Constant-G_m Biasing

Recall that using a PTAT current reference (see top of p. 66 in the notes) to bias a bipolar transistor provides constant transconductance over temperature (and also independent of supply voltage and process). How might we achieve similar results in CMOS analog circuits? Using the beta multiplier current reference.

As you may recall, the beta multiplier current reference (Figure 21.16-17 in your text – p. 69-70 in your notes), when operating in strong inversion saturation, generates an output current described by

$$I_{ref} = \frac{2}{R^2 \beta_1} \cdot \left(1 - \sqrt{\frac{1}{K}}\right)^2$$

Then a MOSFET (let's call it device M_A) biased by this current and operating in strong inversion saturation would have a transconductance described by

$$g_{mA} = \sqrt{2\beta_A I_{ref}}$$

Substituting in for I_{ref},

$$g_{mA} = \sqrt{2\beta_A \cdot \left(\frac{2}{R^2 \beta_1} \cdot \left(1 - \sqrt{\frac{1}{K}}\right)^2\right)} = \frac{2}{R} \cdot \left(1 - \sqrt{\frac{1}{K}}\right) \cdot \sqrt{\frac{\beta_A}{\beta_1}}$$

This result is independent of MOSFET parameters (such as μ and V_{TH}) and supply voltage. The variation in g_{mA} over temperature, however, is directly affected by the temperature coefficient of R. And, of course, the absolute accuracy of R directly impacts the absolute accuracy of g_{mA}.

