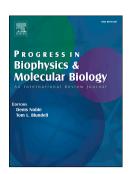# Accepted Manuscript

East-West paths to unconventional computing

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# East-West Paths to Unconventional Computing

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**Abstract**

Unconventional computing is about breaking boundaries in thinking, acting and computing. Typical topics of this non-typical field include, but are

not limited to physics of computation, non-classical logics, new complexity measures, novel hardware, mechanical, chemical and quantum computing. Unconventional computing encourages a new style of thinking while practical applications are obtained from uncovering and exploiting principles and mechanisms of information processing in and functional properties of, physical, chemical and living systems; in particular, efficient algorithms are developed, (almost) optimal architectures are designed and working prototypes of future computing devices are manufactured. This article includes idiosyncratic accounts of 'unconventional computing' scientists reflecting on their personal experiences, what attracted them to the field, their inspirations and discoveries.

## 1. Introduction

The term 'unconventional computing' has no exact definition. Proceeding by inclusiveness we could say that the following research topics are most commonly, but not necessarily, classified as 'unconventional': physics of computation (e.g. conservative logic, thermodynamics of computation, reversible computing, quantum computing, collision-based computing with solitons, optical logic); chemical computing (e.g. implementation of logical functions in chemical systems, image processing and pattern recognition in reaction-diffusion chemical systems and networks of chemical reactors); bio-molecular computing (e.g. conformation based, information processing in molecular arrays, molecular memory); cellular automata as models of massively parallel computing complexity (e.g. computational complexity of non-standard computer architectures; theory of amorphous computing; artificial chemistry); non-classical logics (e.g. logical systems derived from space-time behaviour of natural systems, logical reasoning in physical, chemical and biological systems); smart actuators (e.g. molecular machines incorporating information processing, intelligent arrays of actuators); novel hardware systems (e.g. cellular automata VLSIs, functional neural chips); mechanical computing (e.g. micromechanical encryption, computing in nanomachines, physical limits to mechanical computation).

There are two discipline-wise paths to unconventional computing. First, you are initially trained as mathematician or computer scientist, then you rebel and start pushing the limits of conventional science, and eventually

find yourself outside the well establish tracks. Second, more common, you are trained as chemist, biologist, physicist, then you got involved in computation and got eager to understand the meaning of information and computation in natural systems, and subsequently start realising computing devices in novel substrates. Following the overall goals of this special issue we have aimed to represent a mosaic of snapshots of personal, scientific, spiritual and philosophical experiences of scientists working in the field of unconventional computing. We did not try to answer the question "How?" each of one of them got into the field but rather "Why?" they found themselves doing unconventional computing. Some authors did not even answer "'Why?" because no answer may exist.

To make the compendium of 'paths towards unconventional' representative we have invited authors with backgrounds in different fields of science, various stages of their academic career, and from a wide geographic distribution. They are Cristian S. Calude (Sect. 4), who excels in computability and algorithmic and quantum randomness and was the first to propose the 'unconventional computation; Selim Akl (Sect. 5), who is amongst the fathers of parallel computation, especially sorting, quantum computing, and non-universality; Kenichi Morita (Sect. 2), the guru of reversibility and cellular automata; Yukio-Pegio Gunji (Sect. 3), well known for his unorthodox thoughts on observation and complexity; Hector Zenil (Sect. 6), a pioneer in applications of algorithmic complexity to molecular and computational biology; Andrew Schumann (Sect. 7), who deals with unconventional logic for modelling behaviours; Zoran Konkoli (Sect. 8), known for his unique interdisciplinary contributions to physics and metaphysics of computation; Maurice Margenstern (Sect. 9), famous for hyperbolic cellular automata and computation; José Félix Costa (Sect. 10), excelling in physics and logic of computation; Mark Burgin (Sect. 11), who has advanced super-recursive algorithms, axiomatic complexity and inductive Turing machines; Andrew Adamatzky (Sect. 12), who has designed a range of weird prototypes of unconventional computing devices; Mohammad M. Dehsibi (Sect. 13), who has discovered trends in evolving complexity of Persian language; Richard Mayne (Sect. 14), who has advanced bio-medical foundations of computing; Bruno Marchal (Sect. 15), who has advanced foundations of the physical sciences and the mind-body problem; Yaroslav D. Sergeyev (Sect. 16), who founded the field of numerical computing with infinities and infinitesimals having many applications and a striking importance for foundations of mathematics; Karl Svozil (Sect. 17), who attempted to invent superluminal space travel, and became

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fascinated by the metaphysical debate on (in)determinism; Genaro Martinez (Sect. 18), cellular automata guru; Georgios Ch. Sirakoulis (Sect. 19), an unconventional hardware engineer; Bruce MacLennan (Sect. 20), a prophet of continuous computing; Susan Stepney (Sect. 21), who is making computer science a natural science.

## 2. Kenichi Morita: Unconventional Knowledge

Distinguishing between "conventional computing" and "unconventional computing" is not so easy, since the notion of unconventional computing is rather vague. Some scientist may want to give a rigorous definition of it. But, if he or she does so, then unconventional computing will become less attractive. The very vagueness of the concept stimulates one's imagination, and thus is a source of creation.

In this short essay, related to such a problem, we consider thinking styles of the West and the East. We examine several possibilities of ways by which we can recognize various concepts in the world, and acquire enlightenment from the nature. At first, we begin with the two categories of knowledge in Buddhism. They are "discriminative knowledge" and "non-discriminative knowledge" (however, as we shall see below, discrimination between "discriminative knowledge" and "non-discriminative knowledge" itself is not important at all in Buddhism). Although it is very difficult to explain them, in particular non-discriminative knowledge, by words, here we dare to give some considerations on them.

*Discriminative knowledge* is just the set-theoretic one. Namely, it is a knowledge acquired by classifying things existing in the world. For example, the discriminative knowledge on "cat" is obtained by distinguishing the objects that are cats from the objects that are not cats. Therefore, what we can argue based on discriminative knowledge is a relation among the sets corresponding to various concepts, e.g., the set of cats is contained in the sets of animals, and so on. Knowledge described by an ordinary language (or a mathematical language like a logic formula) is of this kind, since "words" basically have a function to distinguish certain things from others.

*Non-discriminative knowledge*, on the other hand, is regarded as the true wisdom in Buddhism. But, it is very difficult to explain it in words, since words can be used for describing discriminative knowledge. Therefore, the only method by which we can express it is using a negative sentence like

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"Non-discriminative knowledge is not a knowledge that is obtained by distinguishing certain things from others." Actually, non-discriminative knowledge is recognized neither by words, nor by thinking, nor by act. Moreover, it is not even recognizable. This is because all acts such as recognizing, thinking, and explaining some objects necessarily accompany discrimination between the self (i.e., actor) and the object. In Buddhism, everything is empty, i.e., it has no reality in the world in its essence. Hence, there is nothing to be discriminated, and there is a truth that can be gotten without discriminating things. Furthermore, such a truth (non-discriminative knowledge) itself is also empty, and thus does not exist. It may sound contradictory, but this is caused by explaining it by ordinary words.

There is no doubt that discriminative knowledge brings practical convenience to our daily life. Today's science also relies on discriminative knowledge. There, objects to be studied are clearly identified, and their properties are described precisely. By this, science brought us a great success. However, discrimination is considered as a kind of "biased view" in Buddhism. Thus, we should note that such a knowledge is a "relative" one. Namely, when we state a scientific truth, we can only say like "If we assume a certain thing is distinguishable from others based on some (biased) viewpoint, then we can conclude so-and-so on it." We should thus be careful not to overestimate the descriptive power of languages.

It is well known that from the end of 19th century the foundation of mathematics has been formalized rigorously with the utmost precision. It is, of course, based on discriminative knowledge. However, at the same time, problems and limitations of such a methodology were also disclosed. A paradox by Bertrand Russel on the set theory is the most famous one, which first appeared in Nachwort of the Frege's book [1]. Russel's paradox is as follows. Let $R$ be the set of all sets each of which does not contain itself as a member. Is $R$ a member of itself or not? In either case, it contradicts the definition of $R$. Due to this paradox, the naive set theory had to be replaced by some sophisticated ones such as the type theory. The incompleteness theorem by Kurt Gödel [2] also shows a limitation of a formal mathematical system. He proved that in every formal system in which natural numbers can be dealt with, there exists a "true" formula that cannot be proved in this system. He showed it by composing a formula having the meaning "This formula is unprovable."

Nāgārjuna is a Buddhist priest and philosopher who lived in India around 150–250 AD. He is the founder of Madhyamaka school of Buddhism, where

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he developed the theory of emptiness. In his book *Vigrahavyāvartanī* (*The Dispeller of Disputes*) [3], he pointed out "very logically" that false thinking will be caused by relying only on discriminative knowledge. This book is written in the following form. First, philosophers of other schools who believe every concept has a substance (here, we call them philosophical realists) present objections against those of Madhyamaka school. Then, Nāgārjuna refutes all of them.

While philosophers of Madhyamaka school assert every concept has no substance (but they assert "nothing" as we shall see below), the opponents (philosophical realists) say as follows [3].

> If the substance of all things is not to be found anywhere, your assertion which is devoid of substance is not able to refute substance. (Verse 1)

> Moreover, if that statement exists substantially, your earlier thesis is refuted. There is an inequality to be explained, and the specific reason for this should be given. (Verse 2)

Nāgārjuna says:

> If I had any thesis, that fault would apply to me. But I do not have any thesis, so there is indeed no fault for me. (Verse 29)

> To that extent, while all things are empty, completely pacified, and by nature free from substance, from where could a thesis come? (Commentary by Nāgārjuna on Verse 29)

That is, without saying "all things are empty," *all things are empty by nature*, and hence the Nāgārjuna's assertion itself is also empty.

We can see that the observation "If all things are empty, then the assertion 'all things are empty' cannot exist" resembles the second incompleteness theorem "If a formal system in which natural numbers can be dealt with is consistent, then consistency of the system cannot be proved in the system" by Gödel [2]. However, methodologies for obtaining the above observations are quite different. In the former case, non-discriminative knowledge played the crucial role, and thus the observation itself is again empty.

Nāgārjuna launches a counterattack against philosophical realists, who claim "all things have substances," by the following objection.

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> The name "non-existent" — what is this, something existent or again non-existent? For if it is existent or if it is nonexistent, either way your position is deficient. (Verse 58)

It is clear that the above argument is analogous to Russel's paradox. By this, Nāgārjuna pointed out that philosophical realists who rely only on discriminative knowledge have a logical fault. However, as stated in Verse 29, Nāgārjuna asserts nothing in his book.

It will be reasonable to regard discriminative knowledge as conventional knowledge. Then, how is non-discriminative knowledge? Although this kind of knowledge has been argued by philosophers and Buddhists for a very long time, we can say neither conventional nor unconventional. Probably, it is meaningless to make such a distinction. Instead, we consider a question: Can we use non-discriminative knowledge for finding a new way of scientific thinking, and for giving a new methodology of unconventional computing? Since current scientific knowledge is very far from non-discriminative knowledge, it looks quite difficult to do so. However, it will really stimulate our imagination, and may help us to widen the vista of unconventional computing.

I have been studying reversible computing and cellular automata [4] for more than 30 years. Through the research on these topics, I tried to find novel ways of computing, and thus I think they may be in the category of unconventional computing. Besides the scientific research, I was interested in Buddhism philosophy. In 1970's and 80's, I read Japanese translations of several sutras and old texts of Buddhism. They are, for example, *Prajñāpāramitā Sūtra* (*Sutra of Perfection of Transcendent Wisdom*)[1], and *Vimalakīrti-nirdeśa Sūtra* (*Vimalakirti Sutra*), as well as *Vigrahavyāvartanī* (*The Dispeller of Disputes*). All of them discuss emptiness of various concepts and things in the world, but assert nothing. I was greatly impressed by these arguments, which themselves are empty. Although my research results are, of course, given in the form of discriminative knowledge, and thus in the purely Western style, I think such a thought somehow influenced me on my research when exploring new ways for unconventional computing.

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[1] There are several versions of *Prajñāpāramitā Sūtra* that range from a very short one to a very long one. The shortest two are often called *Heart Sutra*, and *Diamond Sutra*.

## 3. Yukio-Pegio Gunji: Observers

Unconventional computing is the computing equipped with an endo-observer or an internal observer (Roessler, Matsuno, Gunji). Formal logic and/or classical and conventional computing is equipped with an exo-observer. A substrate with an endo-observer is called "life". That is a tradition of animism in the Eastern culture.

An observer in computing is defined as an interface connecting computing resource to the external world. If the relation between the computing resource and the external word is uniquely determined, the interface is implemented just as a machine. Otherwise, one is destined to find some ambiguity or indefiniteness in the interface. That is why we generalize interface in the form of an observer. Distinction of exo- and endo-observer is defined with respect to where he or she stands to observe something.

An exo-observer is an observer standing at the edge of the whole perspective. Thus, how to manipulate an object in the perspective is uniquely determined. Grounding an object to the external world is realized at the edge of perspective not at the margin of each object. Imagine "1+2" in arithmetic. The meaning of "1", "+" and "2" is uniquely determined without ambiguity. Ambiguity is nothing but character grounding to the external world. In this sense, each symbol "1", "+" and "2" has no ambiguity at the margin of each symbol. Grounding has not been found till what is adapted to the expression, "1+2". If one counts the two coins added with one coin, the perspective (math) in which "1+2" is well-defined is grounded to the coins in the external world at the edge of the perspective. Similarly, if one counts two pebbles added with one, the perspective is grounded to the external world at the edge of arithmetic.

Writing a sentence or a poem is a kind of computing, although this a computing with an endo-observer (i.e., unconventional computing). Imagine a special expression, "Specially trained beetle". One believes that one can usually determine the meaning of the word, "specially", "training" and "beetle" without ambiguity. Therefore, one believes that the meaning of "specially trained beetle" can be determined just as the combination of meaning. However, what is "trained beetle" and indeed, "specially"? That is an alternative beetle beyond beetle, featured with ominous attribute, which might be appeared in the masterpiece of Hieronymus Bosch. That is the power of literature and/or poetry. Why is it possible? In the strict sense, the meaning of "specially", "training" and "beetle" cannot be uniquely determined. The

"beetle" can be connected to what is not a beetle while "beetle" indicates what is called a beetle. Usually the part of what is not a beetle is hidden and cannot disappear till the special expression, "specially trained beetle" is mentioned. Usually no one notices the part of what is not a beetle in "beetle", but there exists at the margin of "beetle". Each of the words "specially", "training" and "beetle" is linked to an external world. That is why the outside of "specially", of "training" and of "beetle" can be resonated to bring about something ominous. The observer exists at the margin of each word within the perspective of the words. That is why such observers are endo-observers.

Replace words with some materialistic computing resource. One can imagine unconventional computing rather than classical formal computing.

