H Exercises

Exercise III.1 Compute the probability of measuring $|0\rangle$ and $|1\rangle$ for each of the following quantum states:

1. $0.6|0\rangle + 0.8|1\rangle$.
2. $\frac{1}{\sqrt{3}}|0\rangle + \sqrt{2/3}|1\rangle$.
3. $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$.
4. $-\frac{1}{25}(24|0\rangle - 7|1\rangle)$.
5. $-\frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\pi/6}}{\sqrt{2}}|1\rangle$.

Exercise III.2 Compute the probability of the four states if the following are measured in the computational basis:

1. $(e^i|00\rangle + \sqrt{2}|01\rangle + \sqrt{3}|10\rangle + 2e^{2i}|11\rangle)/\sqrt{10}$.
2. $\frac{1}{2}(-|0\rangle + |1\rangle) \otimes (e^{\pi i}|0\rangle + e^{-\pi i}|1\rangle)$.
3. $$(\sqrt{1/3}|0\rangle - \sqrt{2/3}|1\rangle) \otimes \sqrt{2}\left(\frac{e^{\pi i/4}}{2}|0\rangle + \frac{e^{\pi i/2}}{2}|1\rangle\right).$$

Exercise III.3 Suppose that a two-qubit register is in the state

$$|\psi\rangle = \frac{3}{5}|00\rangle - \frac{\sqrt{7}}{5}|01\rangle + \frac{e^{i\pi/2}}{\sqrt{5}}|10\rangle - \frac{2}{5}|11\rangle.$$

1. Suppose we measure just the first qubit. Compute the probability of measuring a $|0\rangle$ or a $|1\rangle$ and the resulting register state in each case.
2. Do the same, but supposing instead that we measure just the second qubit.

Exercise III.4 Prove that projectors are idempotent, that is, $P^2 = P$.

Exercise III.5 Prove that a normal matrix is Hermitian if and only if it has real eigenvalues.

Exercise III.6 Prove that $U(t) \stackrel{\text{def}}{=} \exp(-iHt/\hbar)$ is unitary.

Exercise III.7 Use spectral decomposition to show that $K = -i\log(U)$ is Hermitian for any unitary U, and thus $U = \exp(iK)$ for some Hermitian K.

Exercise III.8 Show that the commutators ($[L, M]$ and $\{L, M\}$) are bilinear (linear in both of their arguments).

Exercise III.9 Show that $[L, M]$ is anticommutative, i.e., $[M, L] = -[L, M]$, and that $\{L, M\}$ is commutative.

Exercise III.10 Show that $LM = \frac{[L,M]+\{L,M\}}{2}$.

Exercise III.11 In Sec. B.5 we proved the no-cloning theorem with single ancillary constant qubit. Prove that the cloning is still impossible if multiple ancillary qubits are provided. That is, show that we cannot have a unitary operator $U(|\psi\rangle \otimes |C\rangle) = |\psi\rangle|\psi\rangle \otimes |D\rangle$, where $|C\rangle$ is an $n > 1$ dimensional vector and $|D\rangle$ is an $n-1$ dimensional vector. (Note that $|D\rangle$ might depend on $|\psi\rangle$.)

Exercise III.12 Show that the four Bell states are orthonormal (i.e., both orthogonal and normalized).

Exercise III.13 Prove that $|\beta_{11}\rangle$ is entangled.

Exercise III.14 Prove that $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ is entangled.

Exercise III.15 What is the effect of Y (imaginary definition) on the computational basis vectors? What is its effect if you use the real definition (C.2.a, p. 112)?

Exercise III.16 Prove that I, X, Y, and Z are unitary. Use either the imaginary or real definition of Y (C.2.a, p. 112).

Exercise III.17 What is the matrix for H in the *sign basis*?

Exercise III.18 Show that the X, Y, Z and H gates are Hermitian (their own inverses) and prove your answers. Use either the imaginary or real definition of Y (C.2.a, p. 112).