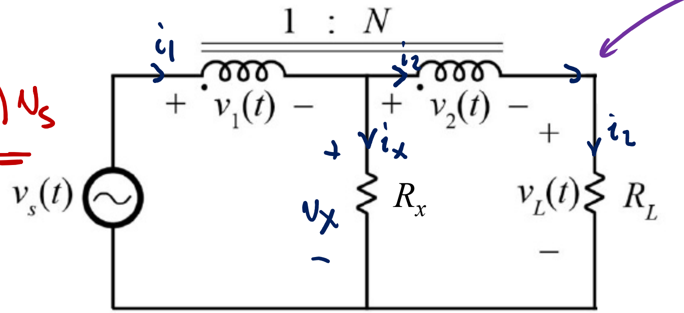
Ideal Transformer Example

Autotransformer (variac)

solve for $v_L(t)$

$v_L = f(N, R_L, R_x)\, v_s$



Transformer equations

$v_1 = \frac{v_2}{N}$

$i_1 = -N i_2$

other elements

$v_x = i_x R_x$

$v_L = i_2 R_L$

KVL:

$v_x = v_2 + v_L$

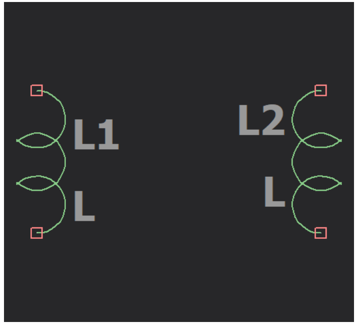
$v_s = v_1 + v_2 + v_L$

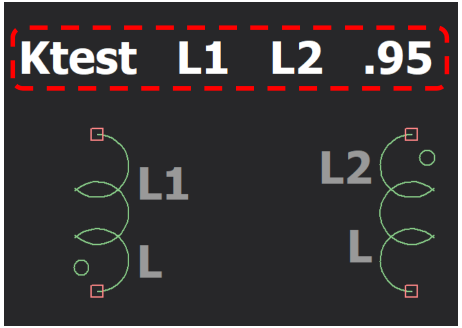
KCL:

