Oracle Turing Machines

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Outline

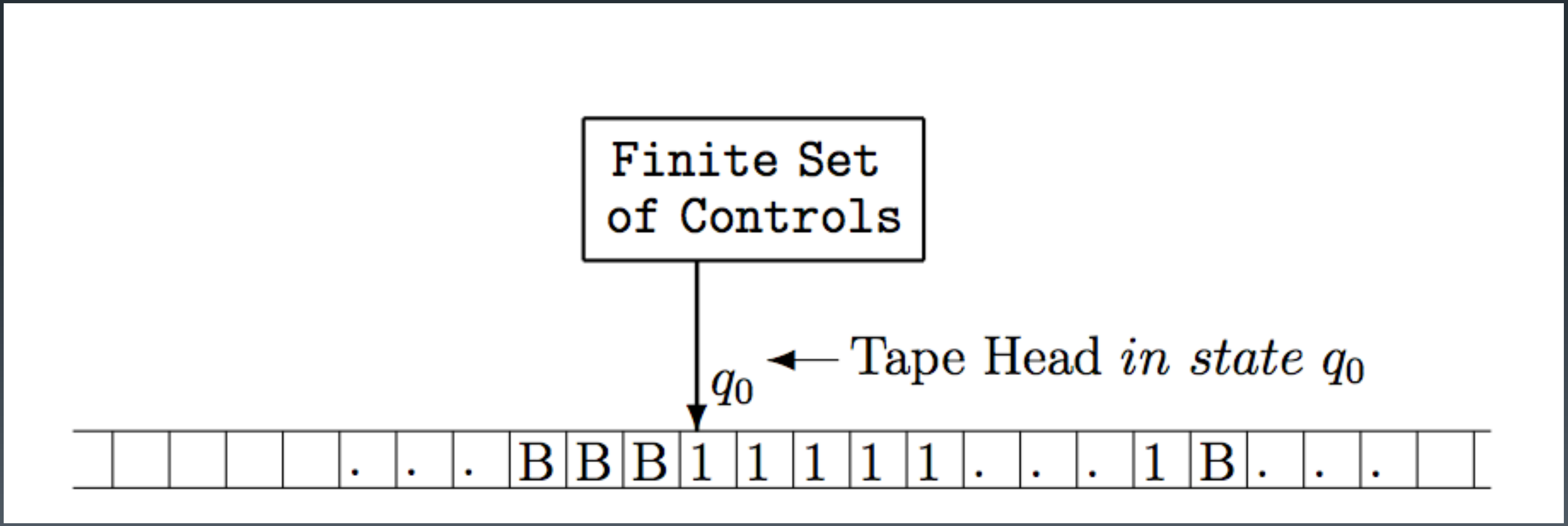
- History of Turing Machines
- Turing Machines Defined
- How a Turing Machine Works
- Turing Machine “Addition” Example
- The Halting Problem Defined
- Examples of Halting Problem
- Oracle Turing Machines Defined
- Applications

History

- Alan Mathison Turing (1912–1954), a mathematician, was influential in the development of computer science.
- Turing first described the Turing machine in his 1936 article "*On Computable Numbers, with an Application to the Entscheidungsproblem*."
- The Turing machine plays a significant role in the creation of the modern computer.

Turing Machines

- A Turing machine is a hypothetical machine.
- Despite its simplicity, the machine can simulate ANY computer algorithm, no matter how complicated it is!

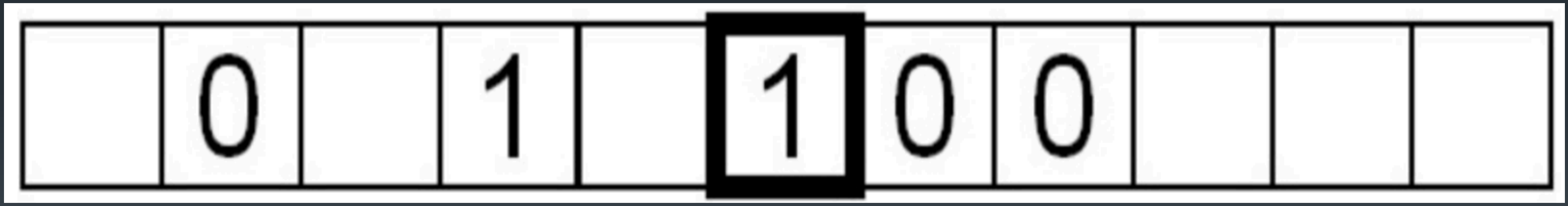

Finite Set of Controls
q_0
Tape Head *in state* q_0
. . . B B B 1 1 1 1 1 . . . 1 B . . .

