Outline

1 Overview

2 High Current Phenomena in PN Junctions
   2.1 Series Resistance
      2.1.1 Physical Origin
      2.1.2 Mathematically Accounting for Series Resistance
   2.2 High-Level Injection
      2.2.1 Revisiting Low-Level Injection
      2.2.2 High-Level Injection

3 Transient Response of PN Junctions
   3.1 Charge Control Approach
   3.2 Turn-Off Transient
   3.3 Turn-On Transient

4 AC Response of PN Junctions
   4.1 Modeling the Admittance of a PN Junction
   4.2 Admittance Under Reverse Bias
   4.3 Admittance Under Forward Bias

1 Overview

In this lecture we finish up the discussion of non-ideal effects in pn junctions by discussing high current phenomena – high level injection and series resistance. We then discuss the temporal response – transient response and ac response – of a pn junction diode.

Topics to cover include:

- High Level Injection
- Series Resistance
- Transient Response
- AC Response

2 High Current Phenomena in PN Junctions

When \( V_A \rightarrow \phi_{bi} \), the forward current in a diode can become quite large. Considering that \( \phi_{bi} \) can be between 4 and 8 times larger than the thermal voltage, \( k_b T / e \), the forward current can be anywhere from \( e^4 \) to \( e^8 \) times larger than the reverse saturation current. At large currents, non-ideal effects due to high-level-injection (HLI) and series resistance become non-negligible and must be addressed in the diode equation.

2.1 Series Resistance

2.1.1 Physical Origin

Series resistance accounts for Ohmic losses that appear in series with the diode current. These lead to parasitic voltages drops that occur internally within the diode, between the anode and cathode of the battery terminals. Usually, series resistance comes from the metal or highly doped regions used to form contacts or the resistance of the quasi-neutral regions.
2.1.2 Mathematically Accounting for Series Resistance

Regardless of where Ohmic losses occur within the diode, their net effect will be perceived as a single resistance added in series to the ideal diode. Therefore, we can define this single resistance as $R_S$:

$$R_S \equiv \text{The net effect of all Ohmic losses in the diode}$$

This additional resistance, $R_S$ contributes additional internal voltage drops within the diode that effectively reduce the applied voltage from its nominal value of $V_A$ to $V_A - IR_S$:

$$V_A \rightarrow V_A - IR_S$$

The diffusion current therefore becomes:

$$I_{diff} = I_0 \left( \exp \left( \frac{eV_A}{k_bT} \right) - 1 \right)$$

$$= I_0 \left( \exp \left( \frac{e(V_A - IR_S)}{k_bT} \right) - 1 \right)$$

From this expression, it is clear that the additional term $IR_S$ has an increasing effect as its magnitude increases. The effect of increasing $IR_S$ is to effectively reduce $V_A$. It is as if the operating point shifts to a lower “effective” value. Since, a lower $V_A$, effective or not, reduces the current, the total current is expected to reduce.

$$V_{A,eff} \equiv V_A - IR_S$$

$$I_{diff,eff} \equiv I_0 \left( \exp \left( \frac{eV_{A,eff}}{k_bT} \right) - 1 \right)$$

$$\uparrow I \implies \uparrow IR_S \implies \downarrow V_{A,eff} \implies \downarrow I_{diff,eff}$$

This can be shown explicitly by taking the limit as $I \rightarrow V_A/R_S$, which is the limiting value of $I$, since this corresponds to all of the forward voltage drop occurring across the series resistance. As $IR_S$ asymptotically approaches $V_A/R_S$, $V_{A,eff} \rightarrow 0$, therefore we can perform a Taylor series expansion of the exponential term:

$$\exp \left( \frac{eV_{A,eff}}{k_bT} \right) - 1 \approx \frac{eV_{A,eff}}{k_bT}$$

The approximation for the diffusion current in this limit is therefore given by:

$$I_{diff,eff} \approx I_0 \frac{eV_{A,eff}}{k_bT}$$

As one might have guessed without doing any math, in the limit that the series resistance dominates the diode characteristics, one obtains a linear I-V characteristic! The diode effectively behaves as a resistor. In practice, diodes are never so poorly designed as to behave like resistors, and we typically exhibit only a slight reduction (or droop) in the IV characteristics above a certain applied potential as shown in Figure 1.
2.2 High-Level Injection

High-level injection is the antithesis to low-level injection. Under high-level injection, the excess carrier concentrations we “inject” under forward biasing conditions are large enough to modify the electrostatics of majority carriers in the quasi-neutral regions.