$i_1 = i_2 + i_x$

Spice Mutual Inductance and XFs

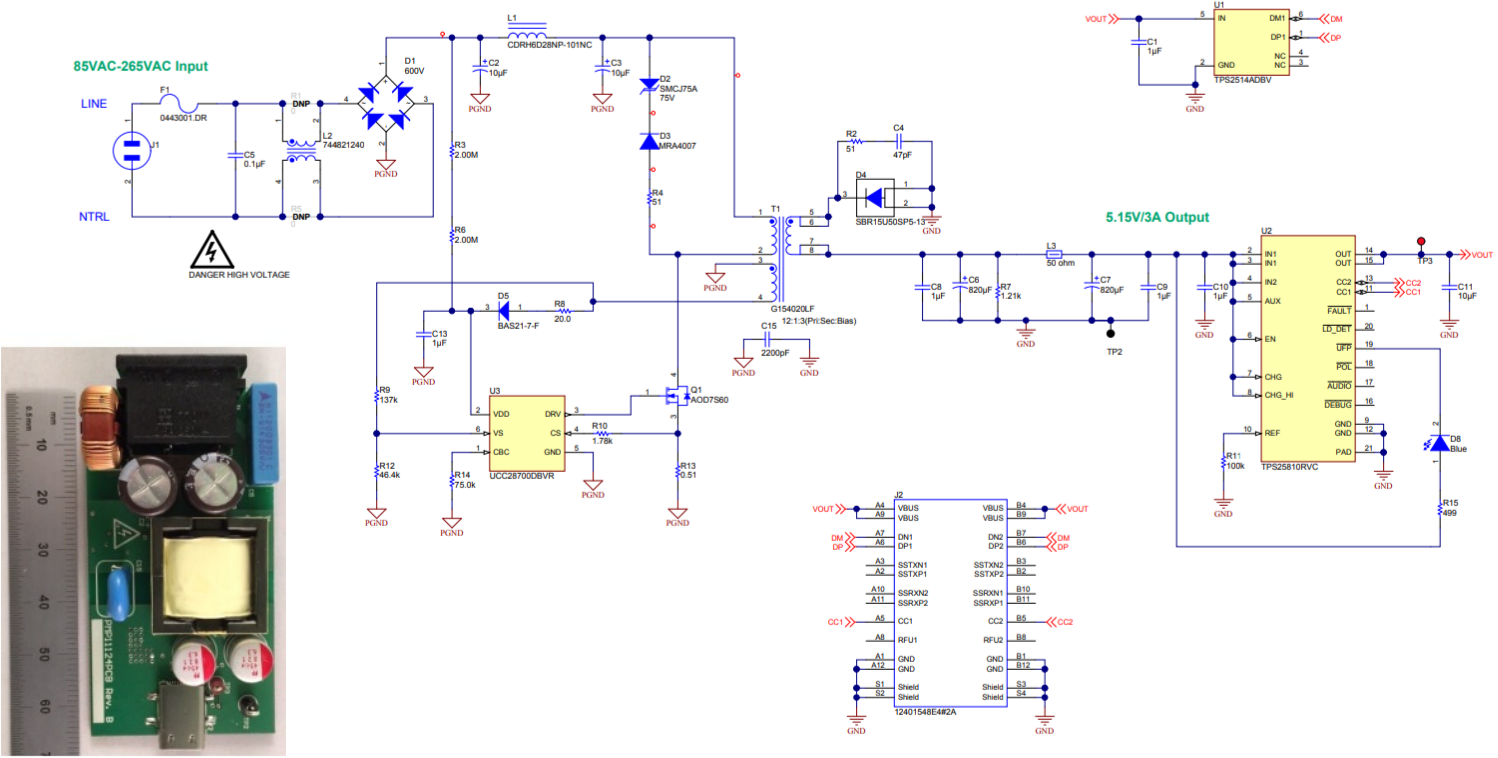
Coupled Inductors





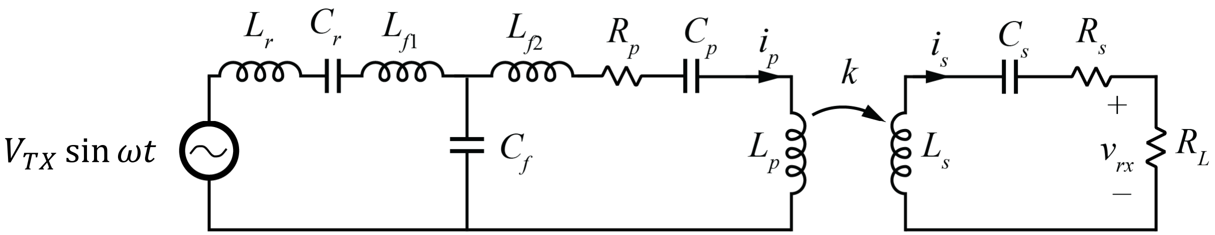
Ideal Transformer

15W USB Adapter Reference Design

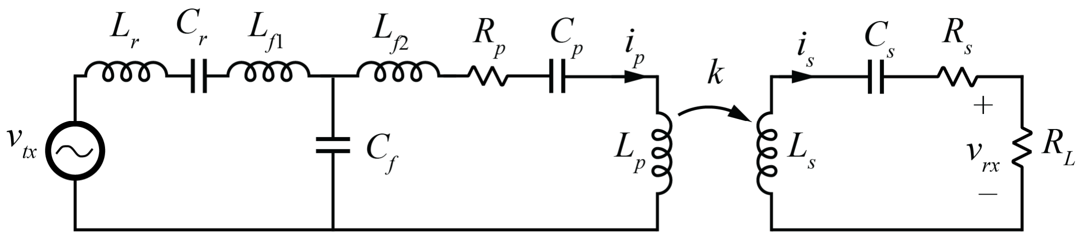


CHAPTER 10: SINUSOIDAL STEADY-STATE

Motivation



Motivation

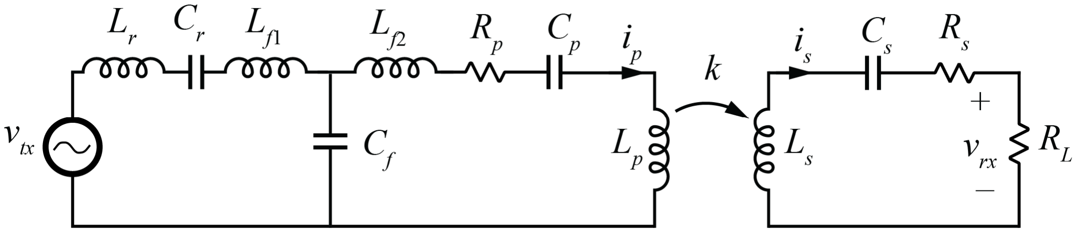


Numerical **Analytical**

Time Domain **Frequency Domain**

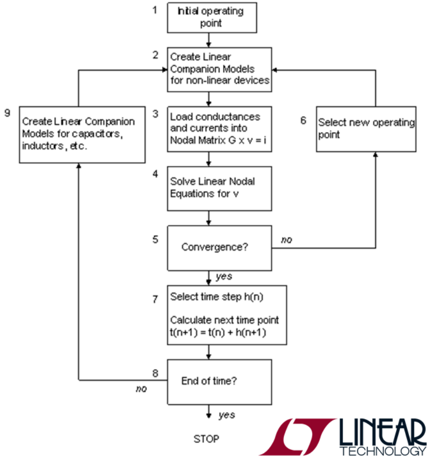
$$
\left\{
\begin{aligned}
\frac{dA}{dt} &= \beta_A \cdot \frac{D^n}{K_{DA}^n + D^n} \cdot \left(1 - \frac{C^n}{K_{CA}^n + C^n}\right) - \alpha_A \cdot A \\
\frac{dX}{dt} &= \beta_X \cdot \frac{A^n}{K_{AX}^n + A^n} - \alpha_X \cdot X \\
\frac{dC}{dt} &= \beta_C \cdot \frac{(A+X)^n}{K_{AXC}^n + (A+X)^n} - \alpha_C \cdot C \\
\frac{dB}{dt} &= \beta_B \cdot \frac{C^n}{K_{CB}^n + C^n} \cdot \left(1 - \frac{D^n}{K_{DB}^n + D^n}\right) - \alpha_B \cdot B \\