Turing Machines

- For simplicity, we assume that this machine can only process 0, 1 and Blank
- The machine has a head which is positioned over one of the squares on the tape
- The head can perform the following three basic operations:
  - **Read** the symbol on the square under the head
  - **Edit** the symbol by writing a new symbol or erasing it
  - **Move** the tape **left or right** by one square so that the machine can read and edit the symbol on a neighboring square

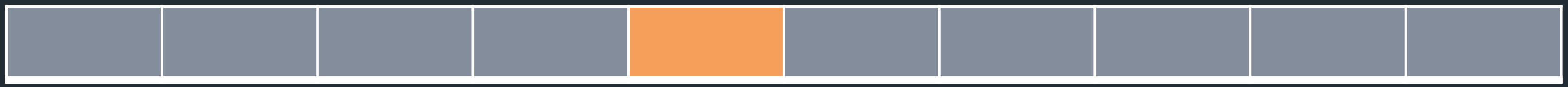
Turing Machines

- A very simple representation of a Turing Machine would consist of a infinitely long tape (memory)
- Each square on the tape is Blank at the start and symbols can be written over it

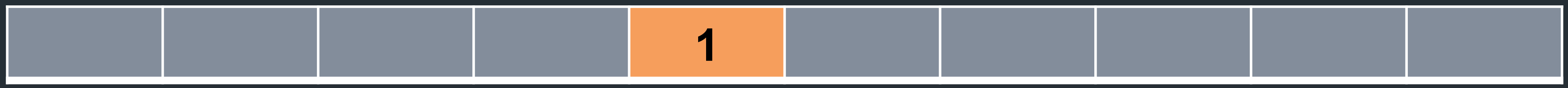


How a Turing Machine Works

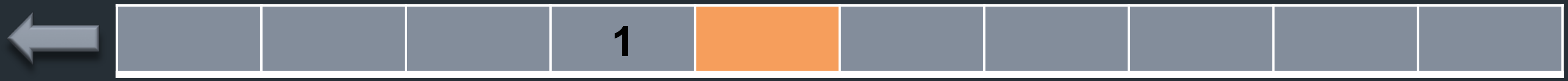
We will try to print symbols "1 1 0" on an initially blank tape



We write a "1" on the square under the head



Next, we move the tape left by one square



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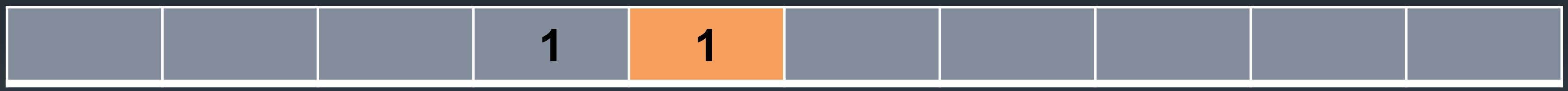
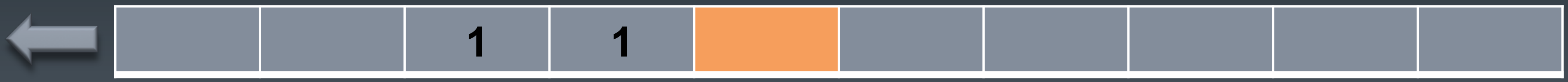
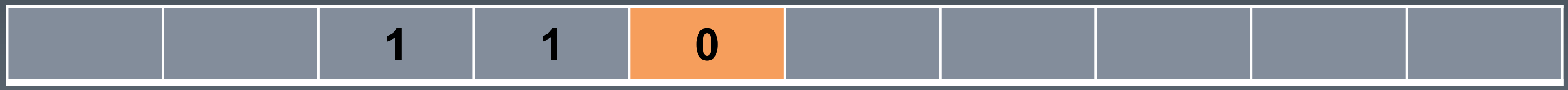


We move the tape left by one square



Finally, write a "0"