We here refer to Bob Rosens idea of life. He first mentioned complex system. In his sense, simple system consists just of formal, efficient and material cause. As for building a house, formal cause corresponds to a blueprint for the design, efficient cause corresponds to works of carpenters, and material cause corresponds to woods, nails and bricks. As for the house building the forth cause exists, the final cause. That corresponds to someones living. Thus, building a house is a complex system because a system is connected to function in the open environment.

We think that the idea remains something to be revised. Note that the final cause is the interface between a system and its environment (external world). That is an observer. If three causes, formal, efficient and material cause are connected to each other without ambiguity, one can find a perspective consisting of three causes as a definite perspective. Thus, the final cause exists at the edge of the perspective. The former three causes can be independently separated from the forth final cause. In this sense the final cause at the edge of the perspective can correspond to the exo-observer. Even if the final cause can participate in the system, the final cause cannot contribute to other three causes within a perspective. In this sense, it is a simple system far from living systems. Instead of it, if the relationship among three causes, formal, efficient, and material causes cannot be uniquely determined and can be opened to the ambiguity, one can find the connection to the external world at the margin of each cause. The connection to the external world erodes each cause, respectively. In other words, dynamic and indefinite relation among formal, efficient and material causes is the final cause and endo-observer. Rosen himself introduced the idea of complex system and the final cause to define life itself. Now we spelled out that the

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final cause is nothing but an endo-observer. If the endo-observer is explicitly found, then the system accompanied with an endo-observer is called a living system. While Rosen tried to formalize living system in a category theory to implement the final cause, as he mentioned, the map from data (material cause) to program or function (efficient cause) is destined to have an inverse map of it. The inverse cannot be uniquely determined and then such ambiguity is opened to the endo-observer. While Rosen involved indefiniteness in formalizing life in a category theory, the indefiniteness can reveal a system with an endo-observer.

## 4. Cristian S. Calude: Cooperation in Rebellion

I was always fascinated by impossibilities and mathematics. Later they merged into mathematical impossibilities, a research topic for many years. Impossibilities appear everywhere, from daily life to science, mathematics and politics. Many impossibilities are just apparent. For example, it is often claimed that having a dispassionate conversation about guns is an impossibility. Impossibilities in science tend to be time-dependent: renowned physicists thought that "heavier than air" flying machines were impossible (W. T. Kelvin), the atom bomb was impossible (E. Rutherford) and black holes were "science fiction" (A. Einstein).

"No triangle can have two right angles" and "the square of 2 cannot be written as a fraction with both positive integers numerator and denominator" are mathematically proven impossibilities. They are forever, as all mathematical impossibilities. Proving a mathematical impossibility is in general more difficult than proving a positive result. For example, to prove that a specific function $f$ mapping natural numbers to natural numbers can be computed by a Turing machine is enough to construct *a Turing machine M* and prove that indeed $M(n)$ computes $f(n)$ for every $n$. Proving that $f$ is not computable by *any Turing machine* is a more difficult task: one has to show that every Turing machine fails to compute $f$, that is, for every Turing machine $M$ there exists a natural $m$ such that $f(m) \neq M(m)$.

Below are a few of the mathematical impossibilities I have pondered over the years.

1. The set of algorithmic random strings is not computable, in fact, it is highly incomputable (immune) – no algorithm can "certify" more than finitely many algorithmic random strings, [5].

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2. The set of reals satisfying the law of large numbers is probabilistically "large", but topologically "small". Similarly, but in a constructive (stronger) sense, the set of Martin-Löf random sequences has measure 1, but it is a meagre set in Cantor's topology, [6].

3. In a quite general topological sense, Gödel's incompleteness is not an exception, but a rather common phenomenon. With respect to any reasonable topology the set of true and unprovable statements of such a theory is dense and in many cases even co-rare, that is "very large", [7].

4. Every computably enumerable Martin-Löf random real is the halting probability of a universal prefix-free Turing machine for which ZFC – arguably the most powerful formal system for mathematics – cannot determine more than its initial block of 1 bits – as soon as you get a 0, it is all over, [8].

5. The halting probability $\Omega_U$ of a universal prefix-free machine $U$ is Martin-Löf random. However, there exists a universal prefix-free machine $U$ such that Peano Arithmetic cannot prove the randomness of $\Omega_U$ based solely on $U$ (which fully determines $\Omega_U$), [9], [10]

Impossibilities highlight limits and with every limit comes the challenge to trespass it. "Heavier than air" flying machines are ubiquitous, the atom bomb was possible and its consequences have been devastating, and on 15 June 2016 the detection of a gravitational wave event from colliding black holes was announced. In mathematics, too, limits can be transgressed. For example, the broken symmetry between measure and category for Martin-Löf random sequences can be restored if we use Staiger's $U^\delta$-topology, [10], a relativisation of the Cantor topology.

*Unconventional computing is about challenging computational limits.*

In 1994 John Casti and I started talking about the eventual decay of Moore's law and the advance of new models of computation, which we called *unconventional*[2]. At that time there was a wide spread belief that the P

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[2]The earliest written reference to the term which I have is from an email sent by Seth Lloyd to John Casti Sat on 27 Jul 1996 17:12:41 in which Seth, answering an email from John, lists some researchers in "unconventional and non-Turing models of computation" [11].

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vs. NP problem[3] will be solved in the negative before the end of the century. This motivated the imperative need to find fast algorithms to solve NP problems, a computational challenge unlikely, if not impossible, to succeed using Turing machines. Another reason was the Turing barrier derived from the Church-Turing Thesis. All computations are extensionally equivalent to Turing machines: is it possible to design new models of computation capable of transgressing Turing's barrier? As a response, in 1998 together with John Casti and Michael Dinneen I started a new series of conferences called *Unconventional Models of Computation*; see [12, 11]. The first conference in the series was organised in Auckland, New Zealand on 6–9 January 1998 by the Centre for Discrete Mathematics and Theoretical Computer Science in Auckland and the Santa Fe Institute.

My interests for the emergent area of unconventional computing sparked from three sources: a) my ongoing work on limits, b) the cooperation with quantum physicist Karl Svozil[4] on discrete modelling of quantum phenomena, see [14, 15, 16, 17] and c) a "rebel" attitude against the mainstream computer science motivated in part by the feeling that although I am part of the community, "I still do not belong".[5] Since then I have been working in trespassing the Turing barrier [18, 19, 20, 21, 22], de-quantisation [23, 23, 24], quantum randomness, [25, 26, 27, 28, 29, 30] and quantum annealing [31, 32, 33]. As one can recognise, these topics are not "main stream"; moreover, the results themselves are not infrequently "swimming against the tides".

## 5. Selim Akl: Nonuniversality

I cannot remember a time when I did not think unconventionally. All my life I tried to see if some things could be done differently. It was always a thrill to explore unconventional wisdom. Since this article is about paths to

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[3]Currently still open.

[4]Author of the influential book [13].

[5]Why such a feeling? Perhaps because of my strong interest in modelling mathematically computational processes. Mathematics is a blend of logical rigour and art, a discipline closer to philosophy and theology than to science and engineering. Like philosophy and theology, mathematics operates with ideas, a universe in which infinity plays a dominant role and beauty is a major criterion of quality. Understanding is more important than knowing or doing (computing). Although breaking barriers is the norm, mathematics is capable of scrutinising its own limits.

unconventional computation, I will restrict my contribution to this topic [6].

In our never-ending quest to understand the workings of Nature, we humans began with the biological *cell* as a good first place to look for clues. Later, we went down to the *molecule*, and then further down to the *atom*, in hopes of unravelling the mysteries of Nature. It is my belief that the most essential constituent of the Universe is the *bit*, the unit of information and computation. Not the cell, not the molecule, not the atom, but the bit may very well be the ultimate key to reading Nature's mind.

Does Nature compute? Indeed, we can model all the processes of Nature as information processes. For example, cell multiplication and DNA replication are seen as instances of text processing. A chemical reaction is simply an exchange of electrons, that is, an exchange of information between two molecules. The spin of an atom, whether spin up or spin down, is a binary process, the answer to a 'yes' or 'no' question. Information and computation are present in all natural occurrences, from the simplest to the most complex. From reproduction in ciliates to quorum sensing in bacterial colonies, from respiration and photosynthesis in plants to the migration of birds and butterflies, and from morphogenesis to foraging for food, all the way to human cognition, Nature appears to be continually processing information.

I had been working on parallel computation since the late 1970s. Because parallelism is inherent to all computational paradigms that later came to be known as "unconventional", the transition from architecture-dependent parallelism to substrate-dependent parallelism was logical, natural, and easy. This is how I embraced quantum computing, optical computing, bio-molecular computing, cellular automata, slime mould computing, unconventional computational problems, and ultimately nonuniversality in computation. My earliest contribution in this direction was made in the early 1990s, when I developed, with Dr. Sandy Pavel, processor arrays with reconfigurable optical networks for such computations as integer sorting and the Hough transform.

Quantum computers are usually promoted as being able to quickly perform computations that are otherwise infeasible on classical computers (such as factoring large numbers). My work with Dr. Marius Nagy and Dr. Naya Nagy, by contrast, has uncovered computations for which a quantum com-

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[6]All works mentioned in this paper are available at: `http://research.cs.queensu.ca/home/akl/`

puter is, *in principle*, more powerful than any conventional computer. One example of such a computation is that of distinguishing among the $2^n$ entangled states of a quantum system of $n$ qubits: This computation can only be performed on a quantum computer.

With Dr. Virginia Walker, I co-supervised three graduate students who built a DNA computer capable of performing a simple form of cryptanalysis. They also put to the test the idea of double encoding as an approach to resisting error accumulation in molecular biology techniques such as ligation, gel electrophoresis, polymerase chain reaction (PCR), and graduated PCR.

With Dr. Sami Torbey I used the two-dimensional cellular automaton model to provide unconventional solutions to computational problems that had remained open for some time, namely: (i) Density classification, that is, given a two-state grid, does it contain more black or more white cells? (ii) Planar convex hull, that is, given a set of $n$ points, what is the convex polygon with the smallest possible area containing all of them? The first problem was solved using a "gravity automaton", that is, one where black cells are programmed to "fall" down towards the bottom of the grid, while the second was solved by programming the cells to simulate a rubber band stretched around the point set and then released. We also used cellular automata to solve a coverage problem for mobile sensor networks, thus bringing together for the first time two unconventional computational models.

One of the dogmas in Computer Science is the concept of computational universality: "Given enough time and space, any general-purpose computer can, through simulation, perform any computation that is possible on any other general-purpose computer." Statements such as this are commonplace in the computer science literature, and are served as standard fare in undergraduate and graduate courses alike. I consider it one of my most important contributions to have shown that such a Universal Computer cannot exist.

I discovered nonuniversality because of a challenge. While giving an invited talk on parallel algorithms, a member of the audience kept heckling me by repeatedly interrupting to say that anything I can do in parallel he can do sequentially (on the Turing Machine, to be precise). This got me thinking: Are there computations that can be done in parallel, but not sequentially? It was not long before I found several such computations. The bigger insight came when I realised that I had discovered more than I had set out to find. Each of these computations had the following property: For a problem of size $n$ they could be solved by a computer capable of $n$ elementary operations per time unit (such as a parallel computer with $n$ processors), but could not be

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solved by a computer capable of fewer than $n$ elementary operations per time unit. This contradicted the aforementioned principle of simulation, and as a consequence also contradicted the principle of computational universality. Thus parallelism was *sufficient* to establish nonuniversality in computation. I later proved that parallelism was also *necessary* for any computer that aspires to be universal.

Specifically, in order to obtain my result on nonuniversality in computation, I exhibited functions of $n$ variables that are easily evaluated on a computer capable of $n$ elementary operations per time unit, *performed in parallel*, but cannot be evaluated on a computer capable of fewer than $n$ elementary operations per time unit, regardless of how much time and space the latter is given. An example of such a function is one that takes as input $n$ distinct integers in arbitrary order, and returns these integers sorted in increasing order, such that *at no time during the computation three inputs appear in decreasing order*. Nonuniversality in computation is the computer science equivalent of Gödel's Incompleteness Theorem in mathematical logic.

And thus the loop was closed. My journey had taken me from parallelism to unconventional computation, and from unconventional computational problems to nonuniversality. Now, nonuniversality has brought me back to unconventional computation. All said, I trust that unconventional computation has provided a perfect research home for my character and my way of thinking, and has uncovered a wondrous world of opportunities for my inventiveness and creativity.

It is relevant to mention in closing that the motto of my academic department is *Sum ergo computo*, which means *I am therefore I compute*. The motto speaks at different levels. At one level, it expresses our identity. The motto says that we are computer scientists. Computing is what we do. Our professional reason for being is the theory and practice of Computing. It also says that virtually every activity in the world in which we live is run by a computer, in our homes, our offices, our factories, our hospitals, our places of entertainment and education, our means of transportation and communication, all. Just by the simple fact of living in this society, we are always computing. At a deeper level the motto asserts that "Being is computing". In these three words is encapsulated our vision, and perhaps more concretely our *model* of computing in Nature. To be precise, from our perspective as humans seeking to comprehend the natural world around us, the motto says that computing permeates the Universe and drives it: Every atom, every molecule, every cell, everything, everywhere, at every moment, is performing

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a computation. To be is to compute.

What a magnificent time to be a computer scientist! Computing is the most influential science of our time. Its applications in every walk of life are making the world a better place in which to live. Unconventional computation offers a wealth of uncharted territories to be explored. Indeed, natural computing may hold the key to the meaning of life itself. What more can we hope for?

## 6. Hector Zenil: Causality in complexity

The line between unconventionality, dogmatism, indeed even esotericism is very fine and critical, even in science. Turing, for example, challenged his own concept, and came up with the idea of an oracle machine to explore the implications of his challenge, though he never suggested that such a machine existed. He continued challenging conceptions with his ideas about thinking machines and processes in biology that could be closely simulated by mathematical equations, yet never suggested that machines could (or could not) think as humans do, which is why he designed a pragmatic test. Nor did he ever suggest that biology followed differential equations. Einstein, in turn, kept looking for ways to unify his gravitational and quantum models of the world, kept challenging the idea of the need for true randomness in quantum mechanics, but fell short of challenging the idea of a static (non-expanding) universe. Successful theories cannot, however, remain forever unconventional, but people can.

My first unconventional moment, of a weak type, came when I faced the philosophical conundrum regarding the practice and the theory of computation: could the kind of mechanical description introduced by Turing be generalized not only to the way in which humans (and now digital computers) perform calculations but to the way in which the universe operates? Contrary to what many may think, this is not an unconventional notion; physics points in the direction of a Turing-universe, where elementary particles cannot be further reduced in size or type. Such particles have no other particularity to them, no distinctive properties; they are exactly alike (except for its *spin*), indistinguishable, just as cells on a Turing machine tape are indistinguishable except in terms of the symbol they may contain (equivalent to reading the spin direction). Moreover, classical mechanics prescribes full determinism, and the necessity of quantum mechanics to require or produce

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true indeterministic randomness is contested by different interpretations (e.g. Everett's multiverse).