\frac{dY}{dt} &= \beta_Y \cdot \frac{B^n}{K_{BY}^n + B^n} - \alpha_Y \cdot Y \\
\frac{dD}{dt} &= \beta_D \cdot \frac{(B+Y)^n}{K_{BYD}^n + (B+Y)^n} - \alpha_D \cdot D
\end{aligned}
\right\}
$$

Motivation



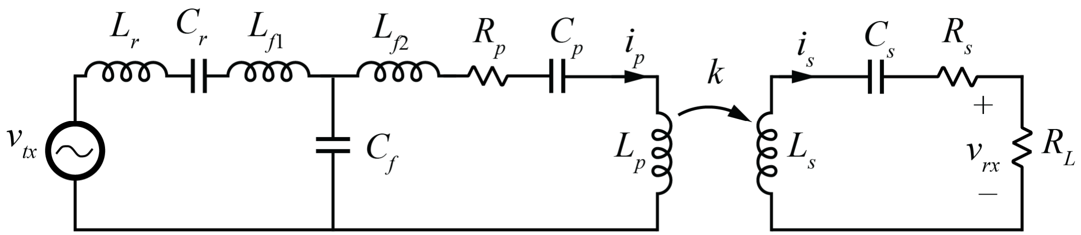
Numerical | **Analytical**

Time Domain | **Frequency Domain**



$$
\begin{cases}
\dfrac{dA}{dt} = \beta_A \cdot \dfrac{D^n}{K_{DA}^n + D^n} \cdot \left(1 - \dfrac{C^n}{K_{CA}^n + C^n}\right) - \alpha_A \cdot A \\
\dfrac{dX}{dt} = \beta_X \cdot \dfrac{A^n}{K_{AX}^n + A^n} - \alpha_X \cdot X \\
\dfrac{dC}{dt} = \beta_C \cdot \dfrac{(A+X)^n}{K_{AXC}^n + (A+X)^n} - \alpha_C \cdot C \\
\dfrac{dB}{dt} = \beta_B \cdot \dfrac{C^n}{K_{CB}^n + C^n} \cdot \left(1 - \dfrac{D^n}{K_{DB}^n + D^n}\right) - \alpha_B \cdot B \\
\dfrac{dY}{dt} = \beta_Y \cdot \dfrac{B^n}{K_{BY}^n + B^n} - \alpha_Y \cdot Y \\
\dfrac{dD}{dt} = \beta_D \cdot \dfrac{(B+Y)^n}{K_{BYD}^n + (B+Y)^n} - \alpha_D \cdot D
\end{cases}
$$

