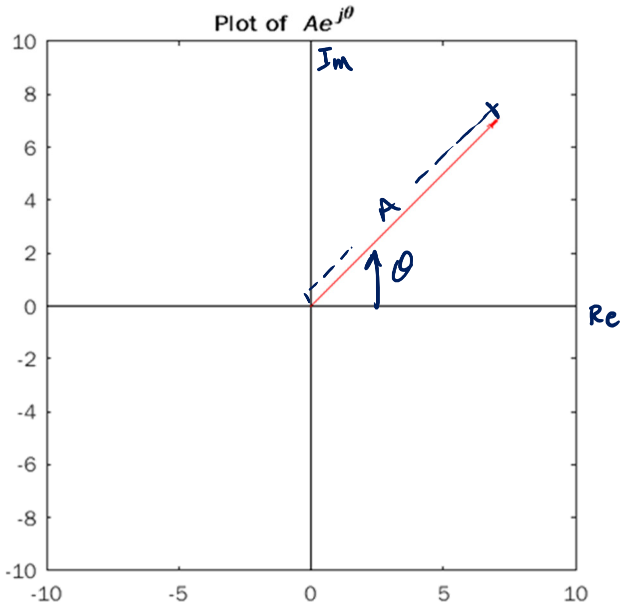


Plot of $Ae^{j0}$
Im
Re
A
θ
10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-5
0
5
10

# Phasor Transformation

"Phasor" is a complex number that represents a sinusoid

→ useful in analyzing single-frequency sinusoidal circuits in steady-state

$A\cos(\omega t + \varphi)$

- $A$ amplitude
- $\varphi$ phase
- $\cancel{\omega}$ ~~frequency~~ → for single-frequency circuits
- $\cancel{\cos}$ ~~sinusoidal~~ function → by convention, always use cos
- $\cancel{t}$ ~~time~~ → steady-state, single-frequency

$$A\cancel{\cos}(\cancel{\omega t} + \varphi) = \cancel{\mathrm{Re}}\{A\cancel{e^{j\omega t}}\, e^{j\varphi}\}$$

phasor transform

$$\longrightarrow A e^{j\varphi}$$

# Phasor Notation

$v(t) = A\cos(\omega t + \varphi) = \mathrm{Re}\{A e^{j\omega t} e^{j\varphi}\}$

transform @ $\omega$

$\underline{V} = A e^{j\varphi} \longleftrightarrow A \angle \varphi$ (short hand)

Bold in book
underline in lecture

$i(t) = B\sin(\omega t + \theta)$
$= B\cos(\omega t + \theta - 90°)$

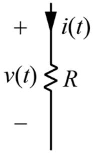
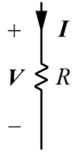
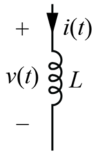
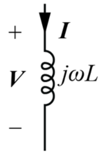
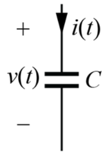
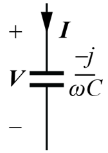
transform

$\underline{I} = B e^{j(\theta - \frac{\pi}{2})} \longleftrightarrow B \angle (\theta - \frac{\pi}{2})$

Comments:

- Phasor transform works for all voltage / current sources & signals
- Everything must be at same frequency $\omega$
- Using complex numbers in transformation
  - No "t" in any phasor expression
  - No complex # in time domain

# Phasor Circuit Elements

| Time Domain | Phasor Domain |
| --- | --- |
|  |  |
| $v(t) = i(t)R$ | $\underline{V} = \underline{I}R$ |
|  |  |
| $v(t) = L\dfrac{di}{dt}$ | $\underline{V} = j\omega L\,\underline{I}$ |
|  |  |
| $i(t) = C\dfrac{dv}{dt}$ | $\underline{V} = \dfrac{-j}{\omega C}\underline{I}$ |