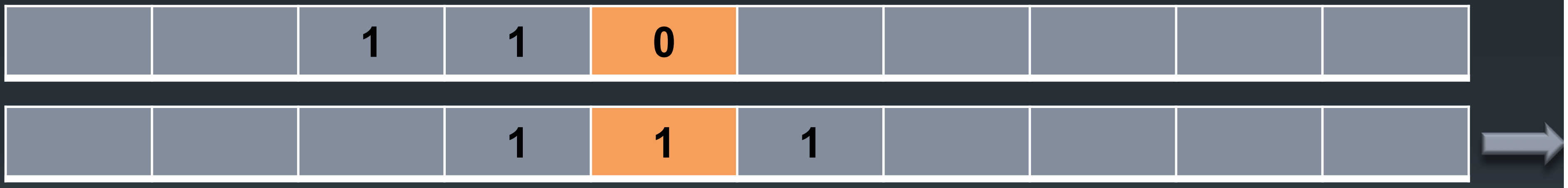
How a Turing Machine Works

- Let’s run a simple program of inverting the bits we have written before i.e. from “1 1 0” to “0 0 1”
- We will resume from where we left off i.e. the position of the head remains unchanged

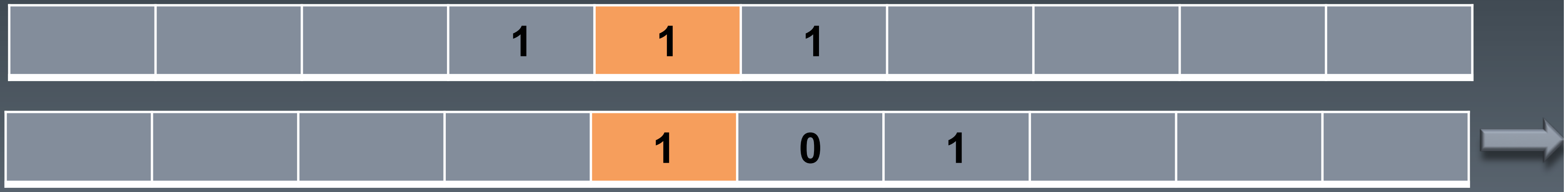
How a Turing Machine Works

Symbol Read	Write Instruction	Move Instruction
Blank	None	None
0	Write 1	Move tape to the right
1	Write 0	Move tape to the right

The current symbol under the head is "0", so we write "1" and move the tape to the right by one square



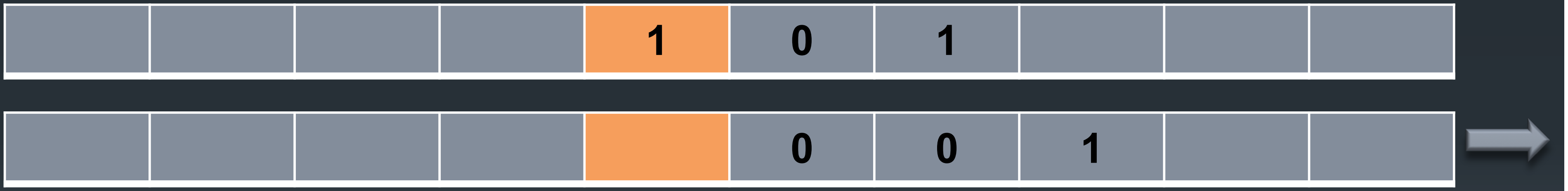
The current symbol under the head is "1", so we write "0" and move the tape to the right by one square



How a Turing Machine Works

Symbol Read	Write Instruction	Move Instruction
Blank	None	None
0	Write 1	Move tape to the right
1	Write 0	Move tape to the right

The current symbol under the head is "1", so we write "0" and move the tape to the right by one square



The ma a "Blank".
How does th g the program?

Does not HALT!!