2.2.1 Revisiting Low-Level Injection

Under low-level injection, the concentration of excess minority carriers is large enough to modify the quasi-Fermi level of minority carriers but too small to modify the quasi-Fermi level of majority carriers. We expressed this mathematically as follows, for the n-side of the junction:

\[ n = n_0 + \Delta n \approx n_0 \]
\[ p = p_0 + \Delta p \approx \Delta p \]

...as well as the p-side of the junction:

\[ n = n_0 + \Delta n \approx \Delta n \]
\[ p = p_0 + \Delta p \approx p_0 \]
If this isn’t crystal clear at this point, we can be more explicit by quantifying the above expressions in terms of the quasi-Fermi levels:

$$E_{F,n} = E_i + k_b T \ln \left( \frac{n}{n_i} \right) = E_i + k_b T \ln \left( \frac{n_0 + \Delta n}{n_i} \right)$$

$$E_{F,p} = E_i - k_b T \ln \left( \frac{p}{p_i} \right) = E_i - k_b T \ln \left( \frac{p_0 + \Delta p}{n_i} \right)$$

Using the expressions above for the carrier concentrations in the respective regions, the quasi-Fermi levels on the n-side are:

$$E_{F,n} \approx E_i + k_b T \ln \left( \frac{n_0}{n_i} \right) \approx E_F$$

$$E_{F,p} \approx E_i - k_b T \ln \left( \frac{p_0}{n_i} \right)$$

...and the p-side:

$$E_{F,n} \approx E_i + k_b T \ln \left( \frac{\Delta n}{n_i} \right)$$

$$E_{F,p} \approx E_i - k_b T \ln \left( \frac{p_0}{n_i} \right) \approx E_F$$

Thus, it is clear that, given these assumptions, the electrostatics of electrons on the n-side (majority carriers) and holes on the p-side (majority carriers) remain near their equilibrium values unperturbed by the presence of excess minority carriers. This is why we draw the quasi-Fermi levels in the quasi-neutral regions as flat – spatially constant – extending to the edges of the depletion region, even though there is a voltage applied!

### 2.2.2 High-Level Injection

Under conditions of **high-level injection**, the excess minority carrier concentrations are no longer negligible in comparison to the majority carrier concentrations. We can understand the effects of high-level injection by considering a limiting case in which the excess minority carrier concentrations are much larger than both the equilibrium electron and hole concentrations.

$$n = n_0 + \Delta n \approx \Delta n$$

$$p = p_0 + \Delta p \approx \Delta p$$

Since charge neutrality conditions apply, $\Delta p = \Delta n$ such that $n \approx p$. Revisiting our law of the junction – the non-equilibrium law of mass action.

$$np = n^2 = p^2 = n_i^2 \exp \left( \frac{eV_A}{k_b T} \right)$$

Therefore, the charge carrier concentrations increase with a modified voltage dependence:

$$n = p = n_i \exp \left( \frac{eV_A}{2k_b T} \right)$$
The diffusion current is a linear function of $n$ and $p$ and therefore will depend similarly on the applied voltage in this limiting case.

\[
I_{\text{diff},n} = eD_n \left( \frac{\partial \Delta n}{\partial x} \right)_{x=0} \propto \frac{\Delta n}{L_n} \propto \exp \left( \frac{eV_A}{2k_bT} \right) - 1
\]

\[
I_{\text{diff},p} = -eD_p \left( \frac{\partial \Delta p}{\partial x} \right)_{x=0} \propto \frac{\Delta p}{L_p} \propto \exp \left( \frac{eV_A}{2k_bT} \right) - 1
\]

\[
I_{\text{diff}} = I_{\text{diff},n} + I_{\text{diff},p} \propto \exp \left( \frac{eV_A}{2k_bT} \right) - 1
\]

Thus, on a semi-log plot ($\log(I)$ vs. $V_A$), one observes a reduction in slope from the ideal value of $\frac{e}{k_bT}$, to the value $\frac{e}{2k_bT}$ as shown in Figure 2.

