Quantum Cellular Automata

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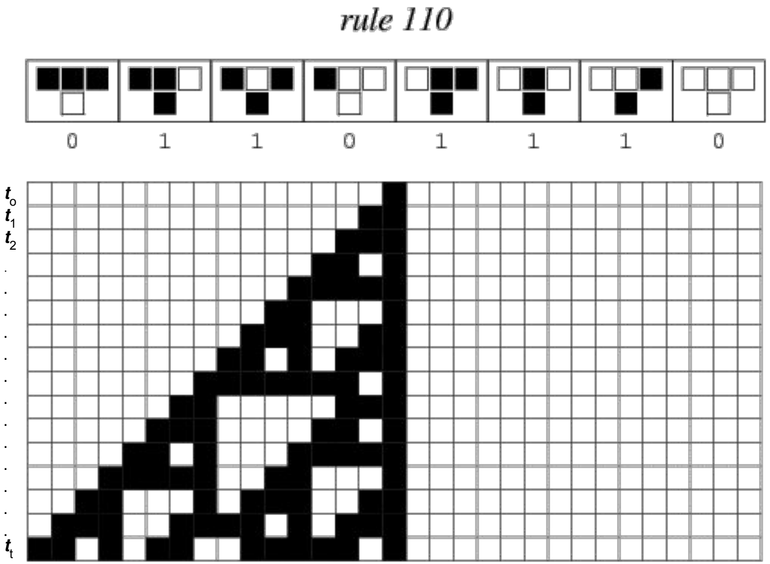
Contents

Classical CA: History

- Conceived in 1950s by von Neumann and Ulam.
- Loose work continued into early 1980s but lacked serious formal analysis and applicability.
- This changed with Stephen Wolfram's "Statistical Mechanics of Cellular Automata" in 1983

Classical CA: Definition

- Requirements:
  - A lattice of cells (potentially infinite, but discrete)
  - A discrete state localized to a cell
  - A rule (function) that maps state of local region about a cell to a new center cell state.



Classical CA: Motivations

- Many physical phenomena occur locally which makes CA intuitive models of physical reality.
    - Diffusion, percolation, phase transitions
- Complex phenomena emerge from simple local interactions making CA a useful tool in the study of complex systems.
    - Chaos, pattern formation
    - Excitable media, biological systems
- Inherent parallelism implies physical implementations with minimal control

Quantum CA: Motivations

- CA have been an enormously useful tool to understand complex behavior in physical systems it is natural to assume quantum CAs would do the same.

- Local quantum control is perhaps the most significant barrier to scalable and practical quantum computing.

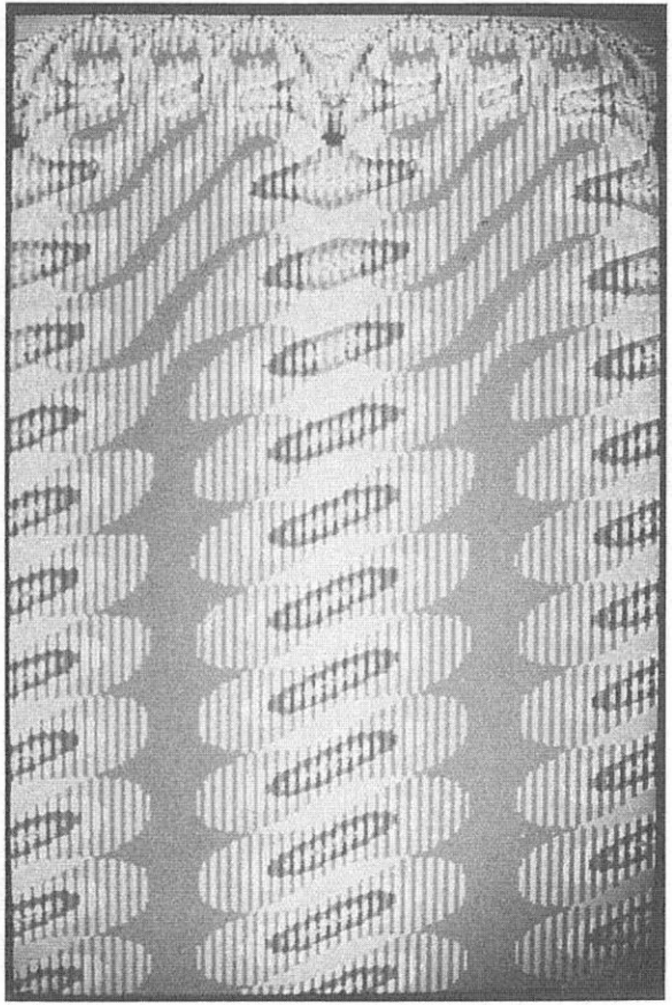
Quantum CA: Considerations

- Quantum states exist in a Hilbert space
- Entangled states are spatially non-local and local interactions are in general non-abelian
- Time translation occurs via unitary operators generated by local interactions (i.e. reversible)
- A QCA definition should be satisfying at both an axiomatic and physical level.

Quantum CA: Early development

- First exploration of QCA was by Grössing and Zielenger in 1988
  - Motivated by exploring the limit of a physically implemented cellular automata.
- The definition turned out to be non-physical due to their use of non-unitary evolution and global normalization scheme.
- Mathematically interesting and demonstrated that a satisfying QCA definition might be nontrivial.
  - Properties of this model were investigated quite thoroughly for ~6 years

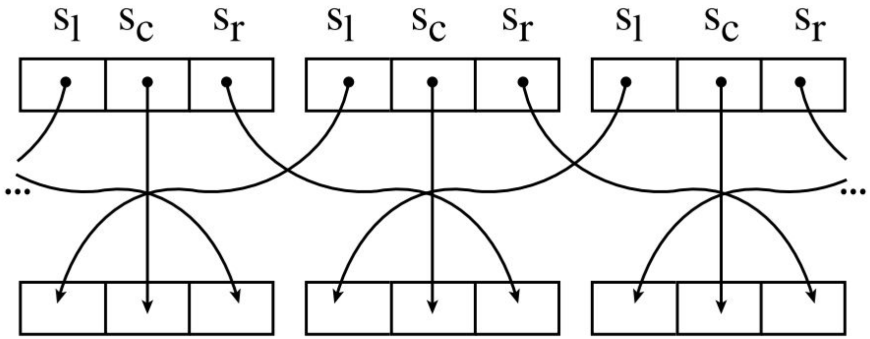
Quantum CA: Early development



Quantum CA: Further Development

- In 1995 Watrous proposed a 1D QCA that he proved to be equivalent to a quantum turing machine (constant slowdown).

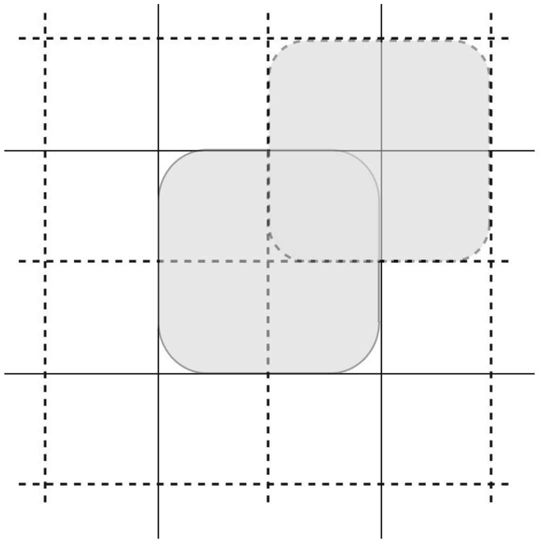
- The definition utilized a cell-partitioning scheme to enable local operations and reversibility.



Quantum CA: Axiomatic Approach

- The first axiomatic approach to QCA was by Schumacher and Werner in 2004
    - Made use of the Heisenberg formalism of QM
    - The update rule is a homomorphism on a local operator algebra

- Used a lattice-partitioning scheme:



Quantum CA: Other approaches

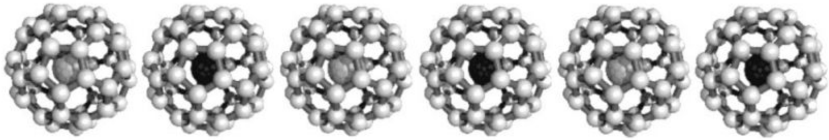
- Many alternative definitions have been proposed for QCA:
  - Block-partitioned QCA (2003)
  - Local Unitary QCA (2007)
  - Hamiltonian QCA (2008)
  - Partitioned Unitary QCA (2018)
- A sequence of papers by Arrighi et al. approach QCA using a similar operator focused formalism. (2008-2012)
  - Demonstrated proofs for n-dimensional QCA that are computationally universal.
  - Showed that all previous QCA were equivalent
  - Believed that cell-partitioned QCA are the most natural.

Quantum CA: Relation to other models

- Quantum Lattice Gases:
  - Choose a rule that describes particle motion on a lattice
  - Take a continuous limit in time and space
  - Can be used to derive Schrödinger and Dirac equations
- Quantum Walks
  - Quantum analogs of classical random walks
  - Many useful graph algorithms have been described recently and it is a very active research area
  - It has been shown that all QW algorithms are inherited by QCA, but not vice versa.

Physical Realizations

- Chain of endohedral fullerene nanoparticles:



- Uses ESR and NMR techniques to locally address each nuclear spin.

- Drawbacks include lack of projective readout and some scaling issues.

Outlook

- The power of classical CAs were easily demonstrated due to their computational simplicity which helped drive their popularity.
  - Lattice-partitioned models might prove useful to model on circuit based quantum hardware

- Potentially useful to explore exotic matter and quantum phase transitions, quantum gravity, or particle interactions.

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