How a Turing Machine Works

The State Table

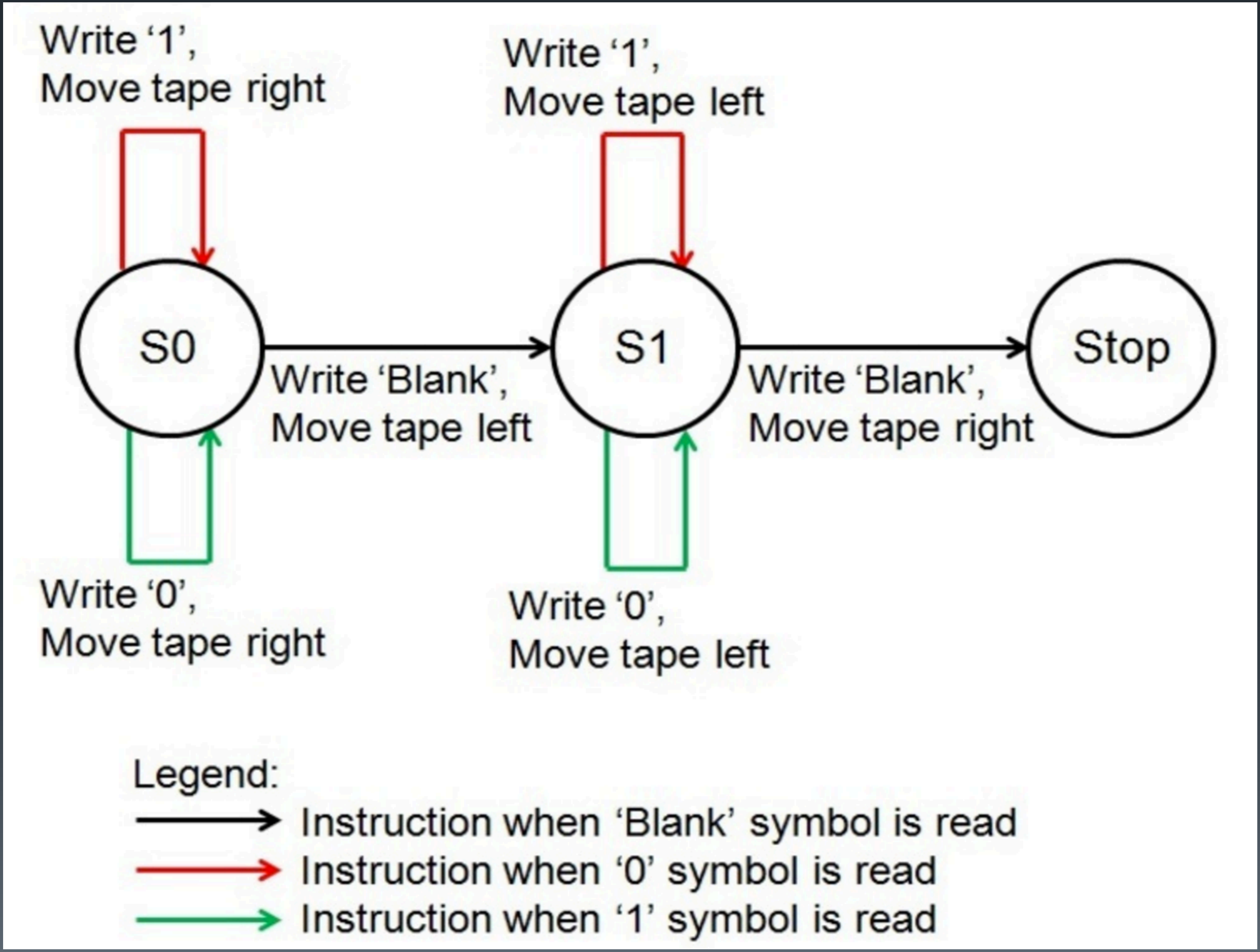
State	Symbol read	Write Instruction	Move Instruction	Next State
State 0	Blank	None	None	Stop state
	0	Write 1	Move tape to the right	State 0
	1	Write 0	Move tape to the right	State 0

How a Turing Machine Works

- Inverting the “0 0 1” state back to initial state of “1 1 0” using a two-symbol, three-state Turing Machine

State	Symbol Read	Write Instruction	Move Instruction	Next State
State 0	Blank	Blank	Left	State 1
	0	1	Right	State 1
	1	0	Right	State 0
State 1	Blank	Blank	Right	Stop state
	0	1	Left	State 1
	1	0	Left	State 1

