# Transcending Turing Computability

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## Outline

**I** Introduction

**II** Assumptions of Turing Computability

**III** Ubiquity of Assumptions

**IV** Transcending Turing Computability

**V** Conclusions

## I. Introduction

- Hypercomputation
- Are neural networks or analog computers more powerful than TMs?
- Ongoing research on computational power of analog computers

### Some Proofs of Hypercomputation

- Pour-El & Richards (1979, 1981, 1982)
- Garzon & Franklin (1989, 1990)