3 Transient Response of PN Junctions

The transient response of a pn junction is both of fundamental and practical importance. Practically speaking, since pn junction diodes can be used to rectify large switching voltages, we are interested in its large-signal transient response to voltages driving the pn-junction from forward bias to reverse bias and vice versa. In particular, we would like to understand the intrinsic delays involved during transitions from the “on” to “off” state (so-called turn-off transient) and the “off” to “on” state (so-called turn-on transient). The intrinsic delays are related to the rates at which we can restore equilibrium within the pn-junction, either through local thermal generation and recombination or through charge transport.

3.1 Charge Control Approach

The charge control approach is central to the theory of the transient response of a pn-junction diode. The charge-control approach focuses on the action of excess minority carriers in the quasi-
neutral regions. Restricting ourselves to the quasi-neutral region, and in the absence of light, the starting point for relating carrier dynamics to quantities of interest are the minority carrier diffusion equations for electrons and holes.

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} - \frac{\Delta n}{\tau_n}
\]

\[
\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau_p}
\]

We can integrate these equations over the respective quasi-neutral regions, recalling that, in the wide-base approximation, the p-side quasi neutral region extends from \(-\infty\) to 0 and the n-side quasi-neutral region extends from 0 to \(\infty\):

\[
\int_{-\infty}^{0} \frac{\partial \Delta n}{\partial t} \, dx = \int_{-\infty}^{0} D_n \frac{\partial^2 n}{\partial x^2} \, dx - \int_{-\infty}^{0} \frac{\Delta n}{\tau_n} \, dx
\]

\[
\int_{0}^{\infty} \frac{\partial \Delta p}{\partial t} \, dx = \int_{0}^{\infty} D_p \frac{\partial^2 p}{\partial x^2} \, dx - \int_{0}^{\infty} \frac{\Delta p}{\tau_p} \, dx
\]

These can be re-arranged as follows:

\[
\frac{\partial}{\partial t} \left( \int_{-\infty}^{0} \Delta ndx \right) = \int_{-\infty}^{0} \frac{\partial}{\partial x} \left( D_n \frac{\partial n}{\partial x} \right) \, dx - \frac{1}{\tau_n} \left( \int_{-\infty}^{0} \Delta ndx \right)
\]

\[
\frac{\partial}{\partial t} \left( \int_{0}^{\infty} \Delta pdx \right) = \int_{0}^{\infty} \frac{\partial}{\partial x} \left( D_p \frac{\partial p}{\partial x} \right) \, dx - \frac{1}{\tau_p} \left( \int_{0}^{\infty} \Delta pdx \right)
\]

... where it has been assumed that the diffusion constants (and also carrier mobility) are spatially invariant. At this point, we can introduce the following new definitions of the total excess minority charge in each region:

\[Q_p \equiv \text{Total Excess Charge due to Holes on the n-side} = eA \int_{0}^{\infty} \Delta pdx\]

\[Q_n \equiv \text{Total Excess Charge due to Electrons on the p-side} = -eA \int_{-\infty}^{0} \Delta ndx\]

...and recalling the definitions for the diffusion current as:

\[I_{\text{diff},n} = AJ_{\text{diff},n} = eAD_n \frac{\partial n}{\partial(-x)} = -eAD_n \frac{\partial n}{\partial x}\]

\[I_{\text{diff},p} = AJ_{\text{diff},p} = -eAD_p \frac{\partial p}{\partial x}\]