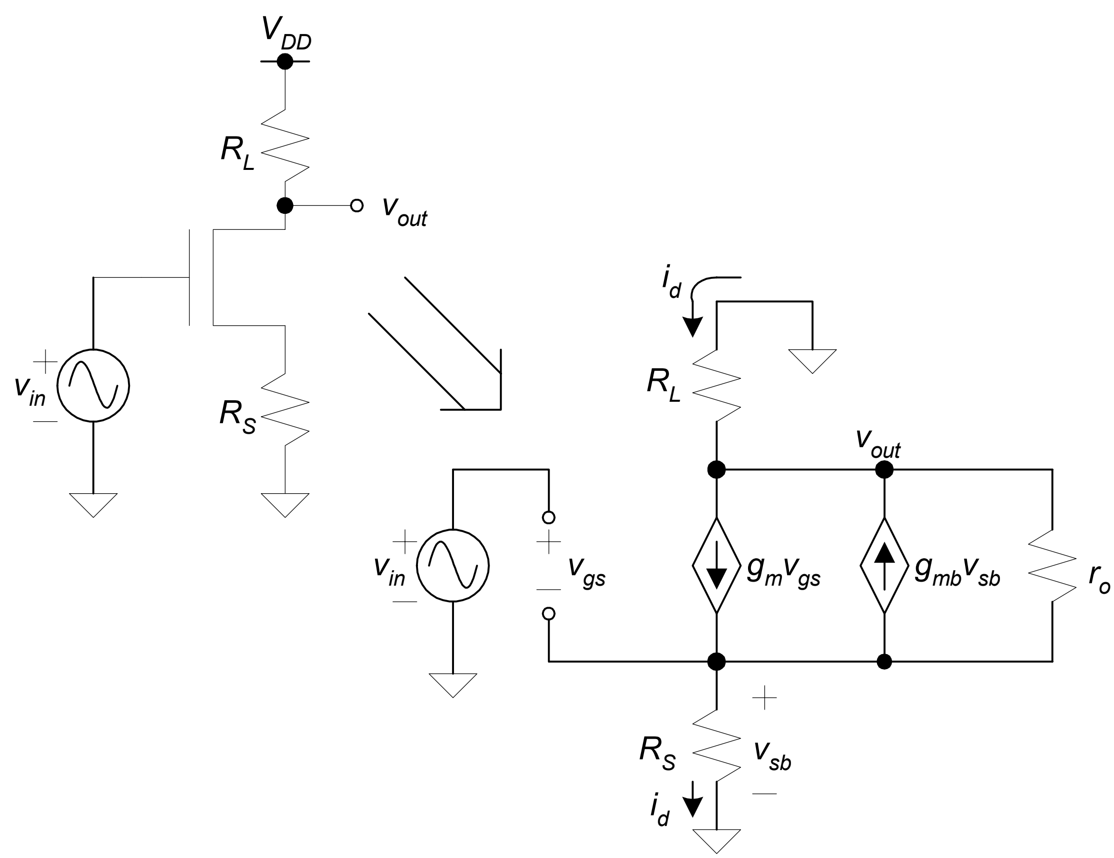
Amplifiers

There are 3 basic amplifier circuit topologies:

1) the inverting amplifier (common-source if $R_S = 0\Omega$),
2) the common-gate amplifier, and
3) the common-drain amplifier (level-shifter/buffer).

To better understand the small-signal attributes of each of these configurations one must be intimately familiar with their respective small-signal models. Let's start by determining the small-signal voltage gain at mid-band for each amplifier topology.

<u>Inverting Amplifier</u> with source degeneration.



From the above mid-band small-signal circuit model we observe that

$$i_d = -\frac{v_{out}}{R_L} = \frac{v_{sb}}{R_S} \Rightarrow v_{sb} = -v_{out}\frac{R_S}{R_L}$$

and

$$v_{in} = v_{sb} + v_{gs}$$

Apply KCL at the drain node,

$$-\frac{v_{out}}{R_L} = g_m v_{gs} - g_{mb} v_{sb} + \frac{v_{out} - v_{sb}}{r_o}$$

Substituting in for v_{gs}, v_{sb}, and using $g_{mb} = \eta g_m$,

$$-\frac{v_{out}}{R_L} = g_m\left(v_{in} + v_{out}\frac{R_S}{R_L}\right) + \eta g_m v_{out}\frac{R_S}{R_L} + \frac{v_{out}}{r_o} + v_{out}\frac{R_S}{r_o R_L}$$

$$-v_{out}\left(\frac{1}{R_L} + g_m(1+\eta)\frac{R_S}{R_L} + \frac{1}{r_o} + \frac{R_S}{r_o R_L}\right) = g_m v_{in}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_m}{g_m(1+\eta)\frac{R_S}{R_L} + \frac{1}{R_L} + \frac{1}{r_o} + \frac{R_S}{r_o R_L}}$$

If $r_o >> R_L$, and $r_o R_L >> R_S$, then

$$\frac{v_{out}}{v_{in}} \cong \frac{-g_m R_L}{1 + g_m(1+\eta)R_S} \qquad \text{at mid-band.}$$

