

Table 2.19

Rule of Inference	Related Logical Implication	Name of Rule
1) $\frac{p \quad p \rightarrow q}{\therefore q}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Rule of Detachment (Modus Ponens)
2) $\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Law of the Syllogism
3) $\frac{p \rightarrow q \quad \neg q}{\therefore \neg p}$	$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$	Modus Tollens
4) $\frac{p \quad q}{\therefore p \wedge q}$		Rule of Conjunction
5) $\frac{p \vee q \quad \neg p}{\therefore q}$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Rule of Disjunctive Syllogism
6) $\frac{\neg p \rightarrow F_0}{\therefore p}$	$(\neg p \rightarrow F_0) \rightarrow p$	Rule of Contradiction
7) $\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Rule of Conjunctive Simplification
8) $\frac{p}{\therefore p \vee q}$	$p \rightarrow p \vee q$	Rule of Disjunctive Amplification
9) $\frac{p \wedge q \quad p \rightarrow (q \rightarrow r)}{\therefore r}$	$[(p \wedge q) \wedge [p \rightarrow (q \rightarrow r)]] \rightarrow r$	Rule of Conditional Proof
10) $\frac{p \rightarrow r \quad q \rightarrow r}{\therefore (p \vee q) \rightarrow r}$	$[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$	Rule for Proof by Cases
11) $\frac{p \rightarrow q \quad r \rightarrow s \quad p \vee r}{\therefore q \vee s}$	$[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$	Rule of the Constructive Dilemma
12) $\frac{p \rightarrow q \quad r \rightarrow s \quad \neg q \vee \neg s}{\therefore \neg p \vee \neg r}$	$[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s)] \rightarrow (\neg p \vee \neg r)$	Rule of the Destructive Dilemma

**EXAMPLE 2.30**

Establish the validity of the argument

$$\frac{p \rightarrow q \quad q \rightarrow (r \wedge s) \quad \neg r \vee (\neg t \vee u) \quad p \wedge t}{\therefore u}$$