How a Turing Machine Works



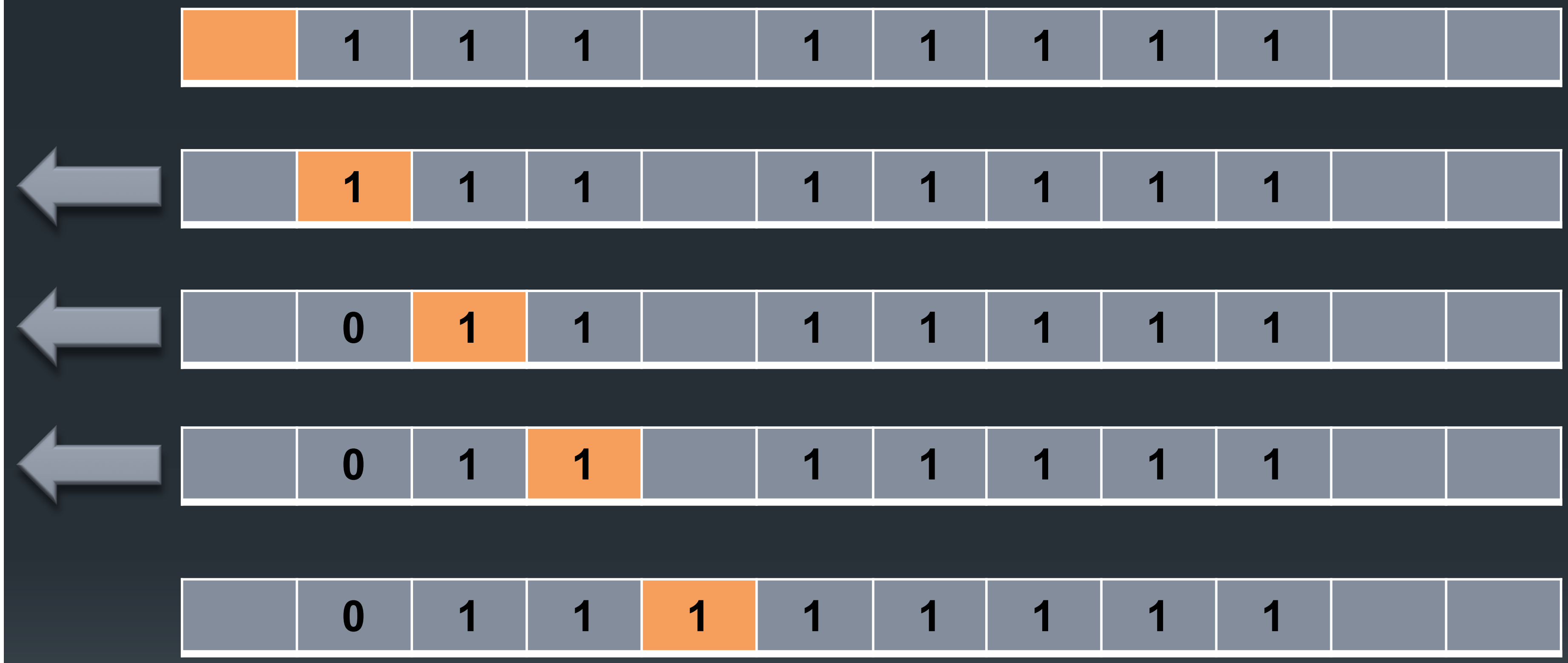
Addition using a Turing Machine

- Let’s add 3 and 5 using a Turing Machine

	1	1	1		1	1	1	1	1		

State	Symbol Read	Write Instruction	Move Instruction	Next State
State 1	Blank	-	Left	State 1
	0	-	Left	State 1
	1	0	Left	State 2
State 2	Blank	1	-	Stop state
	0	1	-	Stop state
	1	-	Left	State 2

Addition using a Turing Machine



The Halting Problem

- In 1936 Alan Turing proved that its not possible to decide whether an arbitrary program will eventually halt or run forever
- This was later called the the Halting Problem by Martin Davis
- It is one of the first examples of a decision problem

The Halting Problem

The **halting problem** is the problem of determining, from a description of an arbitrary computer program and an input, whether the program will finish running or continue to run forever.

For example,

```
while (true) continue
```

Does not halt; rather, it goes on forever in a infinite loop

On the other hand,

```
print “Hello World!”
```

Does halt

The Halting Problem

What about more complex problems?

Why does the Halting Problem matter?

- We often want to know if a program converges (halts), but not possible to provide one algorithm that answers this for all programs.
- Many problems are shown to be undecidable by being reduced to halting problem

Oracle Turing Machines

- An oracle machine is an abstract machine used to study decision problems.
- It can be visualized as a Turing machine with a black box, called an oracle, which is able to decide certain decision problems in a single operation.
- The problem can be of any complexity class. Even undecidable problems, like the halting problem, can be used.

