G Physical realizations

The principal source for this lecture is NC, ch. 7.

G.1 Basic criteria

¶1. We can outline a fundamental set of requirements for any practical physical realization of quantum computing.

¶2. **(a) Scalability:** It's necessary to be able to implement systems of multiple qubits whose coupling and unitary transformation can be controlled.

¶3. Want only a finite number of accessible quantum states.
It's best if the finiteness is a consequence of symmetry, which minimizes decoherence.
E.g., $|\uparrow\rangle, |\downarrow\rangle$.

¶4. **(b) Universal set of quantum gates:** It's necessary to have realizations of some universal set of quantum gates.
Quantum gates are implemented by controlling unitary transformations and qubit coupling.

¶5. It must also be possible to address the individual qubits or pairs of qubits on which the transformations are performed.

¶6. Note that the controller is itself a quantum system, so there is some "back action" because of the interaction of the quantum systems.

¶7. **(c) Longer decoherence times:** It's necessary to maintain phase coherence at least long enough for a quantum gate to operate, and preferably through multiple gates.
Decoherence is also known as *quantum noise*.
This means that the qubits should be isolated from the physical environment.
(E.g., nuclear spins are good in this regard.)

¶8. **(d) Qubit initialization:** It's necessary to be able to initialize qubits into a known state, such as the $|0\rangle$ state or the ground energy state.
There are two primary approaches: cooling into a ground state and

measurement.
Perhaps surprisingly, initialization is a significant problem.
There are two figures of merit:

¶9. The *minimum fidelity* with which an initial state can be prepared.

¶10. The *entropy* of the initial state.
Ideally, the initial states are pure states, which have 0 entropy.
"Generally, input states with non-zero entropy reduce the accessibility of the answer from the output result." [NC 282]

¶11. **(e) Output measurement:** It's necesary to be able to measure the state of certain qubits
(for output and in those cases where an algorithm depends on collapse).

¶12. For measurement, we need to be able to couple to the environment (to thermodynamical degrees of freedom).
This works against longer coherence times.
(E.g., nuclear spins are bad in this regard.)

¶13. There are practical problems, because we are trying to detect quantum-level states and amplify them to the macroscopic level.

¶14. There are "strong measurements" (projective measurements), which involve a switchable strong coupling.

¶15. There are also "weak measurements," which are performed continuously during the computation, which must be short relative to the measurement coupling.

¶16. Another approach is to use ensembles of quantum computers, which give a measurable aggregate signals.
This requires modifications to some algorithms, such as Shor's.

¶17. *Signal to noise ratio* (SNR) is a good figure of merit.

¶18. **Maximum number of operations:** The maximum number of quantum operations can be estimated as follow:
Let τ_{Q} be the decoherence time (seconds).
Let τ_{op} be the operation time (the time to perform elementary unitary transformations involving at least two qubits).

Let $\lambda = \tau_{\text{op}}/\tau_{\text{Q}}$ be the maximum length of a quantum computation. Then $n_{\text{op}} = 1/\lambda$ is the maximum number of operations before decoherence occurs.

¶19. n_{op} estimates range from 10^3 (quantum dot, electron in GaAs) to more than 10^{13} (ion trap, nuclear spin).[13]

G.2 Harmonic oscillator quantum computer

¶1. This is a simple, but infeasible model for a Q computer.

¶2. In the Q domain, the energy of a simple harmonic oscillator is quantized (units of $\hbar\omega$).

¶3. There is a discrete set of energy eigenstates $|0\rangle, |1\rangle, |2\rangle, \ldots$ that can be used to represent qubits.

¶4. **Creation and annihilation operations:** There is a creation operator $a^\dagger$ such that if $|\psi\rangle$ has energy E, then $(a^\dagger)^n|\psi\rangle$ has energy $E + n\hbar\omega$. This defines the energy eigenstates.
$a^\dagger$ and a are also called *raising* and *lowering* operators.

¶5. The following is quoted from NC §7.3:

¶6. **Qubit representation:** Energy levels $|0\rangle, |1\rangle, \ldots, |2^n\rangle$ of a single quantum oscillator give n qubits.

¶7. **Unitary evolution:** Arbitrary transforms U are realized by matching their eigenvalue spectrums to that given by the Hamiltonian $H = a^\dagger a$.

¶8. **Drawbacks:** Not a digital representation! Also, matching eigenvalues to realize transformations is not feasible for arbitrary U, which generally have unknown eigenvalues.