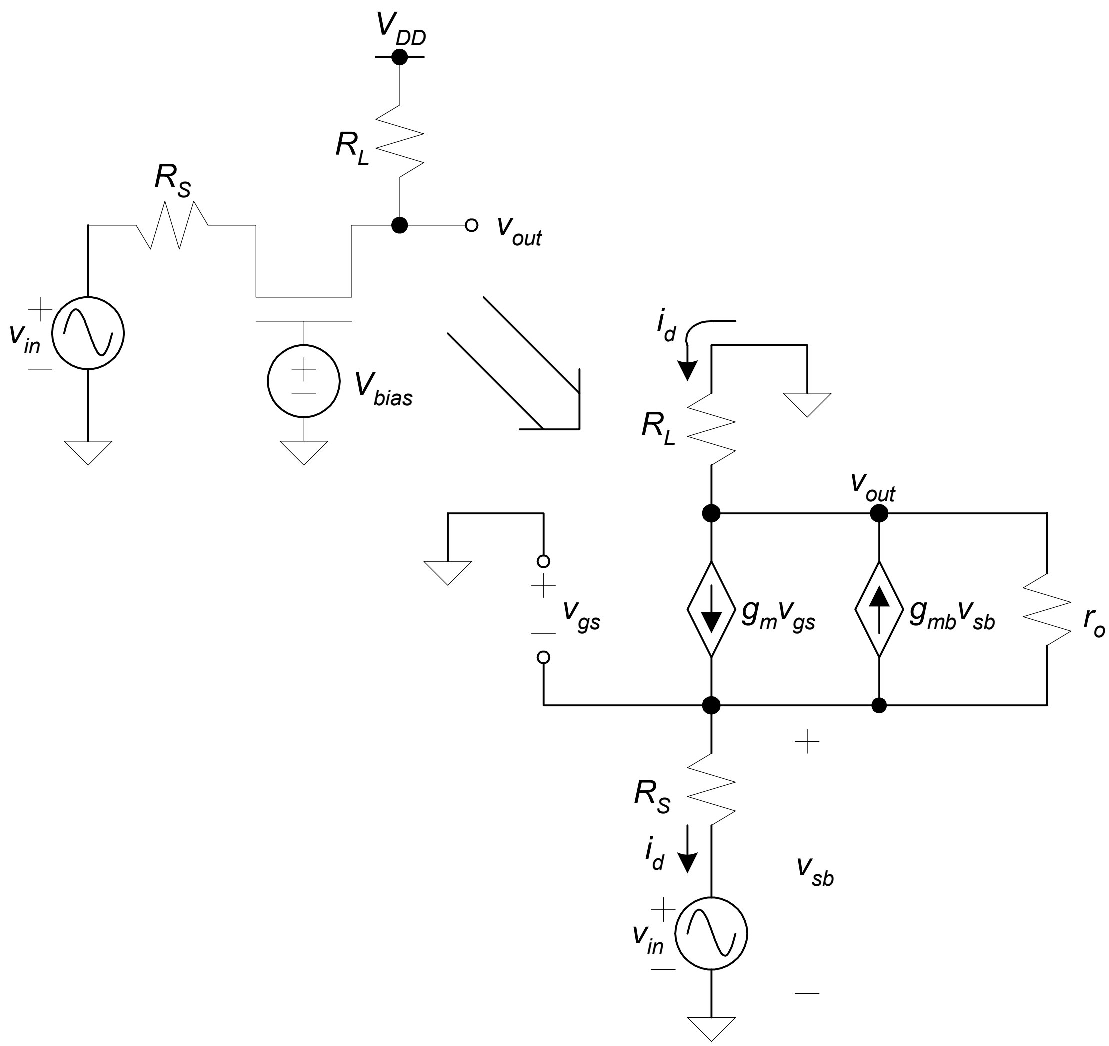
If we can also neglect body effect, then

$$\frac{v_{out}}{v_{in}} \approx \frac{-g_m R_L}{1 + g_m R_S}$$

If $R_s = 0\Omega$ (no source degeneration, i.e., common-source) $\Rightarrow \frac{v_{out}}{v_{in}} \cong -g_m R_L$.

Or, if $g_m R_S >> 1 \Rightarrow \frac{v_{out}}{v_{in}} \approx -\frac{R_L}{R_S}$ (linear gain).

Common-Gate Amplifier



From the midband small-signal circuit model we observe that

$$i_d = \frac{v_{sb} - v_{in}}{R_S} = -\frac{v_{out}}{R_L} \Rightarrow v_{sb} = v_{in} - v_{out}\frac{R_S}{R_L}$$

and

$$v_{gs} = -v_{sb}$$

Apply KCL at the drain node,

$$-\frac{v_{out}}{R_L} = g_m v_{gs} - g_{mb} v_{sb} + \frac{v_{out} - v_{sb}}{r_o}$$