Every model in physics is computational and lives in the *computational universe* [34] (the universe of all possible programs), as we are able to code such models in a digital computer, plug in some data as initial conditions and run them to generate a set of possible outcomes for real physical phenomena with staggering predictive precision. That does not mean that the universe itself is computational, but the correspondence between nature and such computational models has been striking and is at the foundations of science. Such a convergence between simulation and simulated cannot but suggest the possibility that the real phenomenon undertakes similar calculations as the ones carried out by the computers on which the simulation takes place. We may be pushed to believe that the inadequacy of such models in predicting long term weather patterns with absolute precision reflects the limitations of the models themselves, or the divergent nature of the universe with respect to the possibly limited digital carries, or else the fundamental unsoundness of computable models, but we know that the most salient limitation has been the inadequate data–both in quantitative and qualitative terms–that we can plug into the model, as we are always limited in our ability to collect data from open environments, from which we can never attain enough precision without having to simulate every particle in the entire universe, an impossible feat. But we do know that the more data we introduce into our models the better they perform so we have indications of convergence rather than divergence from algorithmic models of the world beyond the limitations of measurement related to non-linear systems.

Computational or not, if anything was clear and not in the least unconventional, it was that the universe was algorithmic in a fundamental way, or at least that in light of successful scientific practice it seemed highly likely to be so. While this is a highly conventional point of view, many may view such a claim as being almost as strong as its mechanistic counterpart because, ultimately, in order to shift the question from computation to algorithms, one must decide what kind of computer runs the algorithms. However, after my exploration of non-computable models of computation [35], I began my exploration of what I call the algorithmic nature of the world, which makes no ontological commitment to some particular specs of a particular kind of computer or of computation. I wanted to study how random the world may be, and what the theory of mathematical randomness may tell us about the nature of the universe and the kinds of data that could be plugged into mod-

els, their initial conditions, and the noise attendant upon the plugging in of the data. This promised to give me a better understanding of whether it was the nature of the data on which a computational model ran that made it weaker and more limited, or whether it was only the quantity of the data that determined the limitations of computable models. And so I launched out on my strong unconventional path by introducing alternatives for measuring and applying algorithmic complexity, leading to exciting deployments of highly abstract theory in highly applied areas. The basic units of study in the theory of algorithmic complexity are sequences, and nothing epitomizes a natural sequence better than the DNA. Because most information is in the connections among genes and not the genes themselves, I defined a concept of the graph algorithmic complexity of both labelled and unlabelled graphs [36, 37]. However, this could not have been done if I had proceeded by using lossless compression as others have [38, 39]. Instead I used a novel approach based upon algorithmic probability [40, 41] that allowed me to circumvent some of the most serious limitations of compression algorithms.

What I used was the theory of algorithmic probability [40, 41], a theory that elegantly reconnects computation to classical probability in a proper way through a theorem called the *algorithmic coding theorem*, which for its part establishes that the most frequent outcome of a causal/deterministic system will also be the most simple in the sense of algorithmic complexity.

When I started these approaches I was often discouraged, as I still sometimes am, and tempted to turn away from algorithmic complexity because 'its *uncomputabilty*' (the reviewers said), that there is no algorithm to run a computation in every case and expect the result of the algorithmic complexity of an object, because the computation may or may not end. But if we were scared away by uncomputability we would never code anything but trivial software.

Once I had the tools, methods and an unbreakable will, I wanted to know to what extent the world really ran on a universal mechanical computer, and I came up with measures of *algorithmicity* [42, 43]: how much the outcome of a process resembles the way in which outcomes would be sorted according to the universal distribution, and of *programmability*[44, 45, 46, 47]: how much such a process can be reprogrammed at will. The more reprogrammable, the more causal, given that a random object cannot be reprogrammed in any practical way other than by changing every possible bit. My colleagues, leading biological and cognitive labs, and I have looked at how the empirical universal distribution that we approximated could be plugged back into all

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sorts of challenges [36, 37, 48, 49, 50, 51, 52, 53] to help with the problem of data collection to generate a sound computational framework for model generation.

When one takes seriously the dictum that the world is algorithmic, one can begin to see seemingly unrelated natural phenomena from such a perspective and devise software engineering approaches to areas such as the study of human diseases [54].

It turns out that the world may be more reprogrammable than we expected. By following a Bayesian approach to proving universal computation [55, 56], we recently showed that class boundaries that seemingly determined the behaviour of computer programs could easily be transcended, and that even the simplest of programs could be reprogrammed to simulate computer programs of arbitrary complexity. This unconventional approach to universality, thinking outside the box, shows that, after the impossibility results of Turing, Chaitin or Martin-Löf, proof can no longer be at the core of some parts of theoretical computer science, and that a scientific approach based on experimental mathematics is required to answer certain questions, such as how pervasive Turing-universality is in the computational universe. We need more daring, unconventional thinkers who would stop fearing uncomputability and carry out this fruitful programme.

While unconventional computing is about challenging some computational limits, the limits I challenge are those imposed by axiomatic frameworks and their quest for only mathematical proofs of ever-increasing abstraction. I rather take proofs from mature mathematical areas to seek for their meaning in disparate areas of science, thereby establishing unconventional bridges across conventional fields.

## 7. Andrew Schumann: Protein Monsters

One of the recent directions in unconventional computing is represented by any biological activity controlled by placing attractants and repellents – some items which are programmed to attract and repel the behaviour. First of all, it is a swarm computing, considering any swarm as a computation medium, because the behaviour of any swarm can be programmed by the localisation of attractants presented as food pieces and repellents presented as dangerous places. Nevertheless, we can program the behaviour of many unicellular organisms in the same way, such as behaviours of *Amoeba proteus*

or plasmodia of *Physarum plycephalum.* On the level of one cell, this controlling is explained by the appearance and disappearance of *actin filaments* or *F-actin.* Actin filaments are connected to the plasma membrane to provide a mechanical support by an actin cortex. If there is an attractant before the cell, actin filaments form a wave to change the cell shape to allow the movement of the cell surface to build a pseudopodium by cross-linked filaments to catch the attractant. If there is a repellent before the cell, actin filaments form a wave to change the cell shape to avoid the repellent.

In swarm computing we use real organisms like ants or slime mould with completely controlling their behaviour. But we may expect that in the future we can control all the chemical reactions responsible for assembling and disassembling the actin filament networks. It means that we would have an "artificial protein monster" whose reactions are programmed by us even at the molecular level.

Conventionally, any device for computations has been regarded as a "mechanical" calculator – a machine designed from inanimate-nature objects and used to perform automatically all the computations in the way of mechanical (later electronic) simulations of calculating processes. The first calculating machine was designed by Blaise Pascal (1623 – 1662) to mechanise calculations. This attempt gave many inspirations for some logicians at that time. So, Gottfried Wilhelm von Leibniz (1646 – 1716) introduced the idea of *characteristica universalis* – the universal computer to mechanise all thinking processes, not only calculations.

Nobody has thought of building up computers from the animate nature. Later, the Leibniz's idea of *characteristica universalis* was theoretically explicated concurrently in the three ways: (i) mathematically by Kurt Friedrich Gödel (1906 – 1978) – the idea of $\mu$-recursive functions; (ii) from the point of view of programming by Alonzo Church (1903 – 1995) – $\lambda$-calculus; and (iii) from the point of view of engineering by Alan Mathison Turing (1912 – 1954) – Turing machines. (i) Gödel's $\mu$-recursive functions are defined by inductive sets. Now, there is a notion of the so-called *corecursive functions* defined by coinductive sets to formally describe any behaviour, even not-algorithmic. (ii) The Church's $\lambda$-calculus can be replaced by process calculi like $\pi$-*calculus* applying corecursuive functions for programming instead of recursive functions. These new calculi are used for simulating different behavioural systems including not-algorithmic and concurrent. (iii) A Turing machine is inanimate in principle. In unconventional computing, designing computers from swarms or designing an "*artificial protein monster*" (in the future) is an at-

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tempt to explain the animal behaviour as such and this attempt is parallel to mathematical theories on corecursive functions and programming languages involving process calculi.

Thus, conventionally there was a philosophical presupposition that the human being is unique who possesses intelligence and all computers can be made just as mechanical (electric) devices simulating the human algorithmic thinking.

However, there is an old tradition of *panpsychism* – a view that all animate things bear a mind or a mind-like quality, too. So, Johann Wolfgang von Goethe (1749 – 1832) stated that nothing exists without an internal intelligence called by him *Seele* (spirit).

The panpsychistic idea of internal intelligence of all things is well expressed in Qaballah, the Judaic mysticism. The Bible verse 'And the spirit of God moved upon the face of the waters' (*Genesis* 1:2) was interpreted as affirming that there exists a spirit (*ruah*, רוח) of the Messiah or a pure man before the world creation (*Genesis Rabbah* 8:1). This spirit is named *'Adam Qadmon* (אדם קדמון). He is the cosmic man or Self and represents 'crown' (*keter*, כתר), the divine will to create everything. From *'Adam Qadmon* emerge the following four worlds: (i) the divine light or pure emanation (*'azilut*, אצילות); (ii) the creation or divine waters (*briy'ah*, בריאה); (iii) the formation or internal essence of all things (*yezirah*, יצירה); and (iv) the action and all the forms of behavior (*'aśiyah*, עשיה).

We find out almost the same description of internal intelligence of all things in the Hindu tradition, as well. The cosmic man or Self is named *Puruṣa* and from him emerge also the same four worlds: (i) the divine light or pure emanation ('the *Agni* [A.Sch.: divine fire] whose fuel is the sun'); (ii) the creation or divine waters ('*parjánya*' or 'clouds whose fuel is the moon'); (iii) the formation or internal essence of all things (contained in 'medicinal plant'); and (iv) the action and all the forms of behaviour (actions started from 'the male which sheds the semen on woman') (*Muṇḍaka Upaniṣad* 2, 1:5; Tr. by S. Sitarama Sastri).

It is quite mysterious why in Judaism and Hinduism (the religions, not connected at all between themselves) there are the similar notions of cosmic men *'Adam Qadmon* and *Puruṣa* with the same four emanations from them.

Hence, according to some religious traditions, such as Judaism and Hinduism, panpsychism holds indeed – it is assumed that intelligence is everywhere. Therefore, their believers suppose that there are many non-human (or even over-human) forms of intelligence in natural processes.

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In accordance with panpsychism, each animate thing is a kind of computer. So, an "artificial protein monster" (*Golem* in Qabbalah) is possible, too. The panpsychist idea cannot be scientific because of its religious roots, but it can be inspiriting for us. It is so surprising that in swarm computing there are some evidences supporting panpsychism. We know that in the neural networks there are the following two mechanisms responsible for perceiving signals: (i) increasing the intensity of the signal by *lateral inhibition*, when inhibitory interneurons inhibit neighbouring cells in the neural network to make the contrast of the signal more visible; (ii) decreasing the intensity of the signal by *lateral activation*, when activation interneurons activate neighbouring cells to make the contrast of the signal less visible. Due to both mechanisms, we deal with some illusions such as the Müller-Lyer one – a geometric illusion in which the perceived length of a line depends on whether the line terminates in an arrow tail (when we face the lateral inhibition effect) or arrowhead (the lateral activation effect).

Hence, the lateral inhibition and lateral activation are two mechanisms of our mind in perceiving signals (in the case of the Müller-Lyer illusion the signals are visual). Nevertheless, the same mechanisms of transmitting signals are discovered (i) on the level of Amoeboid organisms and (ii) on the level of swarms optimising their transport networks. The matter is that both effects are basic for the actin filament networks: (i) among actin filaments, neighbouring bundles can be inhibited to increase the intensity of the signal to make just one zone of actin filament polymerisation active; (ii) neighbouring bundles can be activated to decrease the intensity of the signal to make several zones of actin filament polymerisation active.

Thus, the lateral inhibition and lateral activation can be detected in any forms of swarm networking including social bacteria and plasmodia of *Physarum polycephalum*. The same effects are observed even in the swarm behaviour of alcohol-dependent people [57], i.e. on the level of collective patterns of the human beings. It means that on the level of actin filament networks we have a kind of intelligence that is enough for the adaptation and optimisation of logistics. So, the "artificial protein monster" (*Golem*) consisting of actin filament networks and solving many computational tasks connected to orientation and locomotion is absolutely real. The basic logic for this monster is proposed in [58], [59], [57].

To sum up, panpsychism in computer science means that we design bio-inspired robots by assuming scale-invariant mechanisms that have been conserved across species. In particular, lateral inhibition and lateral activation

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are ubiquitous events that occur over many scales including within the cell during cell polarization, between groups of neuron within the visual cortex to process visual cues, and between active zones of swarms to react to their environments.

## 8. Zoran Konkoli: Following heart

My path to unconventional computation has been a long one. I've noticed that when asked "What sort of research do you do?" most of my fellow colleagues have a prepared answer, but I have always had a problem explaining that. If forced to make a quick statement, I say "I am a physicists with very broad interests" but it is not that simple. Wondering about "mechanics" of nature I finished my undergraduate studies in Physics at Zagreb University (1991). Curious about why chemistry is regular earned me a doctoral degree in the field of Quantum Chemistry at Gothenburg University in 1996. Further musings on whether one can have a theory without details brought me into Statistical Physics. I learned the tricks of trade during three post-docs (1996-2002). In particular, under the influence of John Hertz I came to appreciate two topics, Biological Physics and neural networks, and I slowly moved towards Biological Physics during 2002-2006, and ultimately Theoretical Cell Biology from 2006. I ought to say that I did not turn intentionally to unconventional computation. It had been an intellectual hobby that slowly turned into both a passion and a profession. In the following I will pose several questions that pulled me into the field.

*Computation exists but it cannot be touched:* I still have a vivid picture in my mind when I was shown a set of punch cards and been told that they represent a computer program. There was this wonderful insight of the connection between the physical and the metaphysical: one can touch the machine doing a computation, but one cannot touch the computation per se, and yet it exists. Thus a question:

$$\text{What is computation?} \tag{1}$$

Initially, when I started thinking about it, I was not even sure which type of science could answer such a question. I was ages away from the Church-Turing thesis. It took me a long time to understand what it all meant.

*Seems the whole world can compute:* As I was studying molecular cell biology I've came across a few papers that discussed computational aspects

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of living cells. This motivated me to search for the literature where chemical reactions are studied for computing purposes. Of course, it is hard to miss Adleman's work. But, in addition, I came across a wonderful series of papers by M. Konrad [60] on reaction-diffusion neuron, and a book co-edited by him on molecular computation [61]. The way chemicals systems realize computation is very different from the way CMOS technology is used. Thus after this insight that it does not have to be CMOS, I wondered about the following question:

$$\text{If a living cell can compute, who else can, and why?} \tag{2}$$

I will make a huge leap and talk briefly about Putnam's work on the thesis of computational sufficiency. Hilary Putnam presented a beautiful construct of turning any object into a finite state input/output automaton (sort of a simple computer) [62]. Putnam argued that the ability to compute does not define mind since even a rock has an intrinsic potential to perform any computation. By copying Putnam's argument, a much deeper version of question (1) might be:

$$\begin{array}{l}\text{Since it seems that the whole world can compute, what}\\ \text{does it mean to compute then? For example, is compu-}\\ \text{tation accidental (something that just exists) or essential}\\ \text{(something that exists for a reason)?}\end{array} \tag{3}$$

In very rough terms my interest in unconventional computation interpolates between (2) and (3), and in the next subsection I shall discuss some topics that span this range.

