COSC 311 - Fall 2013 - Homework 5 - Due Sept. 25

1. The real number y is the *derivative* of the function f at the point $\bar{x}$ iff (if and only if) for every real number $\epsilon > 0$ there is a real number $\delta > 0$ such that for every real number x with $0 < |x - \bar{x}| < \delta$,

$$\left| \frac{f(x) - f(\bar{x})}{x - \bar{x}} - y \right| < \epsilon.$$

   Write the *negation* of the proposition that y is the derivative of the function f at the point $\bar{x}$. Your answer should not contain the not-sign ($\neg$) nor the words "not," "no," "none," etc. (You can use either words or symbols.) *Hint:* Look at Example 2.50 in Grimaldi, p. 99.

For each of the following, you are to prove the proposition, but:

(1) Show how you have devised your proof by the naming the methods (Forward-Backward, Construction, Choose, etc.) that you are using and labeling the steps A1, A2, ..., B1, B2, ... as we have done in class. *You will not receive full credit if you do not identify the methods and label the steps!*

(2) After you have invented a proof, *execute* it by writing it out clearly in Euclidean order.

2. A function f is *bounded above* iff there is a real number y such that, for all real numbers x, $f(x) \leq y$. Prove that $f(x) = -x^2 + 2x$ is bounded above.

3. If x is a real number satisfying $x^3 + 3x^2 - 9x - 27 \geq 0$, then $|x| \geq 3$. *Hint:* Express the conclusion as a disjunction.

4. If c is an odd integer, then $n^2 + n - c = 0$ has no integer solution for n.

5. A function f is a *convex function* iff for all real numbers x and y and for all real numbers t with $0 \leq t \leq 1$, it follows that

$$f[tx + (1-t)y] \leq tf(x) + (1-t)f(y).$$

   Prove that if f is a convex function, then for all real numbers $s \geq 0$, $sf(x)$ is a convex function.