Substituting in for v_{gs}, v_{sb}, and using $g_{mb} = \eta g_m$,

$$-\frac{v_{out}}{R_L} = -g_m\left(v_{in} - v_{out}\frac{R_S}{R_L}\right) - \eta g_m\left(v_{in} - v_{out}\frac{R_S}{R_L}\right) + \frac{v_{out}}{r_o} - \frac{v_{in}}{r_o} + v_{out}\frac{R_S}{r_o R_L}$$

$$-v_{out}\left(\frac{1}{R_L} + g_m(1+\eta)\frac{R_S}{R_L} + \frac{1}{r_o} + \frac{R_S}{r_o R_L}\right) = -v_{in}\left[g_m(1+\eta) + \frac{1}{r_o}\right]$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m(1+\eta) + \frac{1}{r_o}}{\frac{1}{R_L} + g_m(1+\eta)\frac{R_S}{R_L} + \frac{1}{r_o} + \frac{R_S}{r_o R_L}}$$

If r_o is very large and $r_o R_L >> R_S$, then

$$\frac{v_{out}}{v_{in}} \cong \frac{g_m(1+\eta)R_L}{1 + g_m(1+\eta)R_S}$$ at mid-band.

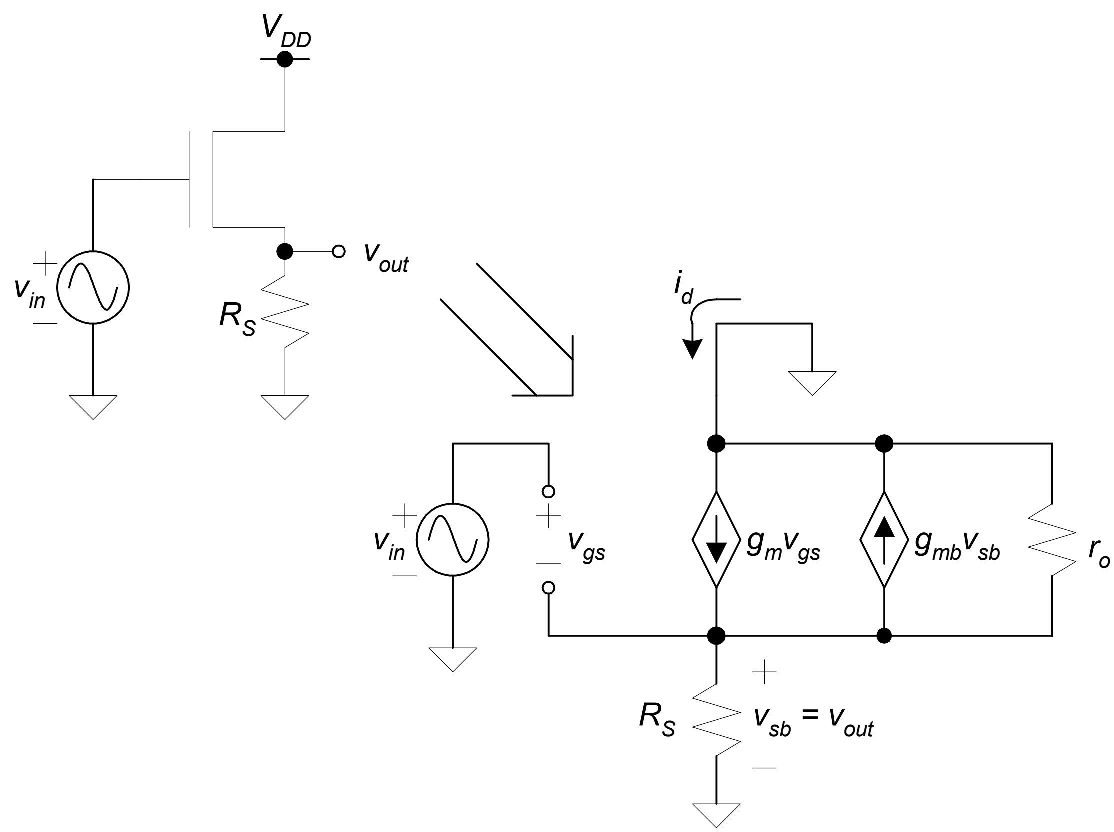
If we can also neglect body effect, then

$$\frac{v_{out}}{v_{in}} \approx \frac{g_m R_L}{1 + g_m R_S}$$

If $R_s = 0\Omega$ (i.e., driven by an "ideal" voltage source) $\Rightarrow \frac{v_{out}}{v_{in}} \cong g_m R_L$.