*Some selected questions on unconventional computation:* There are several ways to rephrase question (3) so that it becomes more specific.

$$\text{Given a physical object, what can it compute?} \tag{4}$$

I have learned that the question above is normally referred to as the implementation problem [62, 63]. Indeed, in unconventional computation we often ask that question. For someone with a background in dynamical systems, and with the interest in computation, a natural question to ask is:

$$\text{Given a dynamical system, what can it compute?} \tag{5}$$

Putnam's construct provides a surprising answer to both of these, as explained earlier. My own contribution to understanding these questions, was the insight that there is a better question to ask, as discussed in [64].

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Putnam's construct has been attacked with the argument that the amount of auxiliary equipment needed to turn a rock into a computer would be unreasonably large. I've managed build a skeleton of a theory that could formalize this issue. While applying the theory (as thought experiments) to several systems (including the rock), to my great surprise, I realized that question (5) does not really makes sense from a rigorous mathematical point of view. However, question (6) does:

> Given a dynamical system, what can it compute *naturally*? (6)

For obvious reasons I refer to question (6) as the *natural implementation problem*. The key insight is that there is a balance between (a) the computation that comes out of the system, and (b) the cost of implementing it, and there is a tipping point, where (b) overpowers (a). This point defines the computation naturally implemented by a system.

*A challenge to my younger self:* Regarding the belief that there is a dynamical system theory for everything: I would like to posit that this might not be the case.

> Are there objects or phenomena around us that we cannot model as dynamical system? If yes, what is the right theory for these systems? (7)

It is possible that the answer to the above question is "no". Every dynamic behavior represents a computation. But, there are computational problems that cannot be solved algorithmically, and accordingly cannot be represented as a dynamical system. Assuming that the computation *per se* is something real, then there might be real objects we do not have a dynamical theory for. The question whether the computation is something abstract (a way to think about the reality) or real (an object one can touch) links the computability and the dynamical system concepts in a peculiar way. Thus, I posit that without understanding the generic dynamical systems - computation interplay we shall never be able to exploit the full horizon of unconventional computation. Further, I wonder whether the model of computation construct has its limits but we are only still not reaching these. Finally, I wonder whether we are ultimately justified in separating the idea of computation from its physical realization.

## 9. Maurice Margenstern: Hyperbolic computation

What are the philosophical, even religious bases of my researches? There are no religious foundation of theses researches as I am an atheist. I was born in a Jewish family but, when my father was rather religious, my mother was not at all, clothing her non-believing with Jewish humour. However, I received a minimal heritage of Jewish tradition making me eager to read the bible. I did that several times, especially the Old Testament. I also read the new one, noticing that it is a completely different story, despite the many references to the Old Scripture in the Gospels. In the bible, I like especially Genesis, Exodus, Job's book, Ecclesiastes and the Song of songs, this marvellous love poem.

To my eyes, the story described at the beginning of Genesis and the Big-Bang theory of modern physics look very similar and their scientific validity is that of White Snow and the Seven Dwarfs. We know reality by a few parameters. What our eyes can see is a very small window of the light spectrum. I think that reality is so rich that a few equations cannot handle it. The equations of our physics are simply approximations of reality. Consequently, I do not subscribe to the idea that reality is utterly mathematics. Why? The latter idea is based on a vision of nature sciences as embedded in the following order: maths contain physics which contains chemistry which contains biology. I do not think that this embedding is correct. If it were, let us go on that embedding chain. Thus, biology contains ethology which contains sociology which contains psychology whose laws describe literature and arts. Nobody believes in that latter sentence. I think the just mentioned chain is false from the very beginning. Nobody knows in which geometry does our universe live. It is funny to notice that NASA desperately wishes that we live in a Euclidean space, arguing that some constant should be null. But that a real number is exactly null is precisely something that no algorithm can check. So that if the whole universe would be Euclidean, we could never be sure of that. Now, it seems that some parts of our solar system, especially around the sun, is not very much Euclidean. To sum up: if we do not know what the geometry of our universe is, how can we be sure that the extent of the physical laws we presently know is global? Another argument is the theory of multiverse whose morale is extremely strange: if it would be true, certainly science cannot predict anything as any prediction does occur in some universe so that we do not know the next universe in which we live at the next time.

In my young studies, I especially liked maths and drawing. As what I did in math classes was much better than my artistic achievements, I turned to math which also satisfied my aesthetically thirst. In maths, I preferred geometry, where I liked both pictures and proofs, two kinds of beauties very different from each other but which I both highly appreciated. By the way, the notion of beauty might indicate us something that escapes standard formalism, although formalism itself may contain a kind of beauty too. Well, for what is usually called beauty, we feel it, we cannot define it. Probably, my turn to maths has something to do with both my non-believing and my feelings to beauty, especially graphical beauty. Up to my thirties, I thought that maths might explain everything. I now know that it is not true, unfortunately, although maths much help us to understand the world. I think theoretical computer science might help us more than maths: in theoretical computer science, models are taken from wider parts of reality than in maths. In particular, computer science models might be more useful for biology than partial differential equations.

My research in the field of cellular automata in hyperbolic spaces came from my fascination to hyperbolic geometry. When I was around 27, among my teachings I had lectures in a school which formed future primary school female teachers, as at that time, in France, there were such schools for men and women separately. Before those lectures, I came upon Meschkowski's small book introducing to hyperbolic geometry. The book is a fascinating introduction to that field. The book gave me an answer to an old question from the time of the public school. We had there rather evolved lectures about geometry, of course, Euclidean geometry. Our lectures about inversion were so elegant that I thought that something was behind, a something about which our teachers were silent. Meschkowski's book gave me the answer: inversion is the tool which allowed Poincaré to model reflection in his disc model of hyperbolic geometry in the plane. Accordingly, in that school I introduced my audience of young ladies to hyperbolic geometry. They were fascinated as I was, but they told me with charming smiles that they understood nothing to these beautiful features I described them.

Life decided that I would return to the subject more than twenty years later, when I was already professor in computer science at the university of Metz. In my books about cellular automata in hyperbolic spaces, I told how I came to that topic. Notice that the initial goal, to devise reversible cellular automata in hyperbolic spaces, was never reached. However, I met something which was much more interesting, which led me to deep results. Interestingly,

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the aesthetic of the figures I used in my research, several referees seemed to share my impression, was of great help. Colours played an important role to grasp the main features of a situation. The aesthetic of the figures was an important motivation to go further.

Another part of my research was raised by finding the border between universality and decidability. Universality means the ability to compute anything which is algorithmically computable. Now, universality entails undecidable problems, which means problems which cannot be solved by any algorithm. That latter situation can be interpreted as a too general specification of the problem. So that if your specification allows you to program a universal device, it means that the specification is not complete. Now, it is possible to program universal devises with small resources. The complexity of viruses are much higher than the complexity of the small universal devices known in computer science. Therapeutic means can be compared to algorithms which decide when the device halts. As the device is universal, such an algorithm cannot exist. This is why the race for more and more efficient antibiotics is hopeless. Viruses and bacterias can be fought by viruses and bacteria only: it is urgent to change the medical strategy.

Now, we should not be pessimistic. Real life shows us that technology, which could not exist without science, is, up to some point, efficient, ignoring here ethical aspects of the issue, so that we do know something and, even, we know more and more although we know that there are a lot of problems which are still unsolved even if some of them are ill posed, a situation which may occur even if we ignore it at the present moment.

That latter point has a link with religion. If we believe in God, no problem, God explains all that we do not know, which does not bring us more concrete knowledge. For me, the assertion that maths are the ultimate reality is exactly of the same kind. That assertion fixes a frame in which, theoretically, we can solve any question, so that we are, intellectually more comfortable. Although I think that material comfort gives better conditions to scientists to make discoveries and to solve problems, I think that "comfortable" views are dangerous in science. They make us forget that doubt is the main tool which allows us to step forward in our endless search of more and more knowledge about the world in which we live, in the spaces, in which our minds like to travel.

## 10. José Félix Costa: Real numbers in computation

This is a short account on how the study of physical measurements guided us into unconventional computation.

If one wonders why the real numbers come into the natural sciences, the most common answer is to say that reality is easier to model and forecast in the continuum, mainly due to the success and the development of Calculus. Thus, when a model of Vannevar Bush's analogue computer (by the end of 1930) was developed by Claude Shannon in [65], it resulted in a system of differential equations of a particular kind, describing a network of mechanical gears and integrators, where input and output were physical magnitudes taking values in the real numbers. In analogue computation inputs are given as initial conditions or, in the general setting of more than one dimension, as boundary conditions. Real numbers may encode non-computable information in different degrees, but the way they are used in Bush's analogue computer does not permit to decipher their potential information content and decide the undecidable. In [66] we show that by means of discontinuous functions and functions with discontinuous derivatives this information content can be retrieved. But, since these functions cannot be realised exactly in the physical world, we conclude that the real numbers have the same role in analogue computation than they have in the physical sciences.

The next step in our journey to understand the role of real numbers in computation was the ARNN model [7] (see [67]), a well known discrete time computational system that computes beyond the Turing model. This feature is common to dynamic systems that are universal and able to extract every digit of the expansion of an internal real-valued parameter. These dynamic systems behave like a technician improving his measurements (using better and better equipment): they can perform a measurement of $O(n^k)$ bits of the binary expansion of a parameter in linear time and use these sequences of bits as advice to decide on inputs of size $n$. By the end of the nineties, the ARNN became a model of what a discrete time dynamic system with real parameters can compute in a polynomial number of steps on the size of the input. (In one way, the fact that the weights are real numbers is not that much conspicuous, since, as "physical" models, neural networks have been treated since the seventies as models of cognition involving real weights (see [68]) either in learning activities (supervised or unsupervised) or in classifi-

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[7]**A**nalogue **R**ecurrent **N**eural **N**et.

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cation tasks.) However, the persistence of real numbers in a computational model can be seen as the possible embedding of the information one wants the system to extract later to help along some computation (see Martin Davis [69, 70]). Nevertheless, the ARNN model exhibits a very interesting structural property: as the type of the weights vary from the integer numbers $\mathbb{Z}$ to the rational numbers $\mathbb{Q}$ to the real numbers $\mathbb{R}$, the computational power of the ARNN increases from the class of regular languages to the class of recursive languages to the class of all languages.

The real numbers can be seen as an oracle or advice to a Turing machine (to a computer). We considered in [71, 72, 73] the experimenter (e.g. the experimental physicist) as a Turing machine and the experiment of measurement (using a specified physical apparatus) as an oracle to the Turing machine. The algorithm running in the machine abstracts the experimental method of measurement (encoding the recursive structure of experimental actions) chosen by the experimenter. In [74, 73] we uncover three types of experiments of measurement to find approximations to real numbers in Physics. Some values can be determined by successive approximations, approaching the unknown value by dyadic rationals above and below that value (see [75] for a universal measurement algorithm relative to two-sided experiments). Fundamental measurement of distance, angle, mass, etc., fall into this class. A second type of experiment was considered, e.g., the measurement of the threshold of a neuron in [76]. We can approach the desired value only from below the threshold (one-sided experiments). A third type of measurement was discussed in [77] relative to experiments where the access to the unknown is derived from the observation of another quantity that vanishes (such like the intensity of light in an experiment to measure some angles in Optiks). We were not able to identify any fourth type of measurement thus far.

In 2008-9, we investigated how much the information encoded into the reals can be retrieved by dynamic systems — the abstract technicians —, performing a measurement, although, intuitively, we knew that, in practice, it cannot be done beyond a few digits. We proved that Turing machines having access to measurements can compute above the Turing limit. However, in the controversial supposition that real numbers exist, no one knows how to engineer such parameters into a dynamic system. It is certainly impossible (see [69, 70]). Good bye to real number based programming! However, natural or artificial systems involving real-valued magnitudes may not be fully simulable. In [78], Michael Manthey questions the reader on how can one even have computation without an "algorithm"?! He answered that the

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classical concept of an algorithm is a specification of a process that is to take when the algorithm is unrolled into time. E.g., he states that "one might compare this feature to the theory of evolution based on natural selection that is a process-level theory for which the existence of some a priori algorithm is problematic". I like this idea as description of what a super-Turing process may be. Thus, in this way, in the limit, evolution can be specified within a (possibly non-computable) real number that encodes the process through time. Intelligent design is then the propaganda of a super-Turing design.

Suddenly, in the beginning of 2009, we realised that the Theory of Measurement (see [79, 80, 81]) did not take into account the physical time needed for a measurement of increasing precision (as a function of precision). A concrete example from dynamics follows (see [72]). Let us assume that we are about to measure inertial mass (according with Newton's laws): if we project a proof particle of known mass $m$ towards a particle of unknown mass $\mu$, then, after the collision, the first particle will be reflected if its mass is less than $\mu$, and it is projected forward together with the particle of unknown mass if its mass is greater than $\mu$. Using linear binary search on the proof particle we can, conceivable, read bit by bit, the value of the unknown. However, the physical time needed for a single experiment is

$$\Delta t \ \sim \ \left| \frac{1}{m - \mu} \right| \ .$$

This means that the time needed to get the $i$-th bit of the mass $\mu$, using the proof particle of mass $m$ of size $i$ (number of bits) is in the best case exponential in $i$! If the standard oracle to a Turing machine is to be replaced by a physical measurement (that in a dynamic system is the ability of reading an internal parameter into the state of the system), then the time needed to consult the oracle is not any more a single step of computation but a number of time steps that will depend on the size of the information that the experimenter already got. The time complexity of a measurement reduces the computational power of dynamic systems with self-advice from their internal parameters. According with [82], this reduction of super-Turing capabilities can be so great that the real numbers add no further power, even assuming that the reals exist beyond the discrete nature of matter and energy. In the best scenario, we are still waiting for some evidence that refutes the following conjecture: *No reasonable physical measurement has an associated measurement map performable in polynomial time.* The ARNN departs from being a realistic physical model in that its dynamics exhibit discontinuous

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derivatives, e.g. not in agreement with conventional neural nets (e.g., as those being trained by the method of backpropagation of errors). With a more realistic (analytic) activation function of the neurons, the time to read the next bit of a real weight is exponential in the number of bits already extracted.

