

C Reversible computing

C.1 Reversible computing as a solution

This section is based on Frank (2005b).

C.1.a POSSIBLE SOLUTION

- ¶1. Notice that the key quantity F_E in Eqn. II.1 depends on the energy *dissipated* as heat.
- ¶2. The $100k_B T$ limit depends on the energy in the signal (necessary to resist thermal fluctuation causing a bit flip).
- ¶3. There is nothing to say that information processing has to dissipate energy; an arbitrarily large amount of it can be recovered for future operations.
 “Arbitrary” in the sense that there is no inherent physical lower bound on the energy that must be dissipated.
- ¶4. It becomes a matter of precise energy *management*, moving it around in different patterns, with as little dissipation as possible.
- ¶5. Indeed, E_{sig} can be increased to improve reliability, provided we minimize dissipation of energy.
- ¶6. This can be accomplished by making the computation *logically reversible* (i.e., each successor state has only one predecessor state).

C.1.b REVERSIBLE PHYSICS

- ¶1. All fundamental physical theories are Hamiltonian dynamical systems.
- ¶2. All such systems are time-reversible. That is, if $\psi(t)$ is a solution, then so is $\psi(-t)$.
- ¶3. In general, *physics is reversible*.
- ¶4. Physical information cannot be lost, be we can lose track of it. This is entropy: “unknown information residing in the physical state.”
 Note how this is fundamentally a matter of *information* and *knowledge*.

What is irreversible is the *information inaccessibility*.
Entropy is ignorance.

C.1.c REVERSIBLE LOGIC

- ¶1. To avoid dissipation, don't erase information. The problem is to keep track of information that would otherwise be dissipated. Avoid squeezing information out of logical space (IBDF) into thermal space (NIBDF).
- ¶2. This is accomplished by making computation *logically reversible*.
(It is already *physically* reversible.)
- ¶3. The information is rearranged and recombined *in place*. (We will see lots of examples of how to do this.)

C.1.d PROGRESS

- ¶1. In 1973, Charles Bennett (IBM) first showed how any computation could be embedded in an equivalent reversible computation. Rather than discarding information, it keeps it around so it can later “decompute” it. This was *logical* reversibility; he did not deal with the problem of *physical* reversibility.
- ¶2. **Brownian Computers:** Or “Brownian motion machines.” This was an attempt to suggest a possible physical implementation of reversible computation.
“the mean free path of the system’s trajectory was much shorter than the distance between neighboring computational states” (see also [B82]).
- ¶3. Therefore: “In absence of any energy input, the system progressed essentially via a random walk, taking an expected time of $\Theta(n^2)$ to advance n steps.”
- ¶4. A small energy input biases the process in the forward direction, so that it precedes linearly, but still very slowly.
- ¶5. Compare “DNA polymerization, which (under normal conditions, such as during cell division) proceeds at a rate on the order of only 1,000 nucleotides per second, with a dissipation of $\sim 40k_B T$ per step.”

This is about 1 eV (see ¶7 below).

Note that DNA replication includes error-correcting operations.

- ¶6. **Energy coefficient:** Since “asymptotically reversible processes (including the DNA example) proceed forward at an adjustable speed, proportional to the energy dissipated per step,” define an *energy coefficient*:

$$c_E \stackrel{\text{def}}{=} E_{\text{diss}}/f_{\text{op}},$$

“where E_{diss} is the energy dissipated per operation, and f_{op} is the frequency of operations.”

- ¶7. “In Bennett’s original DNA process, the energy coefficient comes out to about $c_E = 1\text{eV/kHz}$.”

That is, for DNA, $c_E \approx 40kT/\text{kHz} = 40 \times 26 \text{ meV/kHz} \approx 1 \text{ eV/kHz}$.

- ¶8. But it would be desirable to operate at GHz frequencies and energy dissipation below $k_B T$.

