CS 311 — Fall 2011 — Homework 5 — Due Sept. 23 REVISED

1. The real number y is the *derivative* of the function f at the point $\bar{x}$ iff (if and only if) for every real number $\epsilon > 0$ there is a real number $\delta > 0$ such that for every real number x with $0 < |x - \bar{x}| < \delta$,

$$\left| \frac{f(x) - f(\bar{x})}{x - \bar{x}} - y \right| < \epsilon.$$

Write the *negation* of the proposition that y is the derivative of the function f at the point $\bar{x}$. Your answer should not contain the not-sign ($\neg$) nor the words "not," "no," "none," etc. (You can use either words or symbols.)

For each of the following, you are to prove the proposition, but show how you have developed your proof by the naming the methods (Forward-Backward, Construction, Choose, etc.) that you are using and labeling the steps A1, A2, ..., B1, B2, ... as we have done in class. *You will not receive full credit if you do not identify the methods and label the steps!*

2. A function f is *bounded above* iff there is a real number y such that, for all real numbers x, $f(x) \leq y$. Prove that $f(x) = -x^2 + 2x$ is bounded above.

3. If x is a real number satisfying $x^3 + 3x^2 - 9x - 27 \geq 0$, then $|x| \geq 3$. *Hint:* Express the conclusion as a disjunction.

4. If c is an odd integer, then $n^2 + n - c = 0$ has no integer solution for n.

5. A function f is a *convex function* iff for all real numbers x and y and for all real numbers t with $0 \leq t \leq 1$, it follows that

$$f[tx + (1-t)y] \leq tf(x) + (1-t)f(y).$$

Prove that if f is a convex function, then for all real numbers $s \geq 0$, $sf(x)$ is a convex function.