In the physical world, it is not conceivable that a particle of mass $\mu$ can be set with infinite precision. Measurements should be regarded as information with possible error [8] that take time to consult. The complexity classes involved in such computations bounded in a polynomial number of steps were fully characterised in [74, 76, 77]. In [83], we synthesized our findings stating that in the best scenario the power of the system drops from common computations having access to polynomial long advices to common computations on help by just sublogarithmic long advices. (Moreover, the existence of extra power in any computation with or without advice can only be refuted by an observer — being human or device — in the limit.)

It may also happen that such real values, e.g. supposedly physical constants, may vary through time, adding incomputability to physical observations. This is a step further that we started considering in [84], following this assumption of Peirce:

> [Peirce [85]] Now the only possible way of accounting for the laws of Nature and for uniformity in general is to suppose them result of evolution. This supposes them not to be absolute, not to be obeyed precisely. It makes an element of indeterminacy, spontaneity, or absolute chance in nature.

## 11. Mark Burgin: Wushu

Unconventional computation is treated in literature as an opposition to conventional computation. At the same time, the word unconventional means going beyond conventional or routine. Looking for philosophical roots of this phenomenon, it is possible to find analogous approaches to reality in the philosophy of the great Greek philosopher Socrates and in its further development by another great Greek philosopher Plato. According to dialogues of Plato and other historical sources such as works of Xenophon, Socrates often analyzes conventional concepts and ideas scrutinizing their validity and

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[8]The error in measurement.

aiming to go beyond the conventional understanding [86, 87, 88]. In his communication with other people, Socrates believed his duty was to enlighten himself and fellow-citizens on insufficiency of conventional knowledge and necessity to achieve a higher level of expertise in the pursuit of truth. Although some philosophers thought the goal of Socrates was to demonstrate ignorance of his interlocutors, Socrates was also trying to overcome limitations of perception of words and things opening new ways for innovative insight. The Socratic approach is a way to search for truth by one's own lights. It is an open system of philosophical quest, which allows one exploring the problem from various angles and perspectives.

In a similar way, instead of requiring allegiance to the existing technology or typical procedures, unconventional computing seeks new ways to attain the same goals in a better manner or to do what is impossible to accomplish by conventional means. The Socratic approach to computing asks: Does the best computational models and topmost computing technology of our day offer us the greatest potential for solving the diversity of problems encountered by individuals and society as a whole? Or, may be, the prevailing computational models and computing technology are in fact a roadblock to realizing this potential?

Invention of inductive Turing machines, the first model of algorithms, for which it was mathematically proved that they were more powerful than Turing machines (Burgin, 1987), gives an example of this approach. To invent inductive Turing machines, it was necessary to go out of the box created by Turing machines and sealed by the Church-Turing Thesis. Virtually, there were two boxes. The first box was ideological. Living in this box, computer scientists believed that to get a result from an algorithm, the algorithm had to stop or in some other way to inform the user that the result of computation was already obtained. This condition is actually absent in all informal definitions and descriptions of algorithms. Going out of this artificial box allowed inductive Turing machines to achieve much higher power than Turing machines had [89, 90, 91]. The second box created by Turing machines and sealed by the Church-Turing Thesis was technological. In contrast to real computers, a Turing machine has only a processor and a control device, while computers also have various input and output devices. The incongruence of this box was so evident that it was easy to overcome this obstacle providing an inductive Turing machine with one or several input tapes and one or several output tapes [89, 90, 91]. This theoretical innovation amplified flexibility and increased relevance of inductive Turing machines to real

computers [92].

An important direction in unconventional computing is formed by studying chemical, physical and living phenomena. At the same time, there is historical evidence that many philosophers and other thinkers in ancient Greece contended the concept of technology as learning from and imitating nature. For instance, the principle of learning from nature was central in the medical school of Hippocrates. Democritus suggested an historical evidence of technological development by imitating nature in such areas as house-building and weaving, which were first invented by imitating swallows and spiders building their nests and nets, respectively. According to Plato (Laws, Book X), craftsmen imitate natures craftsmanship when they are producing artifacts [86]. Thus, the Western philosophical tradition supported the approach to building systems following nature or more exactly, natural systems. The same approach we can find in unconventional computing: conventional hardware still outperforms optical and molecular computer. It is also possible to find imitation nature approach in Eastern tradition. However, if Western philosophy accentuates imitation of natural systems (material objects), Eastern tradition concentrates on imitation of natural processes (structural objects).

As it written by An Tianrong and Aiping Cheng in "Tradition Wushu and Competition Wushu"[9]:

> In the long golden river of Chinese cultural history, wushu[10] is a feature of great significance. It is broad and deep and so profound that one cannot see its beginning or its end. It is so broad that one cannot see its edges. Over its five-thousand-year history, it has acquired a theoretical framework that embraces many Chinese traditional cultures (classical philosophy, ethics, militia, regimen, Chinese medicine, and aesthetic, etc.). Its association with Taoism, Confucianism, Buddhism, and hundreds of other Chinese philosophical systems cannot be ignored. Chinese wushu is not only treasured for defense, physical exercise, preventing illness, and longevity, it also best illustrates Chinese behavior, morality, philosophy, and aesthetic expression. It mixes in a philosophy of living and an understanding of the human condition.

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[9]http://www.kungfudragonusa.com/wushu-concept-theories-principles-and-philosophies/
[10]which means martial arts in Chinese

Two of the basic principles of the traditional wushu philosophy may include several versions: the doctrine of no limitation, the doctrine of a harmonious whole and the doctrine of practical use. Wushu philosophy influenced the development of the fighting styles of wushu, also called kung fu or gongfu. One of the popular directions in wushu is formation of fighting techniques imitating animals, reptiles and birds. For instance, the Five Animal martial arts, which supposedly originated from the Henan Shaolin Temple, follows behavior and actions of five living being (animals according to Chinese) - Tiger, Crane, Leopard, Snake, and Dragon. Another selection of five animals, which is also widely used, is the crane, the tiger, the monkey, the snake, and the mantis. Actually, there are more than five animals, reptiles and birds, which give birth to different fighting styles and techniques of wushu. There are such animal styles (techniques) as the Tiger Fist (with its versions Black Tiger Fist and Black Tiger Claw), Panther, Horse, Cobra, Bull, Wolf etc. These styles and techniques are based on creative imitation of the actions and behavior of the corresponding animals, reptiles and birds.

## 12. Andrew Adamatzky: Dissent and inclusiveness

'Unconventional' is "deviating from commonly accepted beliefs or practices"; synonyms of the 'unconventional; are 'dissentient', 'dissenting', 'dissident' [93]. My path to the unconventional is rooted in the 'spirit of dissenting'. I inherited this spirit from my ancestor hieromonk Epiphanius (Adamatzky) and my late father Igor Adamatzky. Epiphanius was famous for his unorthodox thinking and love for science and education. In 1738 Archbishop Gabriel asked Epiphanius to close a church school for poor kids. Epiphanius refused and continued spreading knowledge. For this he was dismissed from Kazan diocese and sent to Solovetsky Monastery Prison. Igor Adamatzky was a well known dissent and writer in Soviet Union [94]. He participated in an illegal organisation aimed to democratise the Soviet society, was tried several times on political legal charges, and founded an organisation of underground writers, artists and musicians (Club-81). His fiction writings emphasise paradoxes of imaginary and reality and praised ideological opposition.

The spirit of dissent led me to dream about the field of science which is egalitarian with no social or academic hierarchies — the field where idolatry is strongly discouraged. This is the unconventional computing: no leaders, no commissions, unions or societies, associations are voluntary and temporary,

expertise is fully distributed, knowledge is produced collectively. Developing theoretical designs and experimental laboratory prototypes of unconventional computing devices is a game. This game is based on creativity, complementarity of skills, unity of minds, and of course arts. The leads to generalised distributed happiness and worry-free recklessness.

The unconventional computing evolved to a

> society to which pre-established forms, crystallised by law, are repugnant; which looks for harmony in an ever-changing and fugitive equilibrium between a multitude of varied forces and influences of every kind, following their own course [95]

What is unconventional computing technically? If a new algorithm is proposed how do we know how to call it: 'new', 'advanced' or 'unconventional'. And how unconventional ideas could emerge in human mind at all if the mind itself is conventional? Would a calculator based on a ternary arithmetic be considered unconventional nowadays? Yes ... Wait, such machine was already built by Thomas Fowler in 1840 [96]. Is quantum computing unconventional? May be or may be not because it is quite an established field and there are quantum computers on the market. As Tommaso Toffoli wrote:

> ... a computing scheme that today is viewed as unconventional may well be so because its time hasn't come yet - or is already gone.

Unconventional computing is a science with no direct links to either past or future. Rather it is a science of the present and of the momentary association:"... the ever-fluid, constantly renewed association of all that exists" [97]. The 'Noosphere' of unconventional computing is shapeless yet ubiquitous. Unconventional computing is a science in flux. Only the present gives us a glimpse of hope through its momentary existence.

## 13. Mohammad Mahdi Dehshibi: Persian philosophy

I interpret role of Eastern-Western philosophies on the shaping of unconventional computing through the prism of complexity of Persian language [98], which reflects intrinsic structure of Persian beliefs in bringing 'order' from 'chaos'.

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*Where was I?* In advance to go through a study in the line of complexity in Persian languages, two questions had to be answered. First of all, how understanding the Persian philosophy could formally organise the ruling mainstream of this study. In addition to, what the main relationship between the evolution of the Persian language and philosophical thoughts is?

The term of Philosophy, literally "love of wisdom," is the infrastructure of critical phenomena such as existence, knowledge, values, reason, mind, and language [99, 100]. According to the Oxford Dictionary of Philosophy, the chronology of the subject and science of philosophy starts with the Indo-Iranians, dating this event to 1500 BC. The interesting fact is that this science is studied during the course of the questioning, critical discussion, rational argument and systematic presentation. Hence, the language plays a critical role [101].

To the human mind, symbols are cultural representations of reality. Every culture has its own set of symbols associated with different experiences and perceptions. Thus, as a representation, a symbol's meaning is neither instinctive nor automatic. Perhaps the most powerful of all human symbols is language.

Eventually, discovering the whole affairs of the universe seems to be an everlasting progress which the pioneer philosophers created the building block of this road, and others try to make this way smoother in a step-wise manner. This is the scientific method of Aristotle, known as the inductive-deductive method. This philosopher used inductions from observations to infer general principles, deductions from those principles to check against further observations, and more cycles of induction and deduction to continue the advance of knowledge [102]. Indeed, in a modern view, Complex Systems which cover both mathematical and philosophical foundations of how micro-structures are evolved through self-organization to form a complex macroscopic collection could keep this manifestation [103, 104, 105].

*Where will I be?* In [98, 106, 107, 108, 109], the dynamics of the complexity of Persian orthography were discovered from different perspectives to understand how the Persian language developed over time. The Pattern Formation paradigm in modelling Persian words, as a complex system, was considered in which L-systems rules were used, and complexity measures of these generative systems were calculated. We argued that irregularity of the Persian language, as characterized by the complexity measures of L-systems representing the words, increases over the temporal evolution of the language.

In Eastern philosophy, there could be found some published rule for some

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phenomena with chaotic appearances, Al-Jafr [11] and Numerology are two examples of this science. Avicenna and Sheikh Baha'I [12] were among Iranian scientists who knew about these sciences. While the published resources in this branch are few, Baha wrote a book which although its central theme was horoscope, it could bring an application for numerology, even if one thinks about that as a sort of entertainment. In what could engage us to think more about this application to find the way for the future work is the process of modelling a complex system. This book contains 25 topics such as prediction of building a house, gender of the child, benefits of a trade and the like. Each topic is associated with a dedicated table $(12 \times 18)$ which each entry contains a character known as Abjad. At the first look, each table is like chaos; however, by selecting an entry and following a definite rule to trace the whole table, some characters are selected. These characters are then divided into two sub-categories of Odd and Even. Surprisingly enough, each subset forms a meaningful verse. Indeed, discovering the routine of changing a random set of characters within a chaotic table into a poetic order, can be considered by unconventional computing methods to better model complex systems.

## 14. Richard Mayne. Union of mind and body

Computing in the abstract sense we are discussing here is not a human creation, but a word we use to relate the link between cause and effect in the world around us. Let us not forget that binary numbers only exist in so far as we choose an arbitrary voltage level within microelectrical circuits to represent bits. That we call the processes that lead to the successful manipulation of data (multiplication of numbers, for example) in a conventional computer 'computation' but the equivalent process in a human something else — cogitation, thought etc. — speaks of the limitations in our knowledge of certain biological, chemical and physical phenomena. Note, however, that both artificial and biological number manipulation are comparable in that

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[11] Al-Jafr is mentioned in the story-line of One Thousand and One Nights and an accurate explanation of al-Jafr is offered by Richard Francis Burton (six volumes 1886–1888).

[12] Sheikh Baha'I was a scholar, philosopher, architect, mathematician, and astronomer who is well known for his outstanding contribution to some architectural and engineering designs in Isfahan, Iran. Designing of the Manar Jonban, also known as the two shaking minarets, was one of his amazing constructions.

they both output the same representation of data. Crucially, our current means of describing principles of biological information processing are typically subjective, whereas *in silico* 'computation' implies a regular, repeatable and fully-defined process. A goal of UC is to define a physical or living system in objective terms that cannot be misinterpreted, i.e. those we are already familiar with as 'computing', in order to enable a better understanding of that system.

This begs the question as to why it is important to attempt to reduce the functioning of beautiful, intricate systems such as live creatures to cold, methodical absolutes. My initial interest in unconventional computing arose through a desire to see a human body as a giant, complex computer, simply so that it could be 'reprogrammed' as a route towards developing novel biomedical diagnostic tests and therapeutic agents. This is, of course, not a new idea: many have noted the similarity of cellular systems to computing systems, e.g. transcription and translation of genes, but to call the functioning of a live system 'computation' requires a further degree of abstraction that a great many scientists shy away from, despite the fact that several of progenitors of modern computing (notably, Turing and Von Neumann) devoted a great deal of time to using computing to better understand biology.

My initial work in UC involved that much-lauded bio-computing substrate, slime mould *Physarum polycephalum*, the virtues of which have been described *ad nauseum* in other texts (see Ref. [110]). My first research role was essentially that of a microbiologist when I was commissioned to load slime moulds with superparamagnetic iron oxide nanoparticles, then study patterns of nanoparticle uptake, intracellular distribution and egress via electron microscopy, for the purpose of making various bio-circuits (see Ref. [111]). I was unaware at the time that this work would lead to a profound change in my understanding of the concepts 'mind' and 'body'.