$v(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{V} = Ae^{j\varphi}$

$i(t) = \frac{A}{R}\cos(\omega t + \varphi) \longrightarrow \underline{I} = \frac{A}{R}e^{j\varphi}$

$v(t) = i(t)R \longrightarrow \boxed{\underline{V} = \underline{I}R}$

$i(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{I} = Ae^{j\varphi}$

$v(t) = AL\omega\cos(\omega t + \varphi + 90^\circ) \longrightarrow \underline{V} = A\omega L e^{j(\varphi + \frac{\pi}{2})}$

$\underline{V} = Aj\omega L e^{j\varphi}$

$\boxed{\underline{V} = (j\omega L)\underline{I}}$

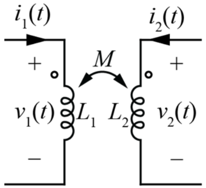
$v(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{V} = Ae^{j\varphi}$

$i(t) = AC\omega\cos(\omega t + \varphi + 90^\circ) \longrightarrow \underline{I} = AC\omega e^{j\varphi}e^{j\frac{\pi}{2}}$

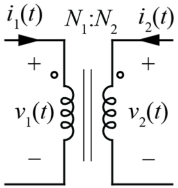
$\underline{I} = Aj\omega C e^{j\varphi}$

$\boxed{\underline{V} = \frac{1}{j\omega C}\underline{I} = \frac{-j}{\omega C}\underline{I}}$

## Time Domain

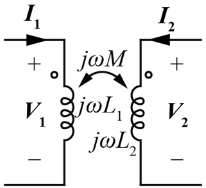



$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$v_2(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

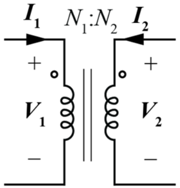



$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$
$$N_1 i_1(t) + N_2 i_2(t) = 0$$

## Phasor Domain



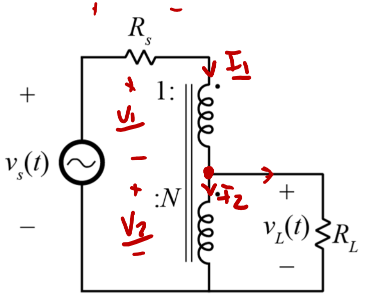

$$\underline{V}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$$
$$\underline{V}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$$




$$\frac{\underline{V}_1}{N_1} = \frac{\underline{V}_2}{N_2}$$
$$N_1 \underline{I}_1 + N_2 \underline{I}_2 = \emptyset$$

# Example Problem

$\omega = 2\pi 60$




Find $v_L(t)$ for $v_s(t) = 170\cos(2\pi 60t)$ and for $R_s = 10\ \Omega$, $N = 0.1$, and $R_L = 50\ \Omega$

$v_L(t) \rightarrow V_L$

$v_s(t) \rightarrow V_s = 170 \angle 0^\circ$ or $170 e^{j0}$ or $170 + j\phi$

$R_s \rightarrow R_s$

$R_L \rightarrow R_L$

$1 + \frac{1}{N} = \frac{N+1}{N}$

$\frac{V_1}{1} = \frac{V_2}{N}$ , $I_1 + N I_2 = \phi$

Node :

$I_1 = I_2 + \frac{V_L}{R_L}$

$\left(1 + \frac{1}{N}\right) I_1 = \frac{V_L}{R_L}$

Loop

$V_s = V_1 + V_2 + I_1 R_s$

$V_s = \left(\frac{1}{N} + 1\right) V_L + I_1 R_s$

$V_s = \left(\frac{N+1}{N}\right) V_L + \frac{R_s}{R_L}\left(\frac{1}{N+1}\right) V_L \longrightarrow$

$$\boxed{V_L = V_s \dfrac{1}{\frac{N+1}{N} + \frac{R_s}{R_L}\frac{1}{N+1}}}$$

compute & inverse transform

$v_L(t) = 15.5 \cos(2\pi 60 t + 0^\circ)$