Or, if $g_m R_S >> 1 \Rightarrow \frac{v_{out}}{v_{in}} \approx \frac{R_L}{R_S}$ (linear gain).

Common-Drain Amplifier (level-shifter/buffer) — the Source Follower



From the midband small-signal circuit model we observe that

$$v_{in} = v_{gs} + v_{out}$$

and

$$v_{sb} = v_{out}$$

Apply KCL at the source node,

$$\frac{v_{out}}{R_S} = g_m v_{gs} - g_{mb} v_{sb} - \frac{v_{sb}}{r_o}$$

Substituting in for v_{gs}, v_{sb}, and using $g_{mb} = \eta g_m$,

$$\frac{v_{out}}{R_S} = g_m (v_{in} - v_{out}) - \eta g_m v_{out} - \frac{v_{out}}{r_o}$$

$$v_{out}\left(\frac{1}{R_S} + g_m(1+\eta) + \frac{1}{r_o}\right) = g_m v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m}{\frac{1}{R_S} + g_m(1+\eta) + \frac{1}{r_o}}$$

If r_o is very large, then

$$\frac{v_{out}}{v_{in}} \cong \frac{g_m R_S}{1 + g_m(1+\eta)R_S} \qquad \text{at mid-band.}$$

If we can also neglect body effect, then

$$\frac{v_{out}}{v_{in}} \approx \frac{g_m R_S}{1 + g_m R_S}$$

which approaches unity, if $g_m R_S >> 1$.

Note that **all** three of the these amplifier configurations have an "effective" transconductance, G_m, described by

$$G_m \approx \frac{g_m}{1 + g_m R_S}$$

To summarize the mid-band small-signal voltage gain results for each of three amplifier configurations: for the **inverting amplifier** with source degeneration,