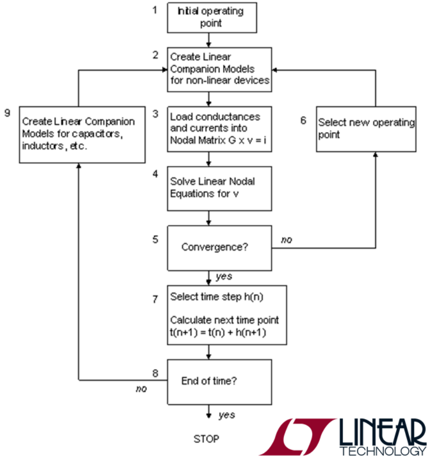
Motivation



Numerical

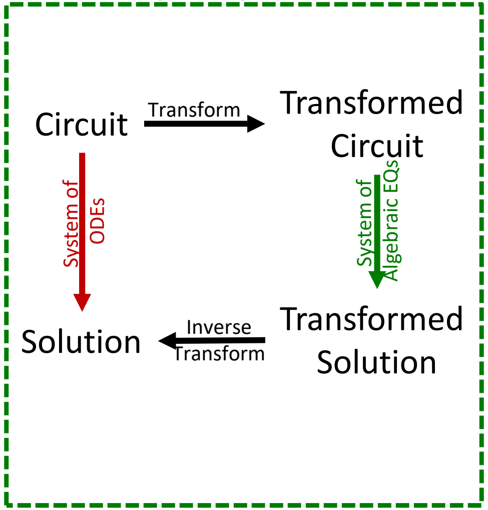
Analytical

Time Domain



$$
\begin{cases}
\dfrac{dA}{dt} = \beta_A \cdot \dfrac{D^n}{K_{DA}^n + D^n} \cdot \left(1 - \dfrac{C^n}{K_{CA}^n + C^n}\right) - \alpha_A \cdot A \\
\dfrac{dX}{dt} = \beta_X \cdot \dfrac{A^n}{K_{AX}^n + A^n} - \alpha_X \cdot X \\
\dfrac{dC}{dt} = \beta_C \cdot \dfrac{(A+X)^n}{K_{AXC}^n + (A+X)^n} - \alpha_C \cdot C \\
\dfrac{dB}{dt} = \beta_B \cdot \dfrac{C^n}{K_{CB}^n + C^n} \cdot \left(1 - \dfrac{D^n}{K_{DB}^n + D^n}\right) - \alpha_B \cdot B \\
\dfrac{dY}{dt} = \beta_Y \cdot \dfrac{B^n}{K_{BY}^n + B^n} - \alpha_Y \cdot Y \\
\dfrac{dD}{dt} = \beta_D \cdot \dfrac{(B+Y)^n}{K_{BYD}^n + (B+Y)^n} - \alpha_D \cdot D
\end{cases}
$$

Frequency Domain



Form of the Solution

N^{th} order circuit with sinusoidal input described by

$$b_N \frac{d^N}{dt^N} v_o(t) + \cdots + b_1 \frac{d}{dt} v_o(t) + b_0 v_o(t) \quad = \quad a_M \frac{d^M}{dt^M} v_i(t) + \cdots + a_1 \frac{d}{dt} v_i(t) + a_0 v_i(t)$$

$$\sum_{i=0}^{N} b_i \frac{d^i}{dt^i} v_o(t) \quad = \quad \sum_{i=0}^{M} a_i \frac{d^i}{dt^i} v_i(t)$$

solution for $v_o(t)$ will be of the form

$$v_o(t) = v_{o,h}(t) + v_{o,p}(t)$$

Transient Response

$v_{o,h}(t)$ is the *homogeneous solution* to the differential equation, the *natural response* of the system, or the *transient response* of the system. For any non-ideal (damped) circuit, $v_{o,h}(t)$ will tend to zero over time

$v_{o,h}(t)$ is the solution to the equation

$$\sum_{i=0}^{N} b_i \frac{d^i}{dt^i} v_{o,h}(t) = 0$$

which will be of the form

$$v_{o,h}(t) = \sum_{i=0}^{N} A_i e^{s_i t}$$

$s_i \rightarrow$ roots of characteristic polynomial

$A_i \rightarrow$ determined by initial conditions

Note: some of the time constants (s_i) of a circuit are independent of the input

Steady-State Response

$v_{o,p}(t)$ is the *particular solution* to the differential equation, the *forced response* of the system, or the *steady-state response* of the system. In general, it does not tend to zero, if non-zero inputs are present.

$v_{o,p}(t)$ is the non-zeroing solution to the equation

$$\sum_{i=0}^{N} b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^{M} a_i \frac{d^i}{dt^i} v_i(t)$$

LTI Systems

For a function $f(\cdot)$

Linearity

if: $v_{o,1}(t) = f\left(v_{i,1}(t)\right)$ $v_{o,2}(t) = f\left(v_{i,2}(t)\right)$

then: $a v_{o,1}(t) + b v_{o,2}(t) = f\left(a v_{i,1}(t) + b v_{i,2}(t)\right)$

Time Invariance

if: $v_o(t) = f\big(v_i(t)\big)$

then: $v_o(t-T) = f\big(v_i(t-T)\big)$

LTI: $v_x(t) + v_y(t)$, $\alpha v_x(t)$, $\frac{dv_x(t)}{dt}$, $\int_0^t v_x(t)dt$ (neglecting ICs)

Not LTI: $v_x(t) \cdot v_y(t)$, $v_x(t)^2$, $|v_x(t)|$

Causality

- Causal systems cannot predict the future
- $v_o(t_0)$ does not depend on values of $v_i(t), \mathrm{t} > t_0$

$$v_{o,1}(t) = f\left(v_{i,1}(t)\right) \qquad v_{o,2}(t) = f\left(v_{i,2}(t)\right)$$

if: $$v_{i,1}(t) = v_{i,2}(t) \quad \forall \quad t < t_0$$

then: $$v_{o,1}(t) = v_{o,2}(t) \quad \forall \quad t < t_0$$

Preview of Frequency Domain