¶9. [Note that we would require a system with 2^n energy levels (like a dial with 2^n positions), as opposed to a digital system with n two-level qubits (like an n-digit display). Cf. unary/analog vs. binary/digital representation.]

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[13] NC Fig. 7.1, p. 278.

G.3 Optical photon quantum computer

¶1. The following is quoted from NC §7.4:

¶2. **Qubit representation:** Location of single photon between two modes, $|01\rangle$ and $|10\rangle$, or polarization.

¶3. **Unitary evolution:** Arbitrary transforms are constructed from phase shifters (R_z rotations), beamsplitters (R_y rotations), and nonlinear Kerr media, which allow two single photons to cross phase modulate, performing $\exp[i\chi L|11\rangle\langle 11|]$.

¶4. **Initial state preparation:** Create single photon states (e.g. by attenuating laser light).

¶5. **Readout:** Detect single photons (e.g. using a photomultipler tube).

¶6. **Drawbacks:** Nonlinear Kerr media with large ratio of cross phase modulation strength to absorption loss are difficult to realize.

G.4 CQED

¶1. The following is quoted from NC §7.5:

¶2. **Qubit representation:** Location of single photon between two modes, $|01\rangle$ and $|10\rangle$, or polarization.

¶3. **Unitary evolution:** Arbitrary transforms are constructed from phase shifters (R_z rotations), beamsplitters (R_y rotations), and a cavity QED system, comprised of a Fabry-Perot cavity containing a few atoms, to which the optical field is coupled.

¶4. **Initial state preparation:** Create single photon states (e.g. by attenuating laser light).

¶5. **Readout:** Detect single photons (e.g. using a photomultipler tube).

¶6. **Drawbacks:** The coupling of two photons is mediated by an atom, and thus it is desirable to increase the atom-field coupling. However, coupling the photon into and out of the cavity then becomes difficult, and limits cascadibility.

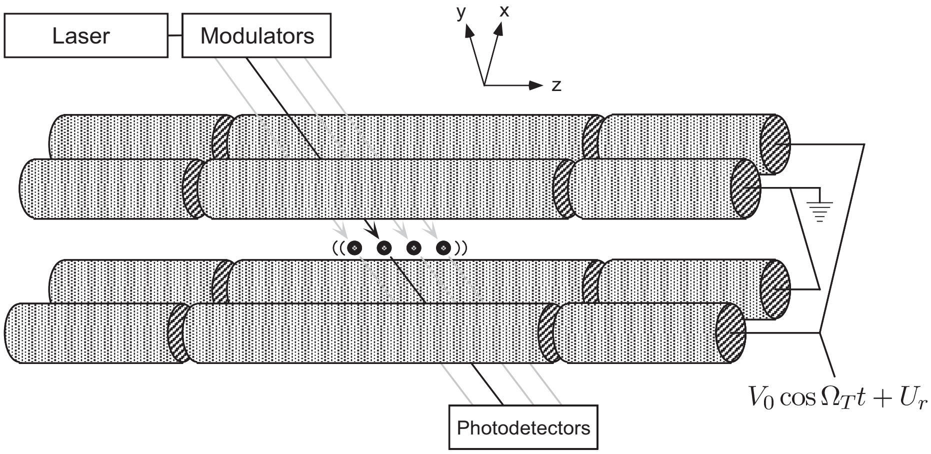

Figure 7.7. Schematic drawing (not to scale) of an ion trap quantum computer, depicting four ions trapped in the center of a potential created by four cylindrical electrodes. The apparatus is typically contained in a high vacuum ($\approx 10^{-8}$ Pa), and the ions are loaded from a nearby oven. Modulated laser light incident on the ions through windows in vacuum chamber perform operations on and are used to readout the atomic states.

Figure III.43: Ion trap quantum computer. [source: NC]

G.5 Ion traps

¶1. **Qubit representation:** Electron and nuclear spin states can be used to represent qubits.

¶2. Since energy difference between spin states is slight compared other energies (such as kinetic), they are difficult to control.
Therefore, they are cooled to lower the kinetic energy.

¶3. Small numbers of charged ions can be trapped electromagnetically (Fig. III.43).

¶4. The collective vibrational modes of the ions is quantized (*phonons*). This is the means by which qubits interact.

¶5. **Spin states:** Angular momentum includes orbital, electron spin, and nuclear spin.

¶6. **Unitary evolution:** Laser pulses (which carry momentum) can be used to switch spin states depending on other spin states.

