Exercise III.43 Show that $\mathrm{CNOT}(H \otimes I) = (I \otimes H)C_Z H^{\otimes 2}$, where C_Z is the controlled-Z gate.

Exercise III.44 Show that the Fourier transform matrix (III.25, p. 137, Sec. D.3.a) is unitary.

Exercise III.45 Prove the claim on page 152 (Sec. D.4.b) that D is unitary.

Exercise III.46 Prove the claim on page 152 (Sec. D.4.b) that

$$WR'W = \begin{pmatrix} \frac{2}{N} & \frac{2}{N} & \cdots & \frac{2}{N} \\ \frac{2}{N} & \frac{2}{N} & \cdots & \frac{2}{N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2}{N} & \frac{2}{N} & \cdots & \frac{2}{N} \end{pmatrix}.$$

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Exercise III.47 Show that if there are s solutions x such that $P(x) = 1$, then $\frac{\pi\sqrt{N/s}}{4}$ is the optimal number of iterations in Grover's algorithm.

Exercise III.48 Design a quantum gate array for the following syndrome extraction operator (D.5.d, p. 162, in Sec. D.5.d, p. 161):

$$S|x_3, x_2, x_1, 0, 0, 0\rangle \stackrel{\text{def}}{=} |x_3, x_2, x_1, x_1 \oplus x_2, x_1 \oplus x_3, x_2 \oplus x_3\rangle.$$

Exercise III.49 Design a quantum gate array to apply the appropriate error correction for the extracted syndrome as given in D.5.d, p. 162 (Sec. D.5.d, p. 161):

bit flipped	syndrome	error correction
none	$\|000\rangle$	$I \otimes I \otimes I$
1	$\|110\rangle$	$I \otimes I \otimes X$
2	$\|101\rangle$	$I \otimes X \otimes I$
3	$\|011\rangle$	$X \otimes I \otimes I$