Oracle Turing Machine

Kaixiang Wang

- Pre-background:
  - What is Turing machine
- Oracle Turing Machine
  - Definition
  - Function
  - Complexity
- Why Oracle Turing Machine is important
- Application of Oracle Turing Machine

Turing Machine

- Turing machine is a mathematical model of computation.
- Turing Machine simulate two steps during calculation:
  - Write or erase a symbol on tape
  - Translate focus(pointer) from a tape block to another
- Church Turing Thesis
  - A function on the natural numbers is computable by a human following an algorithm, ignoring resource limitations, if and only if it is computable by a Turing Machine.

Turing Machine

$$M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle$$

- Q is a finite, non-empty set of states.
- Γ is a finite, non-empty set of tape symbols
- B is a blank symbol.
- Σ is the set of input symbols
- δ is the transition functions.
- q_0 is the initial state.
- F is the final state. Accepting state.

Turing Machine

a machine reverse 0 to 1 and 1 to 0

- (q0, 0) → (q0, 1, R)
- (q0, 1) → (q0, 0, R)
- (q0, b) → (qf, b, R)

0	0	0	1	1	1
^					

1	1	0	1	1	1
		^			

1	1	1	0	1	1
				^	

1	0	0	1	1	1
	^				

1	1	1	1	1	1
			^		

1	1	1	0	0	1
					^

1	1	1	0	0	0

Turing Machine

define a language accept 01 and rewrite to 10

- (q0, 0) → (q1, 1, R)
- (q1, 1) → (q2, 0, R)
- (q2, b) → (qf, b, R)

0	0	0	1	1	1
^					

1	0	0	1	1	1
	^				

0	1
^	

1	1
	^

1	0
	^

Oracle Turing Machine

Definition

definition referenced from Wikipedia

- An oracle machine is an abstract machine used to study decision problems.

- An oracle machine must include a typical Turing machine. AND oracle part:
  - 1. An Oracle Tape.
  - 2. An Oracle head.
  - 3. Two special state: ASK and RESPONSE

Oracle Turing Machine

Definition

definition referenced from Wikipedia

- Oracle machine is a higher hierarchy Turing machine.

- The black box (Oracle part) can be treated like a sub-level machine which solve the certain decision problem in a single operation.

Oracle Turing Machine

Function

- What happens in Oracle part:
  - the contents of the oracle tape are viewed as an instance of the oracle's computational problem;
  - the oracle is consulted, and the contents of the oracle tape are replaced with the solution to that instance of the problem;
  - the oracle head is moved to the first square on the oracle tape;
  - the state of the oracle machine is changed to RESPONSE.

Oracle Turing Machine

Complexity

- The ideal Oracle Turing Machine ignore the time and space complexity of the black box part.
  - Thus the Oracle Turing Machine help understanding and proof of complexity problems.

Oracle Turing Machine

importance

- Unrealistic Machine.

- Help us deal with more theoretic problems.
  - Ex:
    - Halting Problem
    - Turing reduction
    - Complexity problem(P != NP)

The Halting Problem

- Is it possible to determine a machine halt or enter an infinite loop at input i?

The Halting Problem

The pseudocode

```
Int H(procedure, Input); //return 0 if halt, or 1 if infinite loop

Int U(P)
{
    If (H (P ,P) ==1 ) {
        Return 0; //halt
    } else {
        while (1) // infinite loop
    }
}
```

Paradox!

The Halting Problem

- A machine with an oracle for the halting problem can determine whether particular Turing machines will halt on particular inputs, but they cannot determine, in general, whether machines equivalent to themselves will halt. This creates a hierarchy of machines, each with a more powerful halting oracle and an even harder halting problem. This hierarchy of machines can be used to define the arithmetical hierarchy (Börger 1989).

--- definition reference from wikipidia

```
programHalt(program, input) //does program halt
{
Return ture
}
```

The Turing Reduction

- In computability theory, a Turing reduction from a problem A to a problem B, is a reduction which solves A, assuming the solution to B is already known (Rogers 1967, Soare 1987)

definition referenced from Wikipedia

The Complexity Problem

- P and NP problem
  - P problem:

  an algorithm solving the task that runs in polynomial time.