¶7. The phonon state is coupled to the spin state.
Quantum information can be switched between the two.
The two can be used for 2-qubit operations.

¶8. **Initial state preparation:** "Cool the atoms (by trapping and using optical pumping) into their motional ground state, and hyperfine [nuclear spin] ground state."

¶9. **Readout:** "Measure population of hyperfine [nuclear spin] states."

¶10. **Drawbacks:** "Phonon lifetimes are short, and ions are difficult to prepare in their motional ground states."

¶11. **Experimental CNOT implementation:** Experiments have demonstrated CNOT with trapped ^{9}Be ions.

G.6 NMR

¶1. The following is quoted from NC §7.7:

¶2. **Qubit representation:** Spin of an atomic nucleus.

¶3. **Unitary evolution:** Arbitrary transforms are constructed from magnetic field pulses applied to spins in a strong magnetic field. Couplings between spins are provided by chemical bonds between neighboring atoms.

¶4. **Initial state preparation:** Polarize the spins by placing them in a strong magnetic field, then use 'effective pure state' preparation techniques.

¶5. **Readout:** Measure voltage signal induced by precessing magnetic moment.

¶6. **Drawbacks:** Effective pure state preparation schemes reduce the signal exponentially in the number of qubits, unless the initial polarization is sufficiently high.

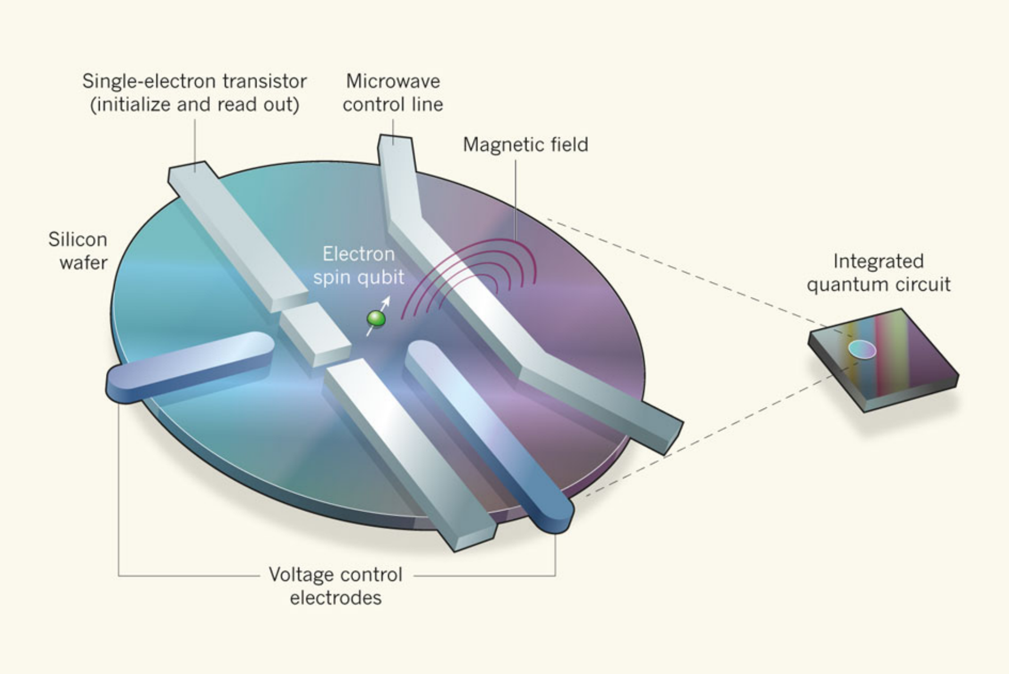

Figure III.44: Single-atom qubit in silicon (drawing). [source: Bassett & Awschalom (2012)]

G.7 Quantum dots

¶1. A *quantum dot* is a nanoscale three-dimensional "box" in which charge quanta can be confined by electrostatic potentials.
The "box" could be in a semiconductor, metal, or even a single molecule.

¶2. Single qubit operations are realized through electrostatic gates, waveguides, and tunnel junctions.

¶3. Multiple qubit operations are realized by controlling long-range Coulomb interactions (which also lead to decoherence).

¶4. Charge cannot be destroyed, but they can be moved around.
Recall conservation in conservative logic (discussed in Ch. II).