Oracle Turing Machines

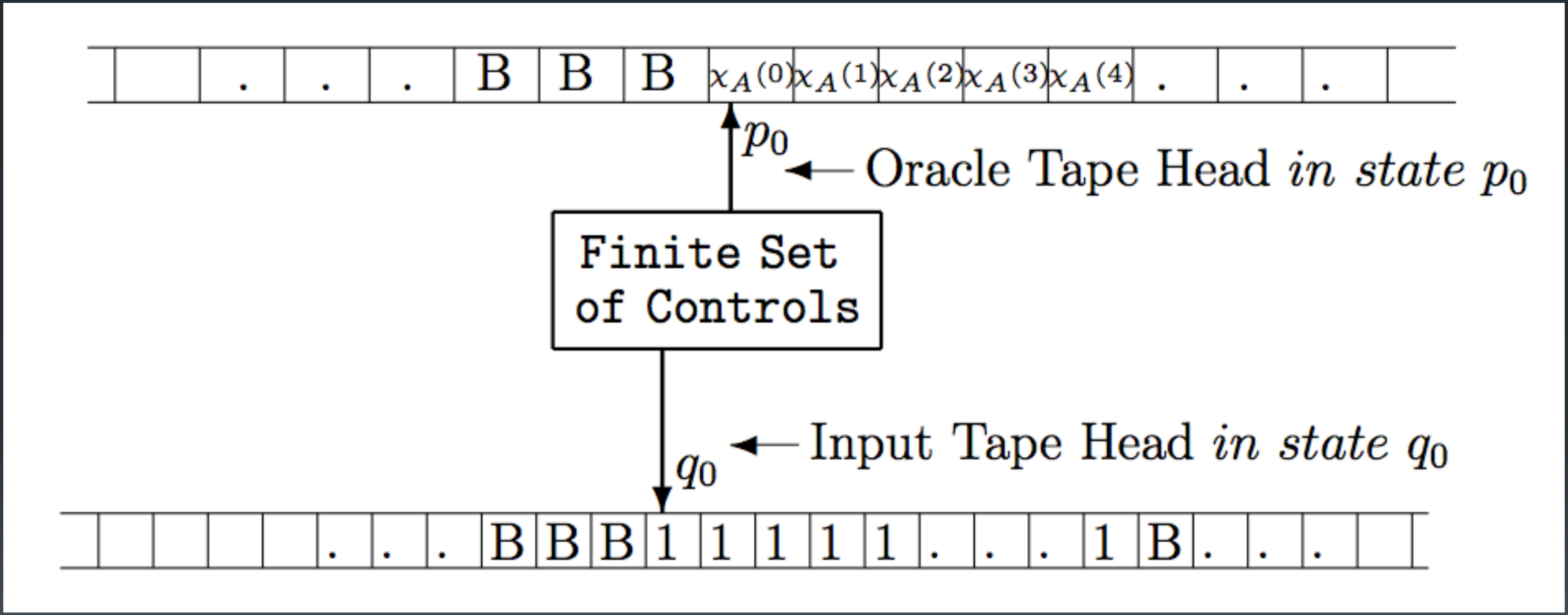
- An oracle machine, like a Turing machine, includes:
  - a **work tape**, **read/write head** and a **control mechanism**
- In addition to these components, an oracle machine also includes:
  - an **oracle tape**
  - an **oracle head**
  - **two special states**: the QUERY state and the RESPONSE state.

Oracle Turing Machines

- From time to time, the oracle machine may enter the QUERY state. When this happens, the following actions are performed in a single computational step:
  - Content of the oracle tape is read
  - the oracle is consulted, and the contents of the oracle tape are replaced with the solution to that instance of the problem;
  - the state of the oracle machine is changed to RESPONSE either of Q_{yes} or Q_{no}

The effect of changing to the QUERY state is thus to receive, in a single step, a solution to the problem instance that is written on the oracle tape.

Oracle Turing Machines



. . . B B B $\chi_A(0)$ $\chi_A(1)$ $\chi_A(2)$ $\chi_A(3)$ $\chi_A(4)$ . . .
p_0
Oracle Tape Head *in state* p_0
Finite Set of Controls
q_0
Input Tape Head *in state* q_0
. . . B B B 1 1 1 1 1 . . . 1 B . . .

Oracle Turing Machines

State	Symbol Read	Write Instruction	Move Instruction	Next State
State 0	Blank	Blank	Left	State 1
	0	1	Right	State 1
	1	0	Right	State 0
State 1	Blank	Blank	Right	Stop state
	0	1	Left	State 1
	1	0	Left	State 1
Q_{yes}	Blank	….	….	….
	0	….	….	….
	1	….	….	….
Q_{no}	Blank	….	….	….
	0	….	….	….
	1	….	….	….

Oracle Turing Machines

- In simple words, OTMs can be seen as computations which use subroutines
- The time spent in these subroutines counts only as **one step**, regardless of complexity
- This makes OTMs an unrealistic computation model

Importance of OTMs

- Used widely in computer science theory, to help us study the relative difficulty between different problems.
- They help us encode the notion of Turing reductions.
- They help identify some barriers to proving results in complexity theory, like proving that P≠NP.

