SUMMARY OF PROOF TECHNIQUES

Proof Technique	**When to Use It**	**What to Assume**
Forward-Backward (page 9)	As a first attempt, or when B does not have a recognizable form.	A
Contrapositive (page 113)	When B has the word "no" or "not" in it.	$NOT\ B$
Contradiction (page 99)	When B has the word "no" or "not" in it, or when the first two methods fail.	A and $NOT\ B$
Construction (page 41)	When B has the words "there is," "there are," and so on.	A
Choose (page 51)	When B has the words "for all," "for each," and so on.	A, and choose an object with the certain property.
Specialization (page 67)	When A has the words "for all," "for each," and so on.	A

SUMMARY OF PROOF TECHNIQUES

What to Conclude	**How to Do It**
B	Work forward from A and apply the backward process to B.
$NOT\ A$	Work forward from $NOT\ B$ and backward from $NOT\ A$.
Some contradiction	Work forward from A and $NOT\ B$ to reach a contradiction.
That there is the desired object	Guess, construct, and so on, the object. Then show that it has the certain property and that the something happens.
That the something happens	Work forward from A and the fact that the object has the certain property. Also work backward from the something that happens.
B	Work forward by specializing A to one particular object having the certain property.

SUMMARY OF PROOF TECHNIQUES

Proof Technique	When to Use It	What to Assume
Forward Uniqueness (page 123)	When A has the key word "unique" in it.	There is such an object, X.
Direct Uniqueness (page 125)	When B has the key word "unique" in it.	There are two such objects, and A
Indirect Uniqueness (page 126)	When B has the key word "unique" in it.	There are two different objects, and A
Induction (page 131)	When a statement $P(n)$ is true for each integer $n \geq n_0$.	$P(n)$ is true for n.
Proof by Cases (page 143)	When A has the form "C *OR* D."	Case 1: C Case 2: D
Proof by Elimination (page 145)	When B has the form "C *OR* D."	A and *NOT* C or A and *NOT* D
Max/Min 1 (page 153)	When A or B has the form "$\max S \leq z$" or "$\min S \geq z$".	
Max/Min 2 (page 153)	When A or B has the form "$\max S \geq z$" or "$\min S \leq z$".	

SUMMARY OF PROOF TECHNIQUES

What to Conclude	How to Do It
X and Y are the same, that is, $X = Y$	Look for another object Y with the same properties as X.
The two objects are equal	Work forward using A and the properties of the objects. Also work backward to show the objects are equal.
Some contradiction	Work forward from A using the properties of the two objects and the fact that they are different.
$P(n+1)$ is true; also prove $P(n_0)$ is true	First prove $P(n_0)$. Then use the assumption that $P(n)$ is true to prove $P(n+1)$.
B B	First prove that C implies B; then prove that D implies B.
D or C	Work forward from A and *NOT* C, and backward from D. or Work forward from A and *NOT* D, and backward from C.
	Convert to "for all $s \in S$, $s \leq z$ or $s \geq z$." Then use choose (if in B) or specialization (if in A).
	Convert to "there is an $s \in S$ such that $s \geq z$ or $s \leq z$." Then work forward (if in A) or use construction (if in B).

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