Note: the minus sign in front of the spatial variable \(x\) in the definition of the electron diffusion current, comes from our choice of device geometry. Electrons diffuse right to left, creating a current.
from left to right. It is straightforward to re-write the differential equations above as:

\[
-\frac{1}{eA} \frac{\partial Q_n}{\partial t} = -\frac{1}{eA} \int_{-\infty}^{0} \frac{\partial I_{\text{diff},n}}{\partial x} dx + \frac{1}{eA} \frac{Q_n}{\tau_n} \\
\frac{1}{eA} \frac{\partial Q_p}{\partial t} = -\frac{1}{eA} \int_{0}^{\infty} \frac{\partial I_{\text{diff},p}}{\partial x} dx - \frac{1}{eA} \frac{Q_p}{\tau_p}
\]

After eliminating the constant factors and performing the integration, noting that the diffusion current must vanish infinitely far from the junctions:

\[
\frac{\partial Q_n}{\partial t} = I_{\text{diff},n} - \frac{Q_n}{\tau_n} \\
\frac{\partial Q_p}{\partial t} = I_{\text{diff},p} - \frac{Q_p}{\tau_p}
\]

Adding these two expressions together yields the following:

\[
\frac{\partial Q}{\partial t} \approx I_{\text{diff}} - \frac{Q}{\tau_{np}}
\]

Where we have defined the following quantities:

\[
Q \equiv Q_n + Q_p \\
I_{\text{diff}} \equiv I_{\text{diff},n} + I_{\text{diff},p} \\
\frac{1}{\tau_{np}} \equiv \frac{1}{\tau_n} + \frac{1}{\tau_p}
\]

Typically, we are restricted to dynamics limited by one type of carrier. In which case we can work directly with the charge-control expressions for electrons or holes without an effective time constant. This expression represents the dynamics of the excess minority charge in the quasi-neutral regions. We can now start to use this expression to evaluate the transient response.

### 3.2 Turn-Off Transient

We begin by analyzing a pn junction in which a forward current, \(I_F\) is initially flowing for some time \(t = -\infty \rightarrow 0\), corresponding to some large forward bias, \(V_F\), applied across the device. At a time \(t = 0\), the applied voltage is switched abruptly from a large forward bias, \(V_F\) to a large reverse bias, \(V_R\). This switching causes the excess charges stored in the quasi-neutral regions to be removed from the device through thermal recombination as well as through conduction. Once the excess charges are removed, the diode current will decay to its steady-state value at the corresponding reverse bias voltage, which should be close to the reverse saturation current of the diode \(-I_0\). We assume that, in both cases, the magnitude of the applied biases are much larger than the forward voltage drop across the diode. This ensures that, immediately after switching, the diode reverse bias current is constant.

\[
I_R \approx \frac{V_R}{R_R}
\]

The diode current during the time period in which the excess charge is being extracted is constant, \(I_{\text{diff}} = -I_R\). The charge control expression can be solved during this time period of charge
The charge $Q(0)$ is the stored charge, immediately after switching. From the charge control expression, this has to be equal to the value which existed prior to switching under steady-state conditions.

$$\frac{\partial Q}{\partial t} = 0$$

$$= I_F - \frac{Q(-\infty)}{\tau_{np}}$$

$$Q(0) = Q(-\infty) = I_F \tau_{np}$$

The charge storage time, $t_s$ is therefore given by:

$$t_s = \tau_{np} \ln \left( 1 + \frac{I_F}{I_R} \right)$$

### 3.3 Turn-On Transient

We begin by analyzing a pn junction in which a forward current, $I_F$ is initially flowing for some time $t = -\infty \to 0$, corresponding to some large forward bias, $V_F$, applied across the device. At a time $t = 0$, the applied voltage is switched abruptly from a large forward bias, $V_F$ to a large reverse bias, $V_R$. This switching causes the excess charges stored in the quasi-neutral regions to be removed from the device through thermal recombination as well as through conduction. Once the excess charges are removed, the diode current will decay to its steady-state value at the corresponding reverse bias voltage, which should be close to the reverse saturation current of the diode $-I_0$. We assume that, in both cases, the magnitude of the applied biases are much larger than the forward voltage drop across the diode. This ensures that, immediately after switching, the diode reverse bias current is constant.