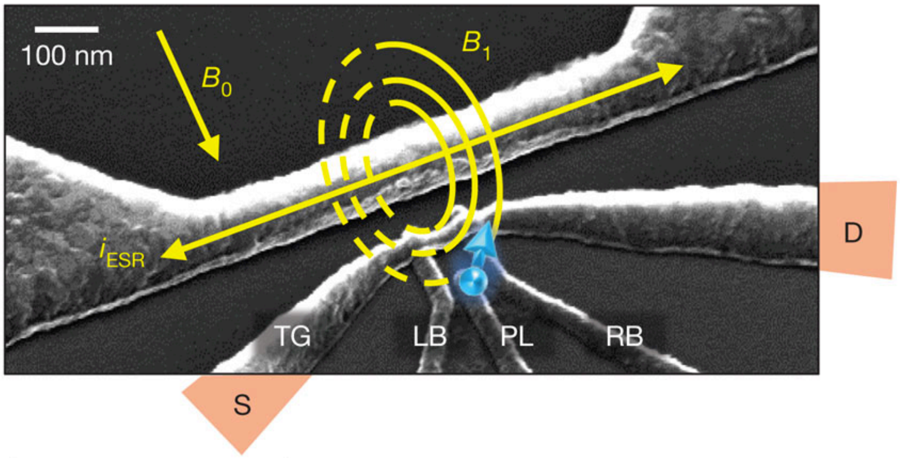

Figure III.45: Single-atom qubit in silicon (scanning electron micrograph). The SET (single-electron transistor) comprses S = source, D = drain, TG = top gate, PL = plunger gate, LB/RB = left/right barrier gates. B_0 is an in-plane external magnetic field. B_1 is the oscillating magnetic field from the microwave transmission line. [source: Pla et al. (2012)]

G.8 Single-atom qubit in silicon

Sources:

- Jarryd J. Pla, Kuan Y. Tan, Juan P. Dehollain, Wee H. Lim, John J. L. Morton, David N. Jamieson, Andrew S. Dzurak & Andrea Morello. A single-atom electron spin qubit in silicon. *Nature* 489, 541–545 (27 September 2012) `doi:10.1038/nature11449`.

- Bassett, Lee C., and Awschalom, David D. Quantum computation: Spinning towards scalable circuits. *Nature* 489, 505–507 (27 September 2012) `doi:10.1038/nature11488`.

- See also *IEEE Spectrum*. `http://spectrum.ieee.org/computing/hardware/physicists-build-first-singleatom-quantum-bit-in-silicon`

¶1. In Sept. 2012, a group at the University of New South Wales (Australia) led by Andrea Morello and Andrew Dzurak demonstrated a system able to read and write the spin of an electron on a single phosphorus atom

embedded in a silicon crystal.
The qubit is coupled to a single-atom transistor (SET) in the Si.
See Fig. III.44 and Fig. III.45.

¶2. They chose this approach because electron spins in Si have very long decoherence times (longer than 1s).

¶3. It also builds on well-developed Si semiconductor fabrication technology.

¶4. At room temperature, P donates a mobile electron to Si, but at liquid-He temperatures, the electron becomes bound to the nucleus, providing a "trapped spin."

¶5. **Initialization & readout:** Only if the electron is in the UP state, it tunnels to the Si transistor, which measures the charge state and resets the spin to DOWN.
This is called "spin-to-charge conversion."
Thus readout initializes the qubit for the next operation.

¶6. **Control:** They use high-power microwaves (30GHz) to control the spin between initialization and readout.
This is done with a nano-transmission line 100nm from the qubit to minimize extraneous signals.

¶7. **Coherence time:** The measured coherence time is about 200μs.
Since a typical microwave driven quantum operation takes about 100ns, this would permit about 1000 operations before decoherence.
It's expected that with a stronger microwave field operations will take less than 1μs, and with improved materials coherence times can be close to a second.
This implies about $n_{\text{op}} = 10^9$ operations per coherence time.

¶8. The system is cooled to 0.1°K to reduce thermal noise.

¶9. **Future:** They are currently working on 2-qubit logic gates.

¶10. **Nuclear spin:** If nuclear spin (as opposed to electron spin) can be harnessed, coherence times may be increased several orders of magnitude.

Use of electron spin to control nuclear spin has been demonstrated, which allows nuclear spin to be used as a “long-term” memory.

¶11. **Multiple qubits:** Atomically precise placement of donor atoms has been demonstrated, which allows them to be placed within 10nm, close enough for coherent transfer of quantum information.