Quantum Probability & Cognition

Comparing classical probability vs. quantum, and QP in human decision making & machine learning

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- Review & Closer Look at QP Characteristics
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        - Example in JMP
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Logic, Classical Probability, & Quantum Probability

- Logic
  - "Classical logic can be seen as a rational way of thinking, *but only in idealized situations in which deductive inference is possible."*
- Classical Probability
  - "CP theory inference can also be seen as rational, *but only idealized situations in which the requirements of unicity match the capabilities of the observers."*
- Quantum Probability
  - "For the real, noisy, confusing, ever-changing, chaotic world, QP is the only system that works in physics and, we strongly suspect, in psychology as well."

Why Quantum Theory?

- Judgement depends on context and perspective
- Assumes no measurable reality
- Assumes no omniscient cognitive agent

- Perhaps analogous to other schools of thought

- Not only applicable to decision-making… potentially useful models of QP theory in other areas of cognition, including *category membership, memory,* and *perception*

Processes of CP vs QP

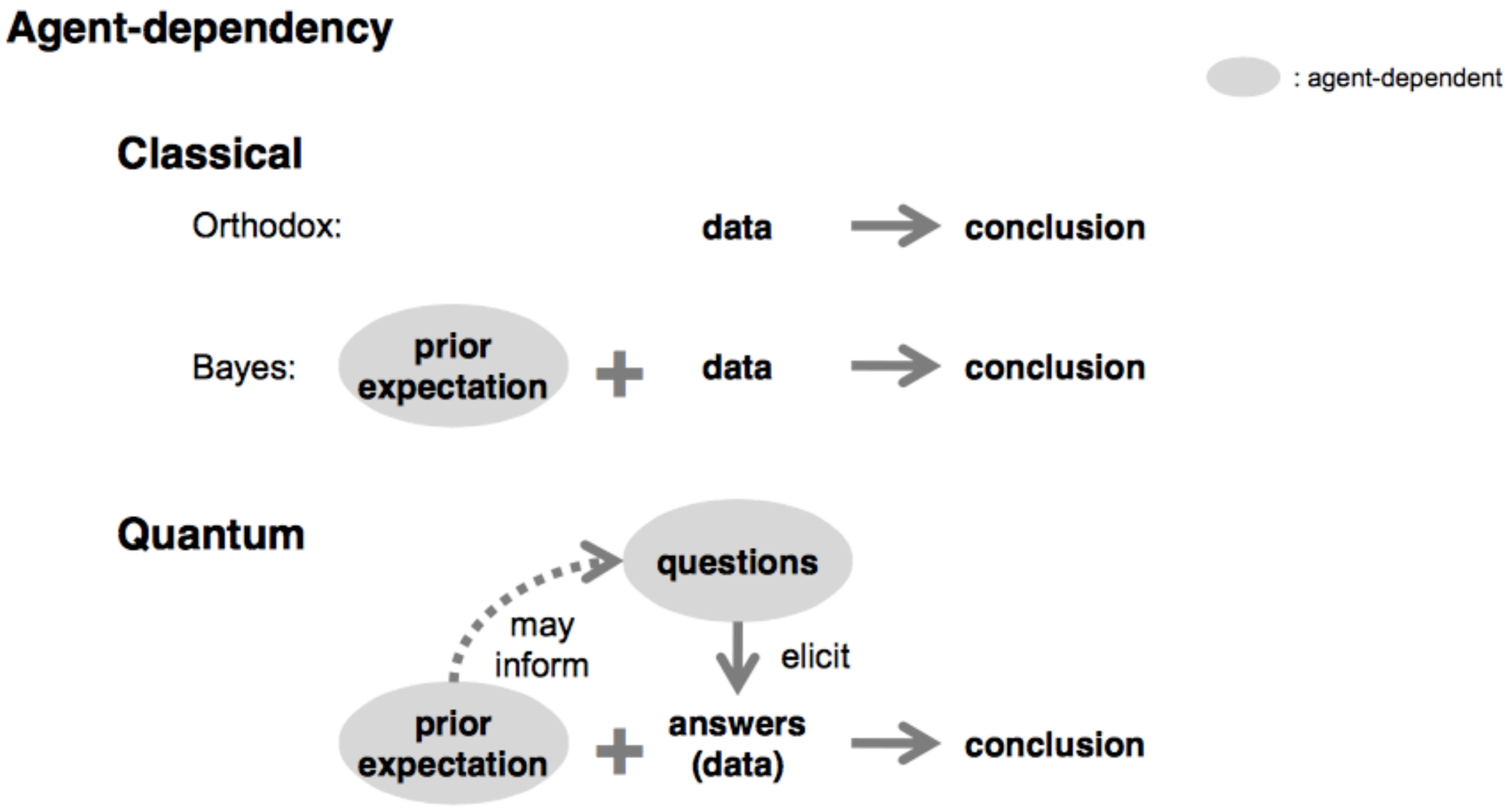

Image: J. Rao, Goethe University, Frankfort

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Review - Shared Ideas in CP & QP

Generic concept	Classical concept	Quantum concept
Proposition system	Sample space	Hilbert space
Hypothesis, proposition	Subset	Subspace
Complement	Set complement	Orthogonal complement
Contradiction, exclusion	Disjointedness	Orthogonality
Granularity	Cardinality	Dimension

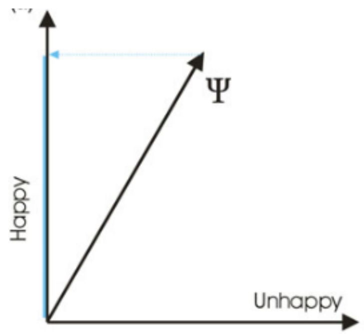
Review- Opposing Ideas in CP & QP

Classical Probability

- Based on set theory
- Two events cannot occur simultaneously - disjoint
- Mixed state:
  - A person is either exactly happy or exactly unhappy, but we don't know which… so we assign some probability to each possibility.
  - There is a definite state that already exists, just unknown.

Quantum Probability

- Based on geometric theory
- Two states cannot occur simultaneously - orthogonal
- Superposition:
  - The person is neither happy nor unhappy, but in an indefinite state regarding happiness, simultaneously entertaining both possibilities.
  - Indefinite state is resolved to a definite (basis) state in decision making.



E. Pothos & J. Busemeyer

CP Failures (Review)

- Order/Context effects
  - Clinton/Gore experiment
- Conjunction Fallacy
  - Linda experiment
  - But in QP, it's not a fallacy!

QP Characteristics - Compatibility

- Compatibility
  - If two questions (A and B) about a system are compatible, it is always possible to define the conjunction between A and B.
  - CP → assumes all questions are compatible… i.e. “Are A and B true?” always has a yes or no answer

- QP theory does not assume all questions are compatible. In QP theory, two questions are *incompatible* if an answer to A implies a superposition state regarding B.
  - Psychological analogy → Considering question A alters the state of our mind. Thoughts about question A inherently lead to other thoughts, which will effect the consideration of question B when it is asked.
  - Incompatibility represented by subspaces at nonorthogonal angles
    - Happiness and Employment example

QP Characteristics - Entanglement

- Independent Events in CP theory
  - Events have no effect on one another
  - In QP, state vector can written as tensor product between two vectors

- Entangled state
  - Extreme form of dependency beyond what can be represented in CP theory
  - Happiness and employment example: an operation that influences happiness unavoidably affects considering employment

QP Characteristics - Time Evolution

Classical Probability		Quantum Probability
• Linear transformation of the initial state	✓	• Linear transformation of the initial state
		• Rotation
• Obeys total probability	✗	• Can violate total probability ◦ Interference terms

Time Evolution Example

- Take happiness & employment example again. Assume questions are compatible. Initial:

$$\begin{aligned} Prob(happy, unknown\ empl.) &= Prob(happy \wedge employed) \\ &\quad + Prob(happy \wedge not\ employed). \end{aligned}$$

- CP Theory after time process:

$$\begin{aligned} &Prob(happy, unknown\ empl., at\ t) \\ &= Prob(happy\ at\ t \wedge employed) \\ &\quad + Prob(happy\ at\ t \wedge notemployed) \end{aligned}$$