$$\frac{v_{out}}{v_{in}} \approx -G_m R_L;$$

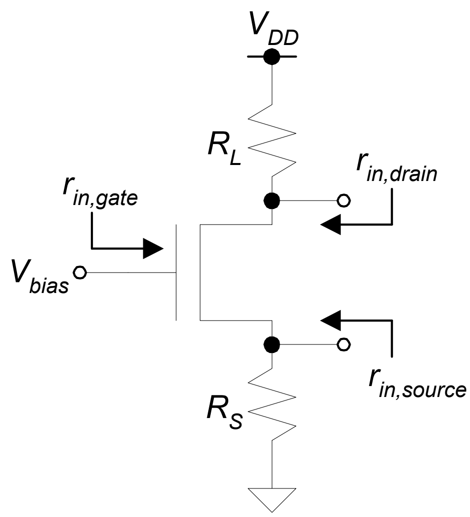
for the **common-gate amplifier**,

$$\frac{v_{out}}{v_{in}} \approx G_m R_L;$$

and for the **common-drain amplifier** (level-shifter/buffer),

$$\frac{v_{out}}{v_{in}} \approx G_m R_S.$$

Mid-band Small-Signal Impedance Analysis

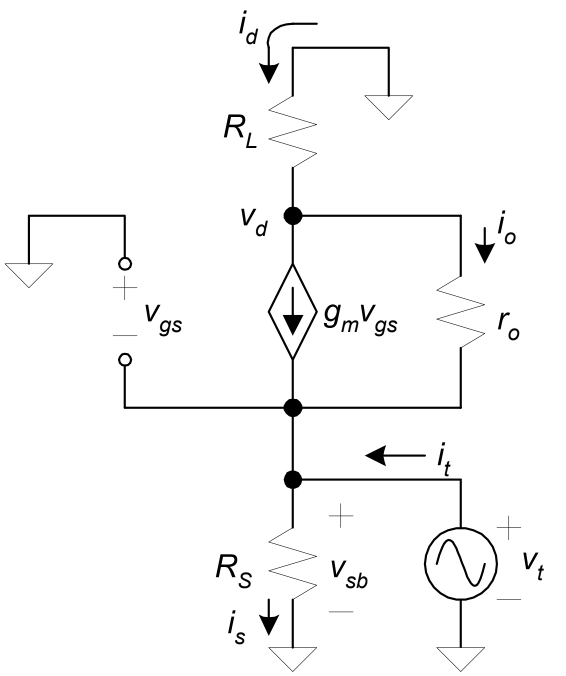


Looking into the gate, $r_{in,gate} \to \infty$. By looking into the drain,

$$r_{in,drain} \approx R_L \,\|\, g_m r_o R_S$$

using the results from previous the current sink/source discussion.

But, looking into the source, consider the mid-band small-signal model for the above circuit. For simplicity, body effect has been neglected.



$$r_{in,source} = \frac{v_t}{i_t}$$

Applying KCL at the source node,

$$i_d + i_t = i_s \Rightarrow \qquad \text{[Eqn. 1]}$$

and at the drain,

$$i_d = i_o + g_m v_{gs} = \frac{v_d - v_t}{r_o} - g_m v_t = \frac{-i_d R_L}{r_o} - \frac{v_t}{r_o} - g_m v_t$$

$$i_d\left(1 + \frac{R_L}{r_o}\right) = -v_t\left(g_m + \frac{1}{r_o}\right) \qquad \text{[Eqn. 2]}$$

Substituting eqn. 1 into eqn. 2:

$$(i_s - i_t)\left(1 + \frac{R_L}{r_o}\right) = -v_t\left(g_m + \frac{1}{r_o}\right)$$

$$\left(\frac{v_t}{R_S} - i_t\right)\left(1 + \frac{R_L}{r_o}\right) = -v_t\left(g_m + \frac{1}{r_o}\right)$$

$$-i_t\left(1 + \frac{R_L}{r_o}\right) = -v_t\left(g_m + \frac{1}{r_o} + \frac{1}{R_S} + \frac{R_L}{r_o R_S}\right)$$

$$\frac{v_t}{i_t} = r_{in,source} = \frac{1 + \frac{R_L}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S} + \frac{R_L}{r_o R_S}}$$

For large R_S,

$$r_{in,source} \cong \frac{1 + \frac{R_L}{r_o}}{g_m + \frac{1}{r_o}}$$

and if $g_m >> \frac{1}{r_o}$, then

$$r_{in,source} \approx \frac{1}{g_m}\left(1 + \frac{R_L}{r_o}\right).$$

Active Loads

Using an active load, i.e., MOSFET(s), to replace a passive load resistor can dramatically reduced the required chip area for the circuit while also helping produce much higher gains (due to the potentially high small-signal resistance an active load can provide).

An active load can be implemented using a gate-drain connected (a.k.a. diode-connected) MOSFET or a current source/sink.

Amplifiers with gate-drain connected active loads tend to achieve large frequency bandwidths but low gain due to their relatively low output impedance. See Figure 22.1 for examples of MOS amplifiers using gate-connected active loads.