This work led into my doctorate on slime mould computing during which I became aware that, although intellectually diverting, treating a whole cell as a single circuit component was a waste of the hardware each cell possesses: here was an organism capable of concurrently processing input from millions of membrane-bound and intracellular receptors, yet we were utilising it as a mere variable resistor (albeit one that would crawl slowly over a circuit board). It transpires that every eukaryotic cell contains a protein skeleton, the 'cytoskeleton', which forms a dense, interconnected network throughout the cell. It was originally thought that this network's purpose was to simply provide mechanical stability for the cell and a means of anchorage for

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various moving parts involved in cell motility. A growing body of evidence has suggested more recently, however, that the cytoskeleton is involved in the transmission of energetic events that constitute forms of cell signalling. These involve, but are not limited to, transduction of mechanical force, conduction of ionic waves and catalysis of propagating waves of chemical reactants in biochemical signaling cascades. A small group of scientists had even suggested that a number of emergent phenomena that occur in higher forms of life (e.g. maintenance of memory within human brain cells) were linked to cytoskeletal signalling processes, see [112]. This work was, to my reasoning, technically sound but had not reached a great degree of acceptance in the wider scientific community.

I opted to study the slime mould cytoskeleton on the basis of it being a network that supports intracellular computation, i.e. a medium for coordinating cellular input and output (sensorimotor coupling). I spent a great deal of time visualising the *P. polycephalum* cytoskeleton, or more specifically, the most predominant protein present in the organism's cytoskeleton, actin.

Our results, in Refs. [113, 114], demonstrated that *P. polycephalum* arranges its actin in dense networks in its pseudopodia (growth cones), whereas actin network topology is more diminutive in caudal regions. This is perhaps to be expected in a tip-growing organism, but we were interested to note that the varying interconnectedness of stress fibre networks approximated proximity graph structures. This was particularly noteworthy as our research group had already demonstrated that slime mould computation could be achieved at the meso-scale through the organism's ability to assume topologies approximating proximity graphs by 'programming' the plasmodium with attractant and repellent gradients. Could we, then, program the organism to assemble micro or even nanoscale circuitry into a moving graph architecture (Kolmogorov-Uspensky Machine)?

By extension, could bio-computation performance be proportional to the available resources, i.e. size, interconnectivity, complexity of data network and speed of signal transduction therein? Hypothetically, this would represent a relatively simple solution to an age-old mystery. Our work on this topic continues at the time of writing. Consider the profound implications this 'cytoskeletal theory of complex behaviour' has on our understanding of concepts such as consciousness. At the time of our first publication on the topic, I suggested that our work was a casual refutation of Cartesian Dualism (or 'mind-body separation') and instead supported a theory of the

mind consistent with one or more of the varieties of physicalism, as we had suggested that virtually every part of an entity's form is involved in doing computation via measurable intracellular interactions between discrete physical quantities. Furthermore, incoming data streams could even be said to influence the structure of of the body, meaning that the mind-body structure is inextricably linked to an entity's actions and environment!

## 15. Bruno Marchal. Computation and Eastern religion

When I was a kid, like many kids, I was terrified by the idea of death, and like many little kids, I was fond of little animals. So when I learned soon that some animals, like the amoebas, where so small that we can't see them, this excited a lot my imagination, and seemed to me to refute many impossibility proofs based on the idea that if we can't see a thing then it does not exist. I discovered both an invisible world, and the relativity of the notion of invisibility. I was taught that we can see them . . . with a microscope, indeed.

All that excitation did not compare with my perplexing feeling when I learned that every 24h the amoeba divides itself. That fact was very crucial to me, as I identified myself to them. The question was: "was my lifetime 24h... or was I immortal?" My argument that an absence of a cadaver favors the absence of death, was not convincing given that I already knew that apparent absence does not entail non-existence. Also, I asked myself "does the amoeba really divide itself, or does the universe or something else divide it? Time passes, and I was lucky to be offered Watson's book "Molecular Biology of the Gene", as well as the paper by Jacob and Monod, which will provide a consistent picture of how, indeed, the amoeba (actually a bacteria) manages to ask the universe to divide itself, solving somehow conceptually the problem, except for the possible still obscure apparent role of chemistry and physics. I was about deciding to be a chemist, or a biologist, but the math teacher in high school drove my curiosity on Cantor's theory of the infinities, which led me to the discovery of Gödel's theorems, and the arithmetical self-reference, and eventually to the celebrate second recursion theorem of Kleene. This will be like a sort of bomb in my mind, because here, it is no more an amoeba which refers to itself relatively to a universe or universal environment, but a word, or a number, relatively to a universal machine, and this, as I will understand later, is an *arithmetical* notion. So it is Gödel's

theorem which will decide me to choose the field of Mathematics as university studies.

With respect to Gödel's theorems, there are three sort of mathematicians. Those who does not care about them, those who love them, and those who hate them, and well, I was told that Gödel was no more in fashion, and things did not get quite well, if not *not well at all*, except for the official diplom.

I fell into depression, and decided to become a Chan Monk instead, stopping meditation only for tea or Chinese calligraphy. I will learn classical Chinese, and read the taoists Lao-Ze, Lie-Ze and Chuang-Ze [115], as well as the immaterialists and materialists of India and Greece. My favorite text was, and still is, "The question of Milinda", which is at the heart of the conjunction of the Eastern and Western insight [116, 117, 118].

Then a miracle occurs: some (illegal) medication worked, and took me out of my "eastern depression", although enriched by a radically new perspective, which will still take some time to develop though. I will came back to my early interest in chemistry, then in quantum mechanics, and quantum logic, and realize eventually that quantum mechanics without wave collapse, like Gödel's theorems [119], are allies to the mechanist idea. The rest will be years of work, in a difficult environment, encouraged by the department of applied science, and by many mathematicians and logicians, but in a unclear opposition of some scientists who seemed both influent, and dogmatic on the materialist issue. I will eventually succeed in defending a PhD thesis with the main result: the *necessity*, when assuming digital mechanism, or computationalism, in the cognitive science, of deriving physics from arithmetic/meta-arithmetic, together with an embryo of that derivation. The key discovery was that, although it is impossible to define the machine's notion of truth and knowledge in its own language, we could still study the logic of a knowledge associated to the machine by its classical definition (found in Plato's Theaetetus). Indeed, Gödel's incompleteness makes "provability" behaving like "belief", so that "knowledge" of any particular arithmetical proposition $A$ can be mimicked by the "true belief" suggested by Theaetetus: (provable($A$) & $A$). A neoplatonist conception of physics, influenced by the greco-indian dream argument ([120]) has to be derivable, if we assume Mechanism, from such modal variants of provability, with the arithmetical interpretations of the atomic propositions restricted to the $\Sigma_1$-sentences—which models computations, and with $A$ weakened by consistent($A$).

A wonderful theorem by Solovay will simplify the task immensely [121]. Solovay showed that a modal logic, G, axiomatizes completely the proposi-

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tional modal logic of the (arithmetically sound) machine for machine which are rich enough to prove/believe sufficiently induction axioms, so that it becomes able to get the important so-called "provable" $\Sigma_1$-completeness:

$$p \to \mathrm{B}p$$

for $p$ interpreted by $\Sigma_1$-sentences. "B" represents Gödel's provability predicate, and "D" will represent the diamond (consistency, not-B-not). This makes the machine somehow aware of its own Turing Universality.

Solovay proved much more, as he found a decidable logic, G*, axiomatizing the true (but not necessarily provable) arithmetical provability logic. This gives a logic (set of formula closed for modus ponens) of the true-but-non-provable formula of provability/consistency logic: G* \ G. That logic, and the intensional variants provide to any sound and rich machine a "theology", in the greco-indian general sense, where "God" is a nickname for Truth. The miracle here is that G* proves the extensional equivalence, and the intensional dissemblance of all the intensional variants of G and G*. This remains true when we limit the arithmetical interpretation of the atomic formula to the $\Sigma_1$-sentences. Albert Visser proved ([122]) that G1 = G + $(p \to \mathrm{B}p)$ axiomatizes correctly and completely the corresponding logic of provability. Here we have the miracle summed up by G1*:

$$\text{G1*prove} \quad p \leftrightarrow \mathrm{B}p \leftrightarrow (\mathrm{B}p \wedge p) \leftrightarrow (\mathrm{B}p \wedge \mathrm{D}t) \leftrightarrow (\mathrm{B}p \wedge \mathrm{D}t \wedge p)$$

The key point is that G1, and G1* confirms this, the *machine-itself*, played by G1, is not allowed to "see" (prove) any of those extensional equivalences. This gives many ways for the machine to see the same arithmetical (and $\Sigma_1$) truth, from different points of view, or, as the neoplatonist named them, the hypostases. Indeed they obeys very different logic, and they fit nicely in a diagram which sums it all. The five modal nuances split into 8, because three of them split along the G/G*, proof/truth, splitting:

$$
\begin{array}{ccc}
 & \mathrm{V} & \\
\mathrm{G1} & & \mathrm{G1*} \\
 & \mathrm{S4Grz1} & \\
\mathrm{Z1} & & \mathrm{Z1*} \\
\mathrm{X1} & & \mathrm{X1*}
\end{array}
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The upper diamond gives, on its middle and right hand sides, the three primary hypostases of Plotinus: V, G1*, S4Grz1, where the One, played by V, represents the arithmetical truth (conceivable as a set of the Gödel numbers of the true closed $(\Sigma_1)$-sentences), and G1* plays the role of the Noùs (the world of ideas), and, finally S4Grz1, the logic of provable-and-true, which miraculously does not inherit the proof/truth, G/G*, splitting, plays the role of the universal (first) person, or "World-Soul" (Plotinus). This notion of subject can be shown coherent with the greco-indian dream arguments, and is also very close to Brouwer's mysticism [123, 124, 125, 126, 127, 128]. This notion of soul, or first person view, makes the universal machine able to defeat all complete effective reductionist theories about itself. The machine's soul is provably not describable by any third person description aviable to that machine (a bit like with the notion of Truth, by Tarski theorem).

This can be used to show that no machine can know which machine she is, or which machine supports its computation, still less which computation(s) support(s) it, making physics into a science of the statistical interference on the computations going through the actual (indexical) state of the machine. This gives rise to a sort of Everett-like, many-dreams, interpretation of physics, which becomes reducible to elementary arithmetic. The key notion is the first person indeterminacy. If we are machine, we are duplicable, and we cannot predict which particular copy will instanciate our first person experience although we can predict it will *appear* to be singular (assuming Mechanism, of course). In particular, the logic of the observable and the sensible should be given by the lower, material, hypostases, with Z1*, the true logic of provable-and-consistent, playing the role of the material hypostases, and X1* (the true logic of provable-and-consistent-and-true) plays the role of the first person sensible materiality. This has been partially confirmed by the fact that S4Grz1 (which is identical to S4Grz1*, they are not distinguishable by G1*), Z1* and X1* gives rise to quantum (intuitionist) logics. We get a transparent interpretation of Neoplatonism in arithmetic, and Plotinus "matter" (the observable) has been shown to obey a quantum logic. That would makes the quantum aspect of nature into a confirmation of the classical mechanist hypothesis in cognitive science, and would lead to an *unconventional*, at least with respect to the widespread Aristotelian materialist belief, reversal between physics and the *classical and canonical* theology of the "virgin" (unprogrammed) universal (in the sense of Church-Turing) machine.

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## 16. Yaroslav D. Sergeyev: Thinking infinities and infinitesimals unconventionally

When Prof. Adamatzky has invited me to contribute to this article discussing unconventional thinking and the roads leading scientists working with non-traditional computational paradigms to their fields I was surprised. However, the idea to try to discover philosophical, cultural, and spiritual sources of the unconventional computing is really original. In the following few pages I first briefly introduce my field – Grossone Infinity Computing – and then describe my personal road to this discovery.

In order to start let us remind an important distinction between *numbers* and *numerals*. A *numeral* is a symbol (or a group of symbols) that represents a *number*. A *number* is a concept that a *numeral* expresses. The same number can be represented by different numerals. For example, the symbols '9', 'nine', 'IIIIIIIII', and 'IX' are different numerals, but they all represent the same number. Rules used to write down numerals together with algorithms for executing arithmetical operations form a *numeral system*.

It is worthwhile to mention that different numeral systems can express different sets of numbers. For instance, Roman numeral system is not able to express zero and negative numbers and such expressions as II – VII or X-XI are indeterminate forms in this numeral system. As a result, before appearing the positional numeral system and inventing zero mathematicians were not able to create theorems involving zero and negative numbers and to execute computations with them. Thus, numeral systems seriously bound the possibilities of human beings to compute and developing new, more powerful than existing ones, numeral systems can help a lot both in theory and practice of computations.

It is interesting that there exist very weak numeral systems allowing their users to express just a few numbers and one of them is illuminating for our study. This numeral system is used by a tribe, Pirahã, living in Amazonia nowadays. A study published in *Science* in 2004 (see [129]) describes that these people use an extremely simple numeral system for counting: one, two, many. For Pirahã, all quantities larger than two are just 'many' and such operations as 2+2 and 2+1 give the same result, i.e., 'many'. Using their weak numeral system Pirahã are not able to see, for instance, numbers 3, 4, and 5, to execute arithmetical operations with them, and, in general, to say anything about these numbers because in their language there are neither words nor concepts for that.

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Notice that the result 'many' is not wrong. It is just *inaccurate*. Analogously, when we observe a garden with 546 trees, then both phrases: 'There are 546 trees in the garden' and 'There are many trees in the garden' are correct. However, the accuracy of the former phrase is higher than the accuracy of the latter one. Thus, the introduction of a numeral system having numerals for expressing numbers 3 and 4 leads to a higher accuracy of computations and allows one to distinguish results of operations 2+1 and 2+2.

In particular, the poverty of the numeral system of Pirahã leads to the following results

$$\text{'many'} + 1 = \text{'many'}, \quad \text{'many'} + 2 = \text{'many'},$$

$$\text{'many'} - 1 = \text{'many'}, \quad \text{'many'} - 2 = \text{'many'}, \quad \text{'many'} + \text{'many'} = \text{'many'}$$

that are crucial for changing our outlook on infinity. In fact, by changing in these relations 'many' with $\infty$ we get relations used to work with infinity in the traditional calculus

$$\infty + 1 = \infty, \quad \infty + 2 = \infty, \quad \infty - 1 = \infty, \quad \infty - 2 = \infty, \quad \infty + \infty = \infty,$$

It should be mentioned that the astonishing numeral system of Pirahã is not an isolated example of this way of counting. In fact, the same counting system, one, two, many, is used by the Warlpiri people, aborigines living in the Northern Territory of Australia (see [130]). Another Amazonian tribe – Mundurukú (see [131]) fails in exact arithmetic with numbers larger than 5 but are able to compare and add large approximate numbers that are far beyond their naming range. In particular, they use the words 'some, not many' and 'many, really many' to distinguish two types of large numbers. Their arithmetic reminds strongly the rules Cantor uses to work with countable and uncountable, i.e., with the numerals $\aleph_0$ and $\mathcal{C}$, respectively. For instance, compare these two records

$$\text{'some, not many'} + \text{'many, really many'} = \text{'many, really many'},$$

$$\aleph_0 + \mathcal{C} = \mathcal{C}.$$

This comparison suggests that our difficulty in working with infinity is not connected to the *nature* of infinity but is a result of inadequate numeral systems used to express infinite numbers. Traditional numeral systems have

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been developed to express finite quantities and they simply have no sufficiently high quantity of numerals to express different infinities (and infinitesimals). In other words, the difficulty we face is not connected to the object of our study – infinity – but is the result of weak instruments – numeral systems – used for our study.