- QP Theory after time process:

$$\begin{aligned} &Prob(happy, unknown\ empl., at\ t) \\ &= Prob(happy\ at\ t \wedge employed) \\ &\quad + Prob(happy\ at\ t \wedge not\ employed) \\ &\quad + Interference(crossproduct)\ terms \end{aligned}$$

Time Evolution Example

- Interpretation of interference:
- Person may think she would be happy if it turns out she is employed (wants the money?)
- Person may think she would be happy if she loses her job (doesn't like the job?)
- BUT if unsure about employment… unhappy. "It is as if these two individually good reasons for being happy cancel each other out." (Busemeyer & Bruza)

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QP & Machine Learning

- Goals in machine learning are to improve artificial intelligence to the quality of human-like (or better) judgement
- Many machine learning algorithms are also called “statistical learning” methods because of their basis in either frequentist or Bayesian theory (classical probability). Additionally, there are many physics-inspired machine learning algorithms.
- If quantum theory has as much (or more) value in human decision-making as CP, then its value to machine learning will hopefully be significant.

Decision Trees

- The basis of many tree-based machine learning algorithms – including bagged trees, boosted trees, and random forests
- Applicable in settings that desire interpretation of the prediction; easier to explain than linear regression
- Closely mirrors human decision making
- NP complete
- www.20q.net

Q30. I am guessing that it is a cell phone?
Right, **Wrong**, **Close**

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29. I guessed that it was a laptop computer? **Wrong**.
28. Does it refract light? **No**.
27. Does it fold up easily? **Yes**.
26. Can you read it? **Yes**.
25. Would you use it in the dark? **Sometimes**.
24. Does it bring joy to people? **Sometimes**.
23. Do you use it with a computer? **Sometimes**.
22. I guessed that it was a cordless phone? **Close**.
21. Can you speak into it? **Yes**.
20. I guessed that it was a telephone headset? **Close**.
19. Is it small? **Yes**.
18. I guessed that it was a telephone? **Close**.
17. Is it white? **Sometimes**.
16. Is it annoying? **No**.
15. Would you pay to use it? **Yes**.
14. Is it found on a desk? **Yes**.
13. Is it pleasurable? **No**.
12. Could you send it in the mail? **Yes**.
11. Would you give it as a gift? **Yes**.
10. Does it have a memory? **Yes**.
9. Does it require specific knowledge to use it? **No**.
8. Can it save your life? **Yes**.
7. Does it have lots of buttons? **Yes**.
6. Does it have pages? **No**.
5. Does it usually have four corners? **Yes**.
4. Is it shiny? **No**.
3. Does it have a hard outer shell? **Yes**.
2. Do most people use this daily? **Yes**.
1. It is classified as **Concept**.

Q16. I am guessing that it is a mobile phone?
Right, **Wrong**, **Close**

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15. Does it have a long tail? **No**.
14. Can it fit in an envelope? **Yes**.
13. Can it think? **Yes**.
12. Do you clean it regularly? **Yes**.
11. Can you put something into it? **Yes**.
10. Can it add? **Yes**.
9. Is it connected to a wire? **No**.
8. Does a basketball player use it? **Yes**.
7. Does it ring? **Yes**.
6. Does it like to be petted? **No**.
5. Does it break if dropped? **Yes**.
4. Is it hard? **Yes**.
3. Is it flat? **Yes**.
2. Would you use it daily? **Yes**.
1. It is classified as **Concept**.

20Q won!

Play Again

You were thinking of a mobile phone.
Is it connected to a wire? You said No, 20Q was taught by other players that the answer is Yes.
Contradictions Detected