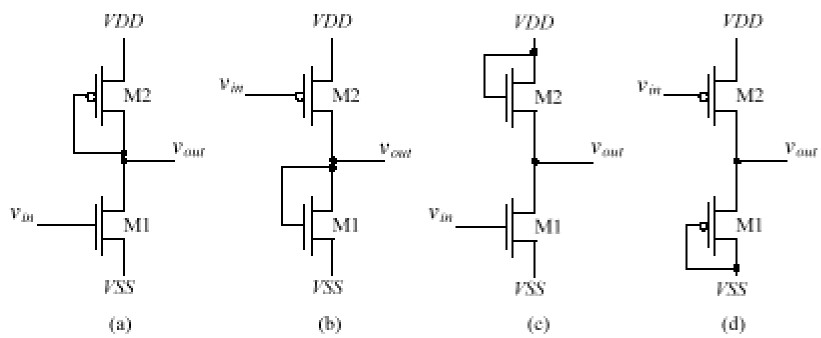


Figure 22.1 The four active load configurations available in CMOS.

Amplifiers with current source/sink active loads tend to achieve higher gain due to the high output impedance, but at the expense of bandwidth.

The cascode amplifier is well suited for achieving both gain and large bandwidth.

Back to the gate-drain connected active load amplifier, let us consider its frequency response in more detail. See the common-source amplifier in Figure 22.5. In this schematic the device capacitances associated with M1 and M2 are explicitly shown. Consequently, the circuit has two *RC* time constants — one at the input and the other at the output.

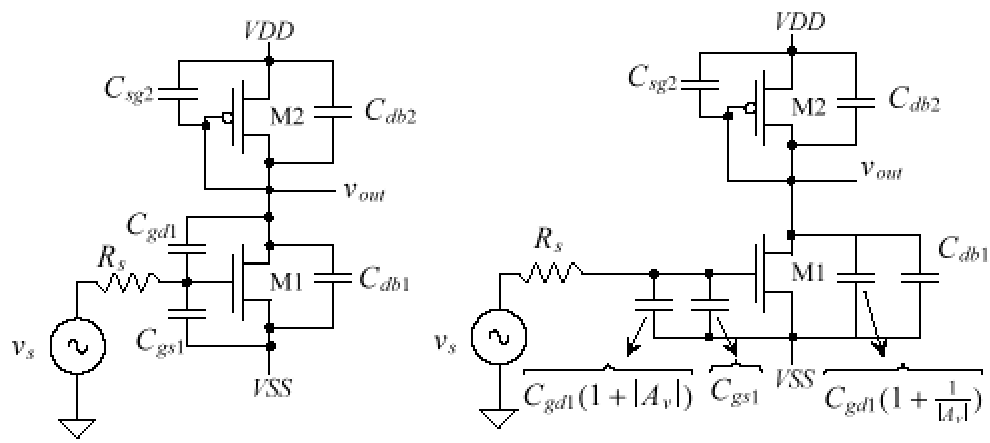


Figure 22.5 Frequency response of the simple common source amplifier with active load.

The input *RC* time constant is given by

$$\tau_{in} = R_s\left(C_{MI} + C_{gs1} + C_{gb}\right)$$

where

$$C_{MI} = C_{gd1}\left(1 - A_v\right) = C_{gd1}\left(1 + \frac{g_{m1}}{g_{m2}}\right)$$

Note the influence of Miller capacitance and driving source resistance, R_S, on this circuit's input time constant, τ_{in}.

At the output,

$$\tau_{out} = \frac{1}{g_{m2}} \cdot \left(C_{gs2} + C_{MO} + C_{db1} + C_{db2}\right)$$

where

$$C_{MO} = C_{gd1}\left(1 - \frac{1}{A_v}\right) = C_{gd1}\left(1 + \frac{g_{m2}}{g_{m1}}\right).$$

Therefore the amplifier's frequency response is described by

$$A_v(f) = \frac{-g_{m1}/g_{m2}}{\left(1 + j\frac{f}{f_{in}}\right)\left(1 + j\frac{f}{f_{out}}\right)}$$

where $f_{in} = (1/2\pi\,\tau_{in})$ and $f_{out} = (1/2\pi\,\tau_{out})$.