Turing Reduction

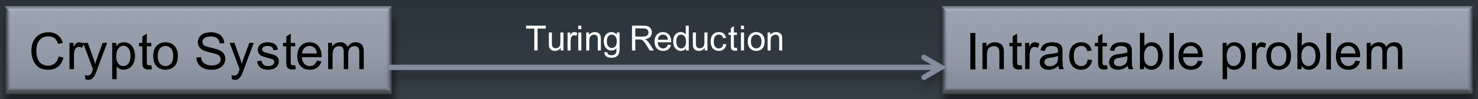
- A Turing reduction from problem A to problem B is a reduction which solves A, assuming the solution to B is already known
- In simpler terms, it is an algorithm that could be used to solve A if it had available to it a subroutine for solving B

Applications

- No practical application
- Theoretical application includes:
  - Relativizing proofs
  - Study the relative difficulty between different problems
  - Turing Reduction

Application in Cryptography

- Reductions play a central role in cryptography
- Designer of a crypto-system wants the system to be difficult to break
- In order to provide a "proof" of security for this scheme, the crypto-analysis problem for the new system is computationally equivalent to a reference problem believed to be intractable



- The existence of such reduction imply the security of the new scheme

References

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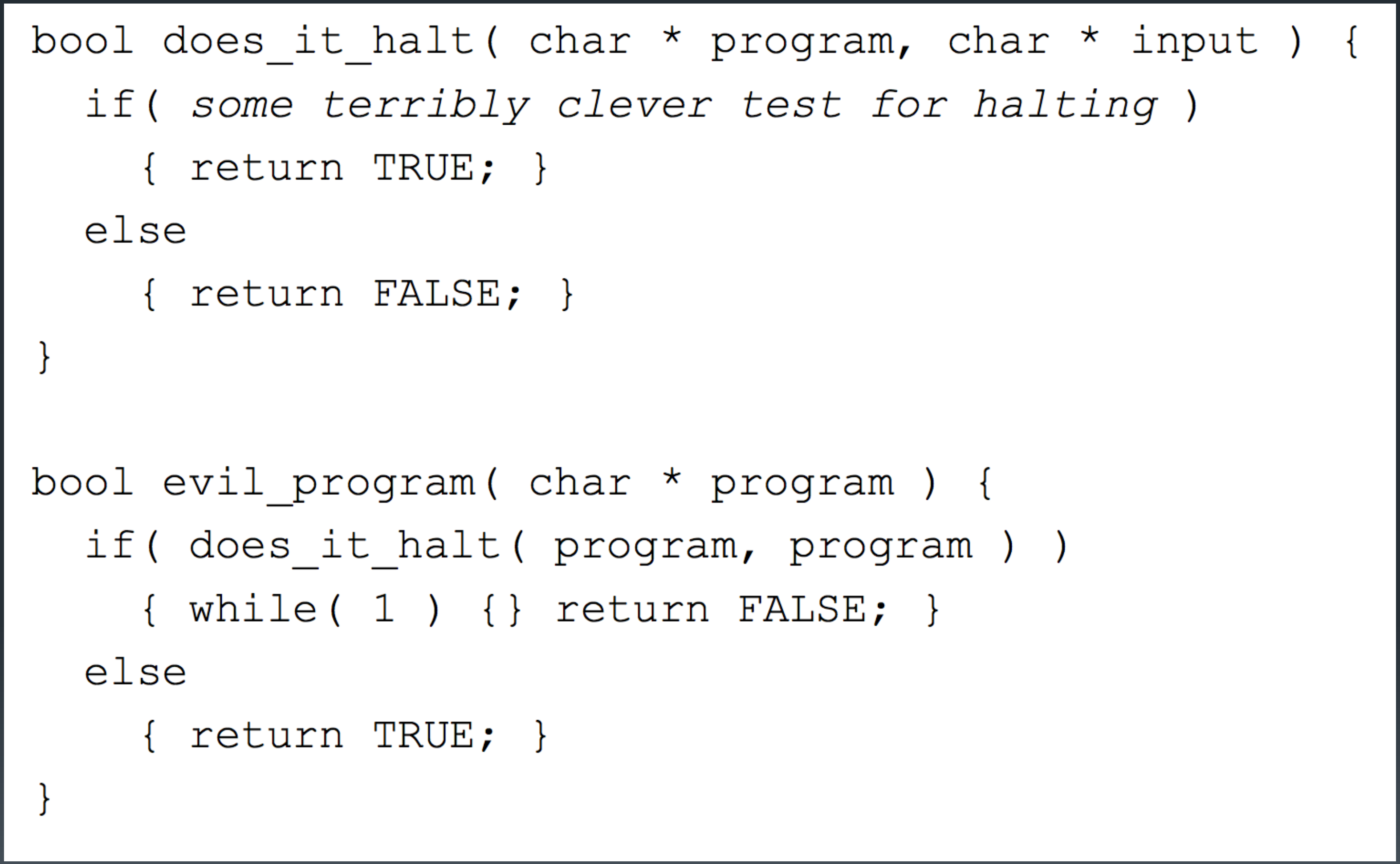
- Recent Advances in RSA Cryptography by Stefan Katzenbeisser, p. 20-21