The field of Grossone Infinity Computing introduced in [132, 133, 134] allows one to look at infinities and infinitesimals in a new way and to execute *numerical* computations with a variety of different infinities and infinitesimals on the Infinity Computer patented in USA (see [135]) and other countries. This approach proposes a numeral system that allows one to use the same numerals in all the occasions we need infinities and infinitesimals. There are applications in numerical solution of ordinary differential equations (see [136, 137, 138, 139]), the first Hilbert problem, Turing machines, and lexicographic ordering (see [140, 141, 142]), hyperbolic geometry, fractals, and percolation (see [143, 144, 145, 146, 147, 148]), single and multiple criteria optimization (see [149, 150, 151, 152]), infinite series and the Riemann zeta function (see [153, 154, 155]), cellular automata (see [156]), etc.

The way of reasoning where the object of the study is separated from the tool used by the investigator is very common in natural sciences where researchers use tools to describe the object of their study and the used instrument influences the results of the observations and determine their accuracy. When a physicist uses a weak lens $A$ and sees two black dots in his/her microscope he/she does not say: The object of the observation *is* two black dots. The physicist is obliged to say: the lens used in the microscope allows us to see two black dots and it is not possible to say anything more about the nature of the object of the observation until we change the instrument - the lens or the microscope itself - by a more precise one. Suppose that he/she changes the lens and uses a stronger lens $B$ and is able to observe that the object of the observation is viewed as eleven (smaller) black dots. Thus, we have two different answers: (i) the object is viewed as two dots if the lens $A$ is used; (ii) the object is viewed as eleven dots by applying the lens $B$. Both answers are correct but with the *different accuracies* that depend on the lens used for the observation.

The field of Grossone Infinity Computing looks analogously at Mathematics that studies numbers, objects that can be constructed by using numbers, sets, etc. Numeral systems used to express numbers are among the instruments of observations used by mathematicians. The powerful numeral system introduced in [132, 133, 134] gives the possibility to obtain more precise

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results in Mathematics (in particular, working with infinities and infinitesimals) in the same way as a good microscope gives the possibility of obtaining more precise results in Physics.

Let us tell now the tale of discovering Grossone Infinity Computing. In November 2002, when I was 39 years old, the Italian Government has invited me to Italy to the prestigious position of Distinguished Professor at the University of Calabria, Italy. So, I have got a possibility to stop writing papers with a high speed that is necessary to survive in the scientific jungle and decided to look out of my field – global optimization – and to think what I could do next in my scientific life. In the same time I have bought a flat in a building that was in construction and decided to organize it following the rules of feng shui in such a way that the flat and its furniture would increase the intellectual force of its owner. I did not know whether this could help in my research but since I had a freedom to organize my flat in any way (including moving internal walls) I have decided to adopt this approach. In particular, the place where the intellectual force should be the strongest was where I have put my bed. It is interesting that due to feng shui, in order to increase the intellect it was necessary to sacrifice some other part of the personality and I have decided to sacrifice the emotional part (in any case, I thought, people think that mathematicians are not able to have emotions).

I then spent several months reading various texts on open problems in mathematics, computer science, and physics. In April 2003, in an evening, one of my friends told me laughing by phone that it was written in my horoscope that during that month I would invent something very interesting. I have laughed also, went in my bed, and try to sleep. Then, being in a border phase between wakeful and sleeping, the idea of how to count different infinities and infinitesimals avoiding the usual paradoxes came in my mind. I have immediately understood its importance and spent the following few months checking the approach and developing it without almost sleeping and eating (I have lost 8 kilograms in 4 months). Every time when I faced a trouble I returned to my bed and was falling into a kind of a trance that helped me to solve the difficulty.

I then have spent several years working on details and looking for applications. Many people have started to adopt this methodology in their research. We have organized several conferences, published many papers, this research was awarded several international awards, etc. More I work in this field, more I am convinced that this new way of computing is in its very first stage. It really opens new horizons in mathematics, computer science, and physics.

## 17. Karl Svozil: Why computation?

Nowadays I might be able to express my long time intuition in a category theoretical form [157]: in short, computation and physics are both categories linked by functors. Thereby category theory serves as a sort of Rosetta Stone [158], making possible a translation among very similar, possibly equivalent, structures – with the functors serving as translators back and forth between the physical and the computational universes. One may even enlarge this picture by other categories like mathematics, and the natural transformations between the possible functors. In what follows I shall rant about computation as a metaphysical as well as metamathematical metaphor. At the same time, computation could also be understood as a narrative designed to navigate and manipulate the impression of what we experience as physical world.

First it should be acknowledged that, on the one hand, although conceptualized with paper-and-pencil operations in mind [159, p. 34], the category of computation, as many structures invented by our minds, including mathematics and theology [160] or our money [161], appears to be "suspended in free thought" – and solely grounded in our belief in it.

On the other hand, there appears to be "physical stuff out there" which at first peek appears to be rather solid and "material." Alas, the deeper we have looked into it, and the better our means to spatially resolve matter became, the more this stuff looked like an emptiness containing point particles of zero extension. Moreover, throughout the history of natural sciences, there appears to be no convergence of "causes," but rather a succession of alternating narrations and (re)presentations as to why this stuff interacts: take what we today call gravity, turning from mythology to Ptolemaian geometry to Newtonian force back to Einsteinian space-time geometry [162]. And this is a far cry from explaining why something exists at all – even if this something might turn out to be primordial chaos, or an initial singularity (possibly hiding other cycles of other universes).

Indeed, it can be expected that, for an embedded observer [163] in a virtual reality, the computational intrinsic "phenomenology" supporting such an agent appears just as "material," and even "quantum complementary like" [164], as our own universe is experienced by us. A surreal feeling is expressed by Prospero in Shakespeare's Tempest, claiming that *"we are such stuff as dreams are made on."* (Some [165] have therefore concluded that science cannot offer much anchor from which to comprehend and cope with the

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absurdities of our existence.) Ought we therefore not be allowed to assume that the category subsumed under the name "physics" contains entities and structures which are not dissimilar to computation?

Second, consider the functors which — like a function — assigns to each entity in the physical world an entity in the computational universe. More specifically, the Church-Turing thesis, interpreted as functor between physics and computation, specifies that every capacity in the physical world is reflected by some computational, algorithmic capacity of what is known today as a partial recursive function, or universal Turing computability. This is a highly nontrivial claim which needs to be corroborated or falsified with every physical capacity we discover. It is, so to say, under "permanent attack" from physics. Although highly likely, nobody can guarantee that it will survive the next day. To give one exotic and highly speculative example: maybe someone eventually comes up with a clever way of building infinity machines with some Zeno squeezed cycles. It is also interesting to note that one might be able to resolve the seemingly contradicting claims of "information is physical" by Landauer, as well as "it from bit" by Wheeler, through perceiving both physics and computation as categories linked by functors.

A universal computer, hooked up to a quantum random number generator (serving as an oracle for randomness) is supposed to be (relative to the validity of orthodox quantum mechanics) a machine transcending universal computational capacities. Claims of computational capacities beyond Turing's universal computability may turn out to be difficult to (dis)prove. One way might involve zero-knowledge proofs or zero-knowledge protocols; but I am unaware of any such criterion [166]. Unfortunately, some such instances, in particular "true randomness" or "true (in)determinism" as claimed by quantum information theory, due to reductions to the halting and rule inference problems, are provable impossible to prove.

The converse functor, mapping entities from universal computation into entities in the physical universe is considered unproblematic. After all, in principle, given enough stuff, universal computers could be physically realized; at least up to some finite means. These finite physical means induce bounds on universal computability [167].

Speaking about computation might be like speaking about physics. And any capacity of one category has to show up in the other one as well. In view of this it is highly questionable if nonconstructive entities such as continua are more than a formal convenience, if not a distractive misrepresentation, of physical capacities.

Let me, in the second part, come to a sketch of the semantic aspect of the categories compared earlier; and just how and why they could have formed.

Suppose that there exist (we do not attempt here to explain why this should be so; for instance due to fluctuations or initial values) two regions in space with a difference in temperature, or, more generally, energy (density). Suppose further that there is some interface, such as empty space, or material structure, or agent, allowing physical dissipative flows from one region into the other, connecting these two regions. Then, as expressed by the second law of thermodynamics [168], there will be an exchange of energy, whereby statistically energy flows from hot to cold through the interface. So far, this is a purely physical process.

Let us concentrate on the interface. More specifically, let us consider a variety of interfaces, and look at their relative efficiency or "fitness" (we are slowly entering an evolution type domain here). Undoubtedly, all things equal, the type of interface with the highest throughput rate of energy per time will dominate the dissipation process: it can "grab the biggest piece of the cake." Finding good or even optimal interfaces might be facilitated through random mutation; thereby roaming through an abstract space of possible interface states and configurations. The situation will become even more dynamic if the relative magnitude of the various processes can change over time. In particular, if a very efficient process (which needs not be the most efficient) can self-replicate. Then a regime emerges which is dominated by the Matthew effect [169] of compound interest: the population of the strongest interface will increase relative to less effective interfaces by the rate of compound interest – which is effectively exponential. This means that the growth rates will at first look linear (and thus sustainable), but later grow faster and faster until either all the energy is distributed or other side conditions limit this growth. Now, if we identify certain interfaces with biological entities we end up with a sort of biological evolution driven by physical processes; in particular, by energy dissipation [170].

How does computation come into this picture? Actually, quite straightforwardly, if we are willing to continue this speculative path: systems which compute can serve as, and even construct and produce, better interfaces for energy dissipation than systems without algorithmics. Thus, through mutation, that is trial-and-error driven by random walks through roaming configurations and state space, the universe, and in particular, self-reproducing agents and units, have learned to compute. This is, essentially, a scenario for the emergence of mathematics and of universal computation.

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## 18. Genaro Martinez: patterns of computation

Having pre-Hispanic ancestors I have been always interested to understand underlying mechanics, and spiritual reasons, in formation of patterns by and orientation of the pyramids, and use of a heave circular monolith as calendar showing solar phases and various astronomic phenomena.

First example can be found at the heart of Aztec culture — Tenochtitlan (the centre of Mexico City). There, *templo mayor* (main temple) has a specific ortho orientation. Also, there is a number of pyramids in the central and southern parts of Mexican Republic, most prominent locations are Teotihuacan (a multi ethnic empire with a Sun and Moon pyramids), and Chichen Itza (a cradle of Maya culture). My aspiration to understand and simulate patterns of pyramids as dynamical systems led me to unconventional computing. I focused mainly in cellular automata theory, thanks to the influence of Prof. Harold McIntosh in the state of Puebla with whom I discussed origins of mathematics in the world and a role of Aztec calendar as an original concept of periodic stages and unique enumeration system. Thus I developed my research around cellular automata representations of patterns and enumeration of patterns as a computational problem [171, 172].

Computability in cellular automata theory is a good example where we can unleash a power of imagination to develop non-conventional devices performing recurrent computations. In our search for novel abstract forms of computations, we find a diversity of representations, which can be interpreted as computations. In this way, the computer science establishes a formal definitions to separate computation from other processes [173, 174]. Examples include pattern formation, swarm behaviour and intelligence, slime mould geometry, wave propagation and other non-linear spatially extended systems [175, 176, 177].

In the unconventional computing we interpret spatio-temporal dynamics of non-linear systems as processes in logical circuits or mathematical machines, including equivalents of Turing machine. A typical quest in the field is the following: given a dynamical system, decide if the system could implement computation or not. Of course, the interpretation depends on the interpreter and 'multi-origin' background of the unconventional computists allows us to consider a wide range of system at nano, micro and macro-levels. The nature of computation [178, 179] or the interpretation of simple programs [180] is to design computing processes and devices structure and function of which are limited only by our imagination.

During the last decade unconventional computing evolved by expanding a range of physical, chemical and biological substrates where conventional computing circuits, e.g. Boolean logical gates, can be realised. I believe the field is now entering a new phase where novel computing paradigms and architectures, inspired by the substrates, will be developed.

## 19. Georgios Ch. Sirakoulis: Computing is understanding

The very first question when scientists come across to the term "unconventional computing" is what exactly the difference is compared to what we know, we apply and we implement, so as to do and produce computation so far. While there are many various definitions and different angles in the topic that try usually to establish a unique connection with the perspectives of such fascinating term, the main problem of "unconventional computing" remains mainly a matter of interpretation and perception also arriving by the subjectivity of the scientist(s) willing to use the term and the conceived ideas on how to produce computation for her(their) problems and tentative applications.

In the case of scientists arriving mainly from the electrical and computer engineering field, as I do, commonly among most of us (especially in the past years or better say in the last few decades) there was a tendency of skepticism what such an exotically considered type of computing, i.e. "unconventional computing" would be in position to deliver to the computing science especially compared to the considered conventional types of computation. However, without a firm definition of the term, scientists from this field were usually frustrated to find common place on their background for the application of such type of computation.

Nevertheless, due to the limitations introduced from the technology and design of computing systems, mainly related with open problems like beyond CMOS technology, more than Moore concept, not von-Neumann architectures, just to name a few of the today's technological and hardware related challenges, the quest for juvenile solutions and corresponding novel types of devices, circuits and systems became a quest of paramount need. Consequently, the unconventional computing related idea, even in the case that was differently speculated by the engineers, started to pave the way for trying to find such solutions to the aforementioned, and most important to the future, open problems, obeying a quite intriguing confrontation; that is for undefined, unpredictable future, we need something beyond the lim-

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its of controversial computing to guide and manifest the tomorrow needed computation.

In my case, after working for many years with Cellular Automata (CAs) as massively parallel computing complex models for the design, development and implementation of novel computing hardware systems, I was also thrilled by the opportunity to deliver alternative non standard computation not only with CAs but also by using novel beyond CMOS models and devices and non von-Neumann architectures, like memristors in crossbar arrays [181, 182, 183], by interfering with biological templates like slime mould and FPGAs [184], by incorporating new processing info with DNA CAs [185], by thinking of non standard logic arriving from species interactions [186], by applying chemical computing for substituting classical Boolean CMOS gates [187], by manipulating swarm robotics [188], by utilizing plants for logic computation [189], etc. Such examples are and should be considered just a few of the various and literally countless examples of unconventional computing.

Moreover, it was just some time ago, when the words of the famous physicist Richard Feynman quoted on his blackboard (as found after his death in February 1988) came apparently to my foreground:

. . . What I cannot create, I do not understand.

Thus, when considering what is the purpose of unconventional computing, to better paraphrase Feynman's sentence, may I dare say: "What I cannot compute, I do not understand", and the idea is that unconventional computing is meant to solve the puzzle and offer the expected solution again and again, now and, in particular, in our future.