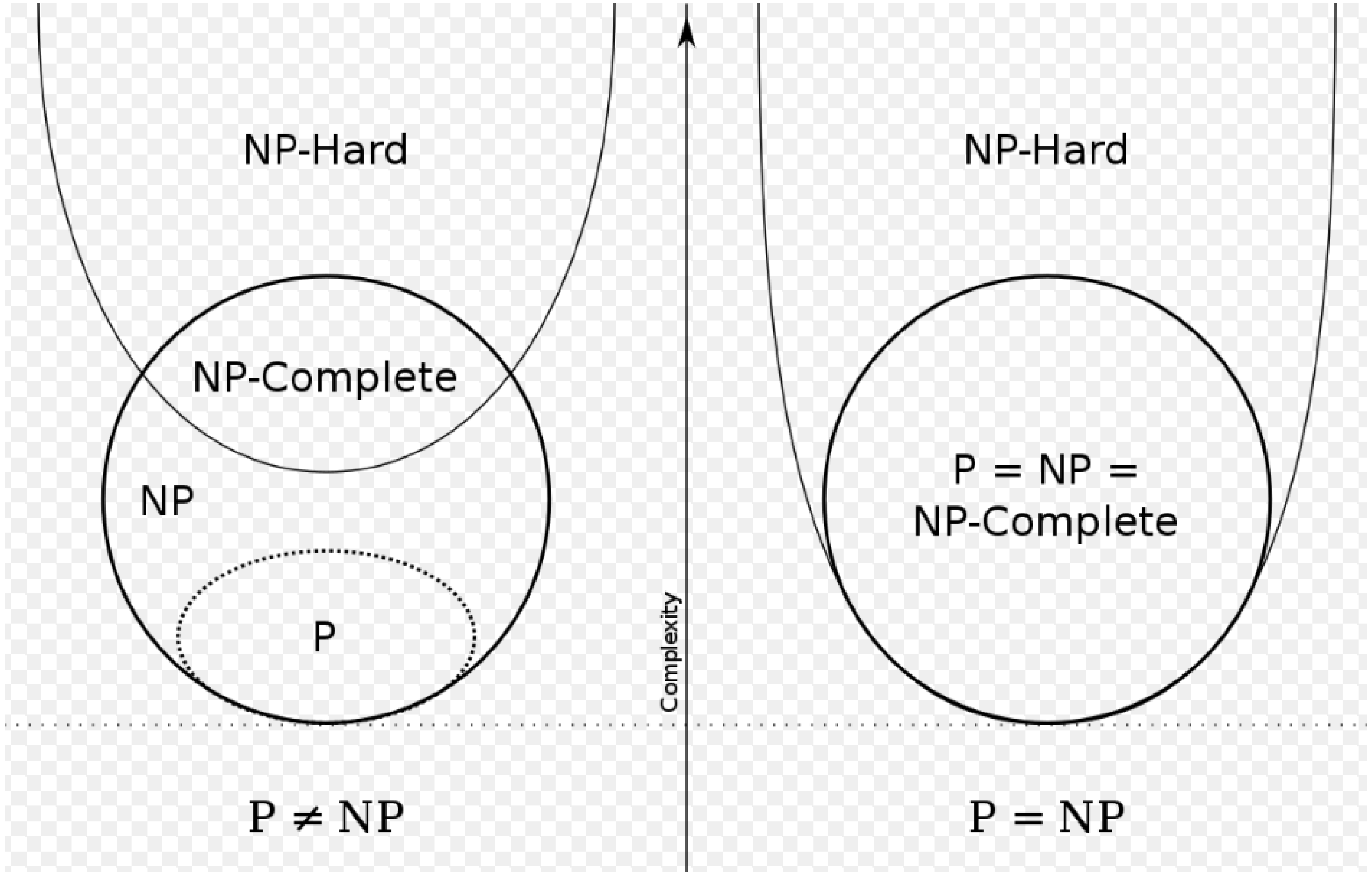
  - NP problem:

  For questions have no known way to find an answer quickly, but if one is provided with information showing what the answer is, it is possible to verify the answer quickly

  - NP hardness:

  Problem at least as **hard** as the hardest problems in **NP.**

The Complexity Problem



Thank you

Questions?