Thank You!

Questions?

The Evil Program

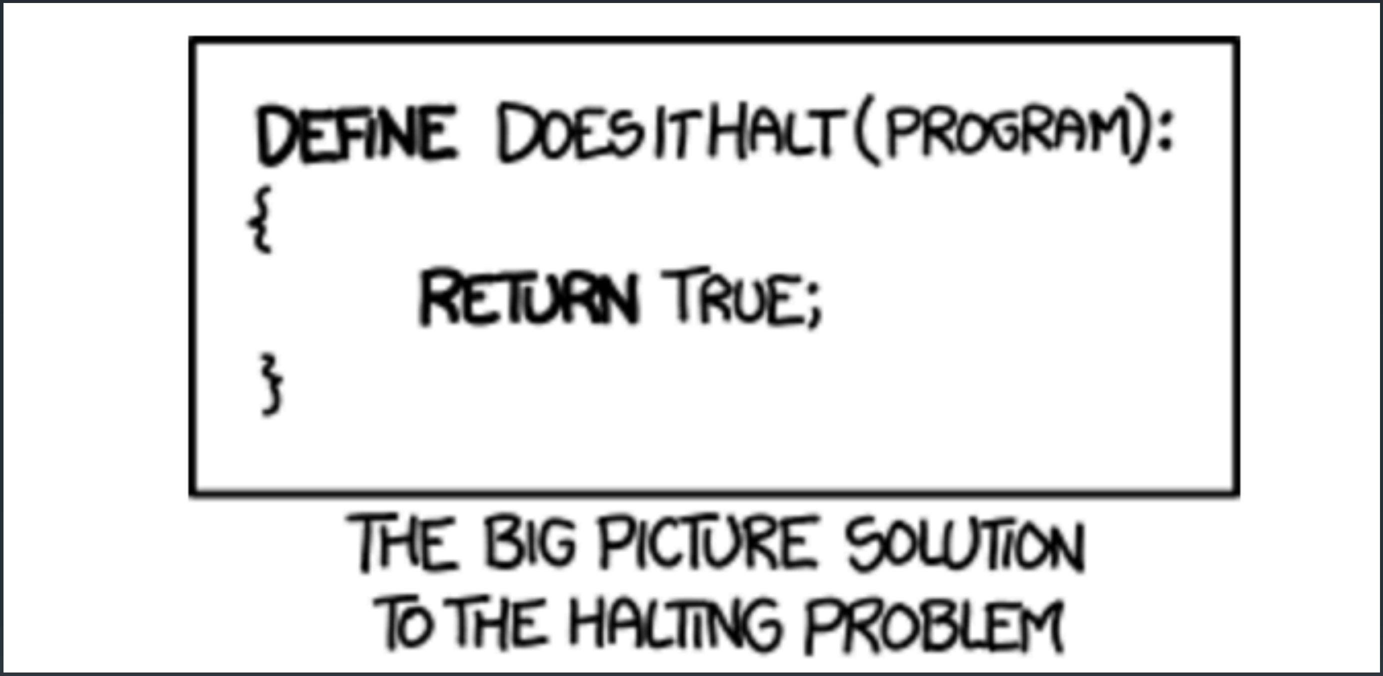


```
bool does_it_halt( char * program, char * input ) {
  if( some terribly clever test for halting )
    { return TRUE; }
  else
    { return FALSE; }
}

bool evil_program( char * program ) {
  if( does_it_halt( program, program ) )
    { while( 1 ) {} return FALSE; }
  else
    { return TRUE; }
}
```

What does `evil_program(evil_program)` do?

Halting Problem Solution



```
DEFINE DOESITHALT(PROGRAM):
{
      RETURN TRUE;
}
```

https://www.explainxkcd.com/wiki/index.php/1266:_Halting_Problem