Adiabatic Quantum Computing

John Reynolds

11/16/2016

What is it?

- Quantum computing that relies on the *adiabatic theorem*
- It involves continuous time evolution of the quantum system

How?

- Uses *Hamiltonians* to solve optimization style problems

Evolution

- From LNUC III.B.2.b
- Evolution of a quantum system is described by a unitary transformation
- $|\psi'\rangle = U(t, t')\,|\psi\rangle = U\,|\psi\rangle$
- We have been using discrete-time circuits
- Continuous time evolution of a quantum system is given by the Schrödinger equation

Schrödinger Equation

- $\hat{E} = \hat{H} = \hat{T} + \hat{V}$
- $\hat{E} = \hat{H}$
- $\hat{E} \left|\psi\right\rangle = \hat{H} \left|\psi\right\rangle$
- $i\hbar \frac{\partial}{\partial t} \left|\psi(t)\right\rangle = \hat{H} \left|\psi(t)\right\rangle$
- $i\hbar \left|\dot{\psi}\right\rangle = \hat{H} \left|\psi\right\rangle$

Hamiltonians

- $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$
- $i\hbar|\dot{\psi}\rangle = \hat{H}|\psi\rangle$
- i is the imaginary unit, $\hbar$ is the reduced planck constant
- $\hat{H}$ is the Hamiltonian operator
- $\hat{H}$ will change based on the situation
- Typically classified by number of particles, number of dimensions, potential energy function..

Example Hamiltonian

- The following is the Hamiltonian of representing the energy of one free particle (V = 0) in one dimension:
  - $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
- Now in 3 dimensions:
  - $\hat{H} = -\frac{\hbar^2}{2m}\nabla^2$
  - $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
- This is because:
  - $\hat{T} = \frac{\hat{p}\cdot\hat{p}}{2m} = \frac{\hat{p}^2}{2m} = (-i\hbar\nabla)(-i\hbar\nabla)(\frac{1}{2m}) = -\frac{\hbar^2}{2m}\nabla^2$
- Remember:
  - $\hat{H} = \hat{T} + \hat{V}$

Adiabatic Theorem

Adiabatic Theorem

A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum.

In other words..

Gradually changing the conditions of a system allows it to adapt its configuration while remaining in. We can choose a simple starting Hamiltonian that we can construct and evolve it into a more complicated Hamiltonian whose ground state represents a solution to our problem.

Adiabatic Quantum Computing

- **Step 1:** Find a Hamiltonian whose ground state represents the solution
- **Step 2:** Prepare a system with a simple Hamiltonian and initialize to the ground state
- **Step 3:** Evolve adiabatically with the Schrödinger equation

Classical Optimization

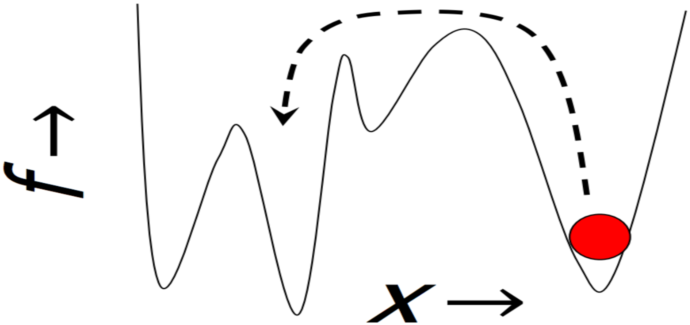

Figure 1: Stuck in local minimum.

Quantum Optimization

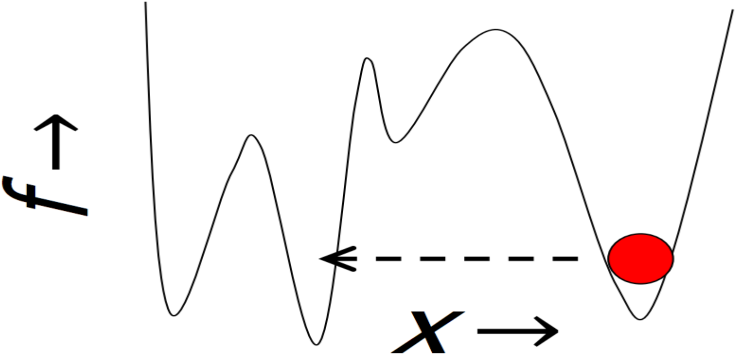

Figure 2: Quantum Tunneling.

