\Phi}{dt} \rightarrow$ Faraday's Law, need time-varying waveforms

When L_1 & L_2 are large enough $\rightarrow$ No energy storage



$$v_1 = \frac{N_1}{N_2} v_2$$

No energy storage, so $v_1 i_1 + v_2 i_2 = \phi$

$$\left(\frac{N_1}{N_2} v_2\right) i_1 + v_2 i_2 = \phi$$

$$N_1 i_1 + N_2 i_2 = \phi$$

$$i_1 = -\frac{N_2}{N_1} i_2$$

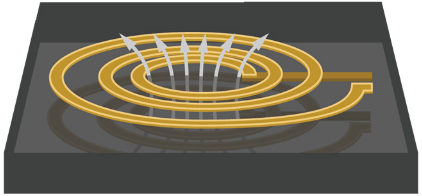
for more than two turns

$$\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3} = \ldots$$

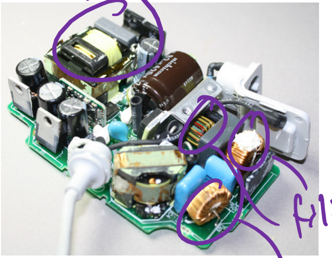
$$N_1 i_1 + N_2 i_2 + N_3 i_3 + \ldots = \phi$$

Applications of Coupled Inductors

common mode choke (transformer)



transformer



filter

transformer