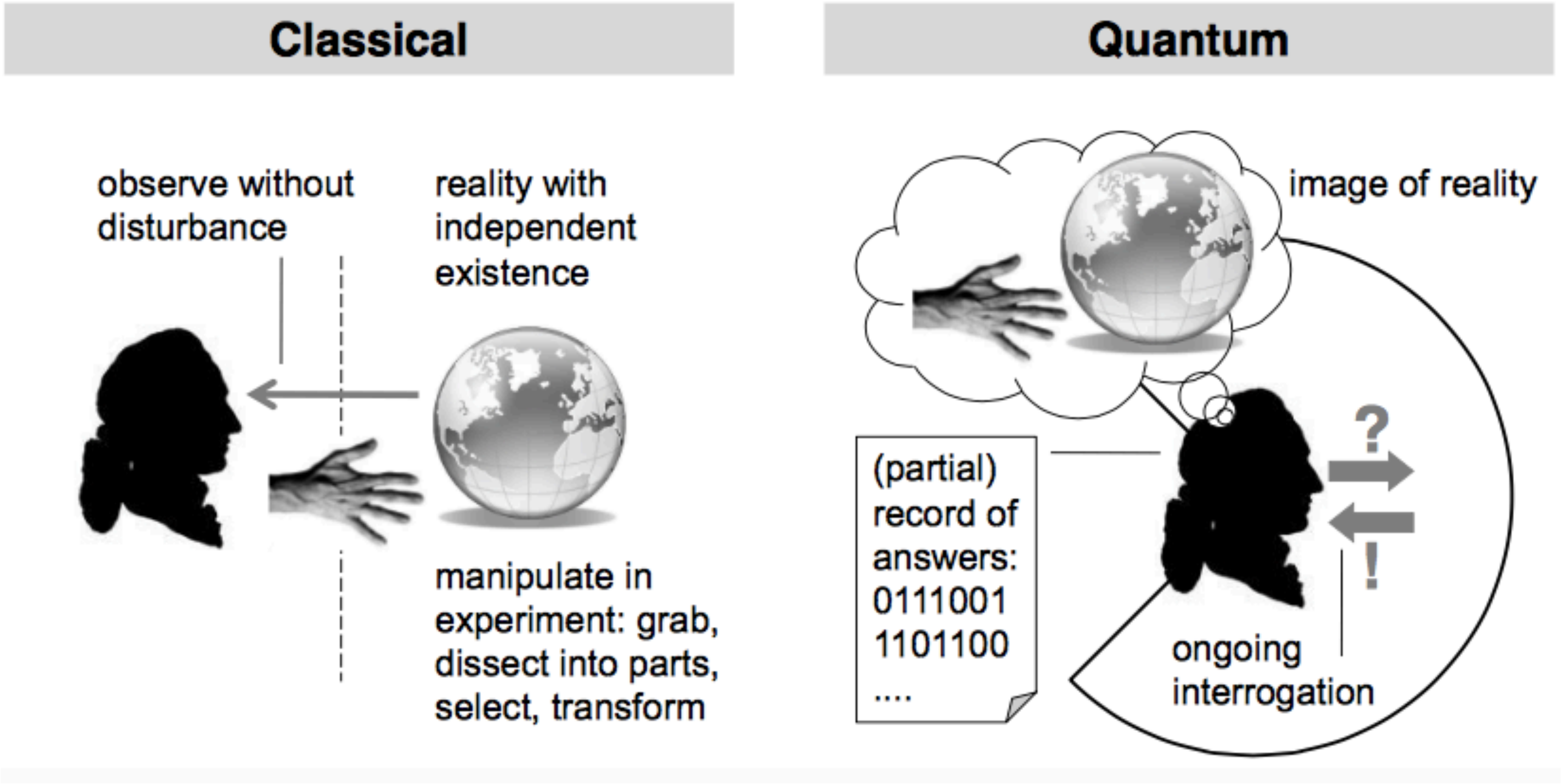


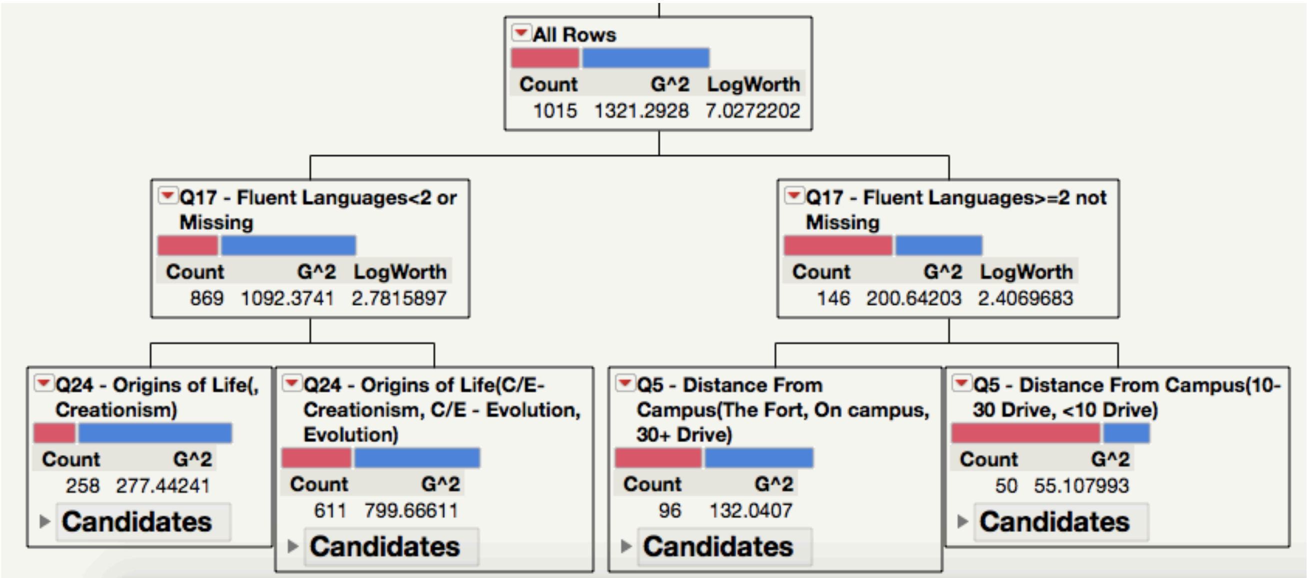
Image: J. Rao, Goethe University, Frankfort

Constructing a Decision Tree

- Top-down algorithm
- Tree structure
- Start with the root node and partition the data with a training set
- Partition by finding the variable that returns the highest information gain (i.e. most homogenous branches)
  - High information gain is equivalent to a decrease in entropy
- Once the tree is made, it can easily be transformed into a set of IF/ELSE rules
- Random Forests: makes use of many decision trees
  - https://www.kaggle.com/c/dsg-hackathon

Decision Trees

- Algorithm: See JMP



- We predict that each observation belongs to the most commonly occurring class of training observations
  - Gini impurity or cross-entropy are used to evaluate the “quality” of a particular split
  - The smaller the value of Gini impurity/cross-entropy, the more “pure” the node is.

Quantum Decision Trees

- Proposed by Songfeng Lu and Samuel Braunstein
- Node is split by Quantum entropy impurity criterion instead of Gini impurity in the "classical" decision tree
  - The smaller the value of quantum entropy, the simpler the decision tree

$S(\rho) = -tr(\rho \log \rho)$

- The quantum entropy impurity of node t

- Entropy quantifies the departure of the system from a pure state
  - Entropy is zero if and only if ρ represents a pure state
  - Entropy is maximized with a maximally mixed state
- Just like a classical decision tree, this measure of entropy will decide the "purity" of a node

Quantum Decision Tree Problems

- Analyzing error of a quantum decision tree
- Pruning methods/strategies to stop splitting
- Training data with quantum noise
- Attributes with differing weights
- The performance of the algorithm has not been carried out experimentally
  - Not convenient, public repositories of data to test the quantum algorithm on

Other Quantum Machine Learning Algorithms

- Quantum Support Vector Machine
  - SVM is similar in understanding to DT – construct a hyperplane separating a multidimensional dataset
- Quantum K-Nearest Neighbors
  - Another classification algorithm – classify by the class of your neighbors
  - Grover's algorithm used to find min distance
- Quantum "Deep Learning"

Investment in Quantum ML

- Microsoft
- https://www.microsoft.com/en-us/research/event/quantum-machine-learning/
- Google – Quantum Artificial Intelligence Lab
- https://research.googleblog.com/2013/05/launching-quantum-artificial.html