Adiabatic 3-Sat

- $C_1 \wedge C_2 \wedge \cdots \wedge C_M$
- $C_i = (x_1, x_2, \cdots, x_3)$
- $H(t) = H_{C_1}(t) + H_{C_2}(t) + \cdots + H_{C_M}(t)$
- $h_C(z_{1C}, z_{2C} \cdots z_{nC})$

$$= \begin{cases} 0 & \text{clause C satisfied} \\ 1 & \text{clause C violated} \end{cases}$$

- $h = \sum_C h_C$

Adiabatic Quantum 3-Sat

- bit z_i is a spin-$\frac{1}{2}$ qubit labeled $|z_i\rangle$ where $z_i = 0, 1$
- $\frac{1}{2}(1 - \sigma_z^{(i)})\,|z_i\rangle = z_i\,|z_i\rangle$
- $\sigma_z^{(i)} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- $H_{P,C}(|z_1\rangle\,|z_2\rangle \cdots |z_n\rangle) = h_C(z_{i_C}, z_{j_C}, z_{k_C})\,|z_1\rangle\,|z_2\rangle \cdots |z_n\rangle$
- $H_P = \sum_C H_{P,C}$
- Note, $H_P\,|\psi\rangle = 0$ only if $|\psi\rangle$ is a superposition of states of the form $|z_1\rangle\,|z_2\rangle \cdots |z_n\rangle$ where $z_1, z_2, \cdots, z_n$ satisfy

Adiabatic Quantum 3-Sat

- $H_B^{(i)} = \frac{1}{2}(1 - \sigma_x^{(i)})$
- $\sigma_x^{(i)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $H_B^{(i)} |x_i = x\rangle = x |x_i = x\rangle$
- $|x_i = 0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $|x_i = 1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Adiabatic Quantum 3-Sat

- $H_{B,C} = H_B^{(i_C)} + H_B^{(j_C)} + H_B^{(k_C)}$
- $H_B = \sum_C H_{B,C}$
- The ground state of H_B is $|x_1 = 0\rangle |x_2 = 0\rangle \cdots |x_n = 0\rangle$
- $|x_1 = 0\rangle |x_2 = 0\rangle \cdots |x_n = 0\rangle = \frac{1}{2^{\frac{n}{2}}} \sum_{z_1} \sum_{z_2} \cdots \sum_{z_n} |z_1\rangle |z_2\rangle \cdots |z_n\rangle$

Adiabatic Quantum 3-Sat

- $H(t) = (1 - t/T)H_B + (t/T)H_P$
- Let s = t/T
- $\widetilde{H}(t) = (1 - s)H_B + (s)H_P$
- Now, evolve according to $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle$
- The ground state of $|\psi(T)\rangle$ for big enough T will be very nearly the ground state of H_P
- Measuring the bits in $|\psi(T)\rangle$ will be a satisfying assignment

Quantum Algorithm

- Choose an easily constructible initial state, which is the ground state of H_B
- A time-dependent Hamiltonian H(t) that is easily constructible
- An evolution time T
- Schrödinger evolution with $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$
- The final state $|\psi(T)\rangle$ will be very nearly the ground state of H_P
- A measurement of $z_1, z_2, \cdots, z_n$ in the state $|\psi(T)\rangle$ will be satisfying (if the formula can be satisfied)

References

- Prof. MacLennan's *Unconventional Computation* course. ©2016, B. J. MacLennan, EECS, University of Tennessee, Knoxville. Version of August 16, 2016.
- E. Farhi, J. Goldstone, S. Gutmann, M. Sipser, "*Quantum C omputation by Adiabatic Evolution*", quant-ph/0001106.