Recall that at room temp. $k_B T \approx 26 \text{ meV}$ (Sec. A ¶6, p. 31).

So we need energy coefficients much lower than DNA.

This is an issue, of course, for molecular computation.

- ¶9. **Information Mechanics group:** In 1970s, Ed Fredkin, Tommaso Toffoli, et al. at MIT.

- ¶10. **Ballistic computing:** F & T described computation with idealized, perfectly elastic balls reflecting off barriers. Minimum dissipation, propelled by (conserved) momentum. Unrealistic. Later we will look at it briefly.

- ¶11. They suggested a more realistic implementation involving “charge packets bouncing around along inductive paths between capacitors.”

- ¶12. Richard Feynman (CalTech) had been interacting with IM group, and developed “a full quantum model of a serial reversible computer” (Feynman, 1986).

- ¶13. **Adiabatic circuit:** Since 1980s there has been work in *adiabatic circuits*, esp. in 1990s.

An *adiabatic process* takes place without input or dissipation of energy.

Adiabatic circuits minimize energy use by obeying certain circuit design rules. “[A]rbitrary, pipelined, sequential logic could be implemented in a fully-reversible fashion, limited only by the energy coefficients and leakage currents of the underlying transistors.”

- ¶14. As of 2004, est. $c_E = 3 \text{ meV/kHz}$, about $250\times$ less than DNA.
- ¶15. “It is difficult to tell for certain, but a wide variety of post-transistor device technologies have been proposed ... that have energy coefficients ranging from 10^5 to 10^{12} times lower than present-day CMOS! This translates to logic circuits that could run at GHz to THz frequencies, with dissipation per op that is still less (in some cases orders of magnitude less) than the VNL bound of $k_B T \ln 2$... that applies to all irreversible logic technologies. Some of these new device ideas have even been prototyped in laboratory experiments [2001].”
- ¶16. “Fully-reversible processor architectures [1998] and instruction sets [1999] have been designed and implemented in silicon.”
- ¶17. But this is more the topic of a CpE course...

C.2 Physical assumptions of computing

Optional

These lectures are based primarily on Edward Fredkin and Tommaso Toffoli’s “Conservative logic” (Fredkin & Toffoli, 1982).

C.2.a DISSIPATIVE LOGIC

- ¶1. The following physical principles are implicit in the existing theory of computation.
- ¶2. **P1. The speed of propagation of information is bounded:** No action at a distance.
- ¶3. **P2. The amount of information which can be encoded in the state of a finite system is bounded:** This is a consequence of thermodynamics and quantum theory.

- ¶4. **P3. It is possible to construct macroscopic, dissipative physical devices which perform in a recognizable and reliable way the logical functions AND, NOT, and FAN-OUT:** This is an empirical fact.

C.2.b CONSERVATIVE LOGIC

- ¶1. “Computation is based on the storage, transmission, and processing of discrete signals.”
- ¶2. Only macroscopic systems are irreversible, so as we go to the microscopic level, we need to understand reversible logic. This leads to new physical principles.
- ¶3. **P4. Identity of transmission and storage:** In a relativistic sense, they are identical.
- ¶4. **P5. Reversibility:** Because microscopic physics is reversible.
- ¶5. **P6. One-to-one composition:** Physically, fan-out is not trivial, so we cannot assume that one function output can be substituted for any number of input variables.
We have to treat fan-out as a specific signal-processing element.
- ¶6. **P7. Conservation of additive quantities:** It can be shown that in a reversible systems there are a number of independent conserved quantities.
- ¶7. In many systems they are *additive* over the subsystems.
- ¶8. **P8. The topology of space-time is locally Euclidean:** “Intuitively, the amount of ‘room’ available as one moves away from a certain point in space increases as a power (rather than as an exponential) of the distance from that point, thus severely limiting the connectivity of a circuit.”
- ¶9. What are sufficient primitives for conservative computation?
The *unit wire* and the *Fredkin gate*.