We now analyze a pn junction in which a reverse bias current $-I_0$ is initially flowing for some
time \( t = -\infty \to 0 \), corresponding to some large reverse bias, \( V_R \), applied across the device. At a time \( t = 0 \), the applied voltage is switched abruptly from a large reverse bias to a large forward bias, \( V_F \). This switching causes excess charges to accumulate in the quasi-neutral regions via thermal generation and recombination and conduction. We begin with the charge control expression as before.

\[
\frac{\partial Q}{\partial t} = I_{diff} - \frac{Q}{\tau_{np}}
\]

For \( t < 0 \), the diode is reverse biased. Therefore, the current flowing in the diode in this time interval is due to the reverse saturation current. Under steady-state conditions:

\[
0 = -I_0 - \frac{Q}{\tau_{np}}
\]

\[
Q(-\infty) = Q(0) = -I_0\tau_{np}
\]

This initial charge is quite small, since the reverse saturation current is a small quantity. As a consequence, one can choose to approximate it to be zero without much loss in generality.

\[
Q(-\infty) = Q(0) \approx 0
\]

After switching from reverse bias to forward bias at \( t = 0 \), the excess charges will accumulate at the edges of the depletion region. At an infinite time after switching, a positive forward current \( I_F \) flows under a new steady-state condition:

\[
0 = I_F - \frac{Q}{\tau_{np}}
\]

\[
Q(\infty) = I_F\tau_{np}
\]

Thus, it is expected that the excess charge increases from \( \approx 0 \) to an asymptotic value of \( I_F\tau_{np} \) as \( t \to \infty \). A reasonable guess at the solution for \( Q(t) \) is therefore:

\[
Q(t) = I_F\tau_{np}\left(1 - \exp\left(-\frac{t}{\tau_{np}}\right)\right)
\]

This can be shown by solving the charge-control expression assuming that the forward current during switching is constant. This assumption is strictly true if a pulsed current source is used to probe the switching dynamics, but it also follows naturally from our assumption that the forward voltage drop across the diode is negligible, which is to say that the forward current is determined primarily by the external circuitry.

\[
I_F \approx \frac{V_F}{R_F}
\]

The steady-state charge is given by:

\[
Q = I_{diff}\tau_{np} = \tau_{np}I_0\left(\exp\left(\frac{eV_A}{k_BT}\right) - 1\right)
\]

If the steady-state value of the applied voltage \( V_A \) is replaced by a time-dependent one \( v_A(t) \), we
have the following:

\[ Q(t) = \tau_{np}I_0 \left( \exp \left( \frac{e v_A(t)}{k_b T} \right) - 1 \right) \]

If we equate this value to the previous expression for the time-dependent solution to the charge-control equation

\[ Q(t) = I_F \tau_{np} \left( 1 - \exp \left( -\frac{t}{\tau_{np}} \right) \right) = \tau_{np}I_0 \left( \exp \left( \frac{e v_A(t)}{k_b T} \right) - 1 \right) \]

Solving this expression for \( v_A(t) \) yields the following:

\[ v_A(t) = \frac{k_b T}{e} \ln \left( 1 + \frac{I_F}{I_0} \left( 1 - \exp \left( -\frac{t}{\tau_{np}} \right) \right) \right) \]

Thus, the forward drop across the diode, \( v_A(t) \) acquires its steady-state value asymptotically, as \( t \to \infty \). It is important to note that this expression was derived by simply replacing a steady-state quantity \( V_A \) with a time-dependent quantity \( v_A(t) \). Replacements such as these are tantamount to assuming that the applied voltage varies quasi-statically such that steady-state equilibrium conditions are maintained at every increment in time. In other words, \( v_A(t) \) must vary slowly with time for this approach to be valid, or equivalently, thermal generation and recombination rates must be sufficiently high to maintain steady-state conditions for electrons and holes.

4 AC Response of PN Junctions

4.1 Modeling the Admittance of a PN Junction

In modeling the AC response of the pn-junction, we assume a periodic sinusoidal function of the form \( v_A(t) = V_0 \sin \omega t \) applied in series to a DC voltage \( V_A \). When \( V_0 \ll V_A \), the AC component is referred to as a small-signal, in which the net effect of \( v_A(t) \) is to generate small oscillations about a bias point defined by \( V_A \). Since we are interested, thus far, in the diode current in response to a voltage, we choose a formulation of the diode in terms of its admittance, \( Y \), defined as follows:

\[ Y = G + j\omega C \]

\( G \equiv \) Diode Conductance

\( C \equiv \) Diode Capacitance

In the sections that follow, we will analyze the admittance under conditions of forward and reverse bias.

4.2 Admittance Under Reverse Bias

Under conditions of reverse bias, the depletion width widens and most of the voltage drop \( V_R \) occurs across the depletion region, since this region is devoid of free carriers and thus contributes a large effective resistance (small conductance) to the overall response.

\[ G \equiv \frac{\partial I}{\partial V_A} \approx \frac{\partial}{\partial V_A}(-I_0) = 0 \]
As a consequence, the admittance of a diode under reverse bias is functionally equivalent to a capacitor, albeit a special type of capacitor in which the capacitance varies as a function of voltage – varactor. Approximating the depletion region as a parallel plate capacitor:

\[ C_{\text{dep}} = \frac{\varepsilon_s A}{W_{\text{dep}}} \]  

... this expression is commonly referred to as the junction capacitance, depletion-layer capacitance. The voltage dependence comes from the voltage dependence of the depletion width \( W \).

\[ C_{\text{dep}} = \frac{\varepsilon_s A}{\sqrt{2\varepsilon_s(\phi_{bi} - V_A)}} \]  

Proof of this functional dependence on voltage is evaluated experimentally by measuring capacitance vs. voltage (aka CV) plotting \( \frac{1}{C_{\text{dep}}} \) vs. \( V_A \), which should produce a line with slope and intercept defined as:

\[
\text{slope} = -\frac{2}{eN\varepsilon_s A^2} \\
\text{y-intercept} = \frac{2}{eN\varepsilon_s A^2}\phi_{bi} \\
\text{x-intercept} = \phi_{bi}
\]

These relationships are useful for experimentally measuring the built-in potential (using the x-intercept) and the doping concentration (using the slope). Thus, step-junction diodes can be used as a simple device to probe electrical properties of materials.

**How might I measure the doping concentration vs. position using CV data?**

Small-Signal Admittance of a PN Junction Under Reverse Bias:

\[ Y = j\omega C_{\text{dep}} = j\omega \frac{\varepsilon_s A}{\sqrt{2\varepsilon_s(\phi_{bi} - V_A)}} \]

### 4.3 Admittance Under Forward Bias

Under forward biasing conditions, the excess charge carriers build-up at the depletion region boundaries. Small oscillations about a given biasing point defined by \( V_A \) lead to oscillations in minority carrier concentrations. Unlike the case of reverse biasing, these oscillations are non-negligible under forward bias. Since the presence of minority carriers on either side of the junction is due to diffusion, the admittance under forward bias is referred to as a diffusion admittance. The diffusion admittance, \( Y_D \), defined in terms of diffusion conductance, \( G_D \), and diffusion capacitance, \( C_D \), is shown below:

\[ Y = Y_D = G_D + j\omega C_D \]
Using the definition of admittance, we seek to evaluate the real and imaginary parts of the derivative of the small signal current, \( i \) with respect to the small signal voltage \( v_A \). We start by considering the minority carrier diffusion equations.

\[
\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} \\
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n}
\]

The application of an ac voltage superimposed to a DC bias voltage gives rise to in-phase (conductance) and out-of-phase (reactance) responses of the excess minority carriers.

\[
\Delta p = \Delta \bar{p} + \Delta \tilde{p} \exp(j\omega t) \\
\Delta \bar{p} \equiv \text{DC component of excess hole concentration} \\
\Delta \tilde{p} \equiv \text{AC component of excess hole concentration}
\]

Substituting this expression into the minority carrier diffusion equation yields the following:

\[
\frac{\partial}{\partial t} (\Delta \bar{p} + \Delta \tilde{p} \exp(j\omega t)) = D_p \frac{d^2}{dx^2} (\Delta \bar{p} + \Delta \tilde{p} \exp(j\omega t)) - \frac{1}{\tau_p} (\Delta \bar{p} + \Delta \tilde{p} \exp(j\omega t))
\]

Performing the differentiation and collecting all terms associated with the DC and AC components, and applying steady-state conditions we can write:

\[
0 = \frac{\partial \Delta \bar{p}}{\partial t} = D_p \frac{d^2 \Delta \bar{p}}{dx^2} - \frac{\Delta \bar{p}}{\tau_p} \\
0 = \frac{\partial \Delta \tilde{p}}{\partial t} = D_p \frac{d^2 \Delta \tilde{p}}{dx^2} - \frac{\Delta \tilde{p}}{\tilde{\tau}_p}
\]

Where the \( \tilde{\tau}_p \) is defined as:

\[
\tilde{\tau}_p = \frac{\tau_p}{1 + j\omega \tau_p}
\]

The boundary condition at the depletion region edge \( x = 0 \) becomes:

\[
\Delta p = \Delta \bar{p} + \Delta \tilde{p} = \frac{n_i^2}{N_D^+} \left( \exp \left( \frac{e(V_A + v_A)}{k_bT} \right) - 1 \right)
\]

Since we know that the DC solution has to be...

\[
\Delta \tilde{p} = \frac{n_i^2}{N_D^+} \exp \left( \frac{e(V_A)}{k_bT} \right) \left( \exp \left( \frac{ev_A}{k_bT} \right) - 1 \right)
\]

...after some algebra we can write the AC component as:

\[
\Delta \tilde{p} = \frac{n_i^2}{N_D^+} \exp \left( \frac{e(V_A)}{k_bT} \right) \left( \exp \left( \frac{ev_A}{k_bT} \right) - 1 \right)
\]
The AC amplitude is typically very small during a measurement. In such a case we can approximate the exponential by its first two terms of the Taylor series:

\[ \Delta\tilde{\rho} \approx \frac{n_i^2}{N_D^+} \exp\left(\frac{e(V_A)}{k_BT}\right) \left(\frac{eV_A}{k_BT}\right) \]

The DC diffusion current is given by the usual result:

\[ I_{\text{diff}} = eA \frac{D_p}{L_p} \frac{n_i^2}{N_D^+} \left(\exp\left(\frac{eV_A}{k_BT}\right) - 1\right) \]

\[ = eA \sqrt{\frac{D_p}{\tau_p} \frac{n_i^2}{N_D^+}} \left(\exp\left(\frac{eV_A}{k_BT}\right) - 1\right) \]

By comparison, the AC diffusion current has a form:

\[ i_{\text{diff}} = eA \sqrt{\frac{D_p}{\tau_p} \frac{n_i^2}{N_D^+}} \exp\left(\frac{eV_A}{k_BT}\right) \left(\frac{eV_A}{k_BT}\right) \sqrt{1 + j\omega\tau_p} \]

The admittance is therefore:

\[ Y = \frac{\partial i}{\partial v_A} = G_0 \sqrt{1 + j\omega\tau_p} \]

\[ G_0 \equiv \frac{eI_0}{k_BT} \exp\left(\frac{eV_A}{k_BT}\right) \]

When separating the expression for the diffusion admittance into real and imaginary parts, we obtain the diffusion conductance, \( G_D \) and diffusion capacitance, \( C_D \):

\[ G_D = \frac{G_0}{\sqrt{2}} \left(\sqrt{1 + (\omega\tau_p)^2} + 1\right)^{1/2} \]

\[ C_D = \frac{G_0}{\omega \sqrt{2}} \left(\sqrt{1 + (\omega\tau_p)^2} - 1\right)^{1/2} \]