## 20. Bruce MacLennan: A philosophical path

As a philosophically inclined computer scientist, I was very interested in problems in epistemology and the philosophy of science. Therefore in the late 1970s I began reading its literature, attending philosophy of science conferences, and eventually joined both the Philosophy of Science Society and the History of Science Society. As a consequence, I learned the inadequacies of logical positivism, which had been my working philosophy, and began to appreciate the requirements for a more accurate account of human knowledge and cognition. I concluded (along with many others pursuing "naturalized

epistemology") that epistemology could not be developed in an a priori fashion, but needed to take account of our scientific knowledge, including human psychology and neuroscience.

About this time I read the revised edition of Hubert Dreyfus' *What Computers Can't Do* (1979), which applied a phenomenological critique to symbolic AI. He showed how contemporary approaches to knowledge representation and cognition were based on long-discredited epistemology and would suffer the same limitations. His book was widely condemned by the AI community, but much of the criticism came from ignorance (or uninformed dismissal) of twentieth-century continental philosophy. What was often overlooked, moreover, was that in addition to his critique, Dreyfus had made several positive suggestions about the sorts of physical systems that might exhibit genuine intelligence. Among the take-aways: Heidegger had important insights into skilled behavior; most concepts are not defined by necessary and sufficient conditions, but are more like Wittgensteinian "family resemblances"; cognition is more often imagistic than discursive; understanding takes place against a background of unarticulated and largely unarticulatable common sense; there are many things that we understand simply by virtue of having a body; and brains do not work like digital computers.

Since it became apparent to me that contemporary AI was built on inadequate theories of knowledge and cognition, I designed and taught a graduate-level course, "Epistemology for Computer Scientists," which surveyed Western epistemology from the pre-Socratics to contemporary debates. This developed into a book *Word and Flux: The Discrete and the Continuous in Computation, Philosophy, and Psychology*, which eventually became two volumes. Volume 1 was titled *From Pythagoras to the Digital Computer: The Intellectual Roots of Symbolic Artificial Intelligence* and traced the descent of symbolic AI from the origins of Western philosophy to contemporary issues in cognitive science, AI, and the theory of computation. Volume 2 was intended to present alternative theories of knowledge, drawing especially from continental philosophy, including Heidegger, Polanyi (tacit knowledge), Merleau-Ponty (phenomenology of perception), the later Wittgenstein, Jung (archetypes), Maturana (autopoiesis), Varela (neurophenomenology), Lakoff and Johnson (metaphorical thought), field theories in psychology (gestalt psychology, Nalimov, Lewin, Pribram), and new theories of the embodied mind. I also intended to explain the new foundation provided by the theory of artificial neural networks and massively parallel analog computation, and to outline the implications of this theory for our understanding of knowledge

in general and for our understanding of the mind and of science in particular. Unfortunately, I did not quite complete vol. 1 and barely started vol. 2, but the background research has informed most of my work since the late 1980s. *Ars longa, vita brevis!*

It became apparent that if AI were to succeed, research would have to begin with the brain, since it clearly operated by different principles than digital computers and traditional symbolic AI. Since the latter (so called "Good Old-Fashioned AI") was rooted in formal logic with its (often implicit) background of assumptions, I concluded that the "new AI" that was emerging from connectionism, neural network research, and neuroscience would require new concepts of knowledge representation and processing [190]. In particular, massively parallel analog information representation and processing in cortical maps inspired my research in *field computation*, in which information is represented in spatially continuous distributions of continuous data (or in discrete spatial arrays sufficiently dense to be treated as a continuum) [191]. This was intended as a design for future neurocomputers with very dense arrays of analog computational elements (which also invites optical and quantum implementations), but also as a mathematical model of cortical information processing.

As I continued to explore analog computation (more accurately termed *continuous computation*), I began to see how pervasive were the ideas and assumptions of discreteness, not only in computer science, but also in the foundations of mathematics, logic, linguistics, and psychology. Therefore I adopted a research strategy: wherever I found something that was discrete, I would consider the implications of assuming instead that it was continuous. Instead of taking the discrete as basic and assuming that apparently continuous phenomena were actually discrete, I would turn it on its head, assume the continuous was basic, and treat apparently discrete phenomena as fundamentally continuous.

Some theorists have argued that continuous computation is not computation at all, asserting that Church and Turing defined computation, and that's the end of it. I have argued that, at very least, this is historically incorrect, since it ignores analog computation, which had been as important as digital computation. But it does raise the problem of defining computation: how is it distinguished from other physical processes? I have argued that computation is distinguished by the fact that its function or purpose in a larger system could, in principle, be served as well by a different physical system obeying the same mathematical laws (i.e., it is multiply-realizable

and therefore formal) [192].

A perennial problem is the relative "power" of unconventional computation compared to the Turing machine. Here the philosophy of science comes to our aid, if we remember that the Turing machine is a model, and that each model makes simplifying assumptions that are appropriate for a certain class of questions, its *frame of relevance* [193]. Models give bad (inaccurate, misleading) answers when applied outside of their frames of relevance. I have argued that the interesting questions about many unconventional computing paradigms are outside the frame of relevance of the Turing model, and so, for the most part, such comparisons are meaningless and misleading.

On the one hand, we know Moore's Law is coming to an end; on the other, brain-scale neural computing requires millions or billions of artificial neurons. This has been a concern of mine since I began working on neurocomputers more then thirty years ago. We need to make (analog) computational elements that are sufficiently small, but more importantly, we need to connect them in intricate patterns such as we find in the brain. Here again I think we can apply some ideas from philosophy, in particular, from embodied philosophy and cognitive science, which focus on the essential role that embodiment plays in psychology. In particular, a principal purpose of cognition is to control the physical body, and conversely the brain is able to offload some computational processes to the physical interaction of the body with its environment. By analogy we may define *embodied computation* as "computation in which the physical realization of the computation or the physical effects of the computation are essential to the computation" [194]. The theory of embodied computation provides a basis for using computational principles to design physical systems that have desired physical effects, such as the assembly of complex physical structures. We have been applying this to *artificial morphogenesis*, which applies the embodied computation principles of embryological development to coordinate massive swarms of microscopic agents to assemble complex physical structures.

As I look back at my career in unconventional computing, I realize that it has been guided by philosophical ideas, questions, and methods. What is knowledge and how is it represented in the brain? What are concepts and how are they learned? How do we think, remember, imagine, and communicate? What is the relation of mind and body, and how does this relate to robots and computers? How does nature compute? What are formal processes? What is computation? What are the limits of models? It is important to remember that from its beginning computer science was not merely technology, but

had important connections with philosophy (as is apparent from the work of Turing, Church, Gödel, von Neumann, and others, even back to Leibnitz). Insights from philosophy are still valuable to us; they invite us to question the assumptions of conventional computation, and they suggest new directions for unconventional computation.

## 21. Susan Stepney: Three steps to Unconventional Computing

I came to UCOMP late in my career via a round-about route. I was originally a physicist, but I decided during my post-doctoral research that being a theoretical astrophysicist in the climate of the 1980s UK was not a practical career plan, so I moved to industry. The computer industry in those days was happy to employ someone with a PhD in an arcane technical subject, and some knowledge of Fortran programming (despite me never again writing another line of Fortran). It was there that I learned my computer science, mainly through various formal methods projects: proving correct certain business-critical algorithms, from compilers [195, 196] to electronic cash purse protocols [197, 198]. The mathematical modelling skills I had absorbed as a physicist served me well in this work, but none of the other background I had, none of the physics, none of the link to the real material world, seemed to be relevant. Except on two occasions.

The first occasion was during the compiler proof work. We had a potential client who was very excited about the work, and was interested in us doing something more ambitious for them, to prove the entire stack, from compiler, through the assembler, down to microcode and chip design, so that they could have a "fully proved system". During the discussion, I said something along the lines of: "But of course, you can't prove that the physical system implements the lowest level model correctly. Proof only works for the mathematical models, not for actual physical devices. There might be manufacturing defects, or other problems." Excitement deflated rapidly, and we didn't get that contract. (I was never very good at sales.) Along similar lines, I recall someone at a conference saying "when you can prove that your software works correctly when the device is dunked in liquid sodium, I will use it for the safety interlocks on my nuclear reactor."

The second occasion was during the electronic purse project. Our team was proving the cash transfer protocols obeyed the security properties: no cash made, no cash lost. Another team was working on the cryptography, and we were taking properties of the cryptographic hash function as axiomatic

in our proof [199]. During the development project, "side channel" attack techniques were published. These attack crypto systems not through their mathematical properties, but through measuring behaviours such as timing [200] or power consumption [201] during the algorithm's execution. The very concept of these attacks stunned some mathematical computer scientists, but for those of us with a physics background, it seemed perfectly natural that "breaking the model" [202] of the physical system would lead to such possibilities. Indeed, many safety and security issues can be considered as the system moving outside its model, and hence moving outside the realms of any formal proof.

So when I had the opportunity to move back into academia at the beginning of the new millennium, I was primed to consider physical aspects of computer systems, and decided to start my third career by researching unconventional computation (UCOMP).

I start from Stan Ulam's famous quote: *using a term like non-linear science is like referring to the bulk of zoology as the study of non-elephant animals* [203, 204]; in actuality, non-linear science forms the bulk of natural science. So it may be with UCOMP: it may form the bulk of computer science. However, conventional (or classical) computation (CCOMP) has had much more effort expended, both theoretically, and in engineering computers. Today, UCOMP is broad but (relatively) shallow, whilst CCOMP is narrow, but incredibly deep. What would computation look like if UCOMP were as deep as CCOMP, and there were an integrated theory combining all its aspects? From now on, I will just refer to "computation" (when referring to abstract models) or "computing" (when referring to the actions of physical devices).

Computing is physical [205]. The world is physical: it comprises matter and energy. It contains *information*, physically embodied in the structure and organisation of that matter and energy. And parts of it *compute*: purposefully manipulate and process that physically embodied information. I have been working with colleagues on unpicking what we mean by physical computing: *physical computing is the use of a physical system to predict the outcome of an abstract evolution* [206]. The "abstract evolution" is the desired computation; the physical system is used to compute that evolution.

Is "used" by what, exactly? By the *representational entity*, the entity whose purpose is determining the outcome of the abstract evolution. This representational entity does not need to be a person, but it (almost certainly) needs to be alive [207]. Our definition [206] also allows us to distinguish when

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arbitrary exotic substrates are computing, from when they are being used as scientific experiments to determine their computational potential, and also to highlight the role of engineering a substrate to perform particular computations. It allows *computer science to be seen as a natural science* [208].

This broadens the definition of computing away from "whatever a Turing Machine does". But it does not allow everything. Our view is not a pan-computationalist one: the universe is not computing itself, rocks are not computing arbitrary functions, because there can be no associated representational entity using them for this purpose [209]. Also, there appears to be a deep link between the limits of what physical devices can do, and what (quantum) Turing Machines can do. That the laws of physics constrain computational power is unsurprising; that they appear to constrain it to just what was devised mathematically is remarkable.

Some researchers buck at these constraints, however, and postulate super-Turing computers (more efficient) or even hypercomputers (more effective). However, investigation of these machine designs (the modern day equivalent of perpetual motion machines?) shows that they appear to require one of two properties of the physical world to be changed: the currently understood laws of physics need to be changed (often back to Newtonian laws), or a physical infinity needs to be instantiated (usually of precision or time) [210]. These approaches seem to assume that the model exactly captures the physical system. In side channel attacks (above), the model does not encompass the entire physical system: it neglects features like power consumption. With these proposals, the model is *more* powerful than the physical system: for example, that the model is cast in terms of infinite precision real numbers in no way means that any physical system supports infinite precision quantities and measurements. So hypercomputers seem unlikely.

But hypercomputing is not the only goal of UCOMP. Examining fundamental differences in the assumptions behind CCOMP models and physical systems may help in the design of UCOMP devices that can simulate certain physical processes and complex systems more naturally [211]. Composing a variety of unconventional substrates may also lead to better exploitation of their diverse properties [212, 213]. Biology offers an exciting route to UCOMP, because it is the study of evolved (as opposed to engineered) complex substrates capable of information processing [207]. As well as studying living material, it is worth studying non-living substrates that have sufficiently complex structure and dynamics [214, 215], to explore a diversity of

behaviours that might be analysed with a common model [216].

I have been looking to more complex physics, chemistry, and biology to find new insights into *computational novelty* [217]. This again harks back to the idea of "breaking the model", and realising there is a difference between the model and the physical system. Any sufficiently complex time-evolving system eventually moves outside (breaks) the model we use to capture it, new properties and function emerge, and we then must build a new model. One challenge is how to capture such richness and model-breaking *in silico*: how can a designed computational system move outside its design? Meta-programming is a classical option [218], but UCOMP potentially holds the key, with systems directly exploiting rich physical properties.

In summary, my take on UCOMP is that it enriches computer science by foregrounding the embodied nature of information and computation, and it enriches the natural sciences by foregrounding the informational and processing abilities of complex matter.

## 22. Conclusion

'The inner tangle and the outer tangle —
This generation is entangled in a tangle.
And so I ask of Gotama this question:
Who succeeds in disentangling this tangle?'
(S.i.13, Visuddhimmaga 'The Path of Purification' [219])

We aimed to establish links between spirituality at the intersection of East and West cultures and our personal quests in unconventional computing. What is unconventional computing? Answering the unanswerable? Combining the incompatible? Each one of us might define it differently. Unconventional computing is: going beyond discriminative knowledge (Morita), computing with endo-observers (Gunji), challenging impossibilities (Calude), intrinsic parallelism and nonuniversality (Akl), everywhere-intelligence (Schumann), the art of paradoxes (Konkoli), harmonious wholeness of wushu (Burgin), spirit of dissent (Adamatzky), order emerging from chaos (Dehshibi), infinity (Sergeyev), subcellular nirvana (Mayne), many dreams theology (Marchal), physics of measurement (Costa), patterns of complexity (Martinez), science of "uncomfortable" (Margenstern), continuous computation (MacLennan), physical universe (Svozil), undefined computation (Sirakoulis), finding causality in complexity (Zenil), a natural science (Stepney). Have we addressed the aim of the special issue on whether there is a conjunction of the

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Eastern and West thought tradition in exploring the nature of computation? The essays presented show that each author had different journey towards the unconventional computing. There is no evidence that any cultural tradition or religious or spiritual beliefs underly our styles of thinking. Our spiritual worlds and styles of thinking are highly diverse and nonlinear. This diversity of the 'ecosystem of thoughts' should be cherished and protected. Is there any connection between the Eastern and Western thought traditions as a central and leading element of scientific development? Unlikely. The scientific world is now highly mobile and interconnected, stereotypes and beliefs acquired by us in childhoods already disappeared and we look at the nature of intellectual challenges through the eyes of pragmatic scientists. The Nature is an amazing machinery, defined by laws of physics, chemistry and biology, which leaves little place for unfounded beliefs. Why we are doing unconventional computing? One of possible explanations could be that facing the indifference of nature to our lifes we create our own beacons of light to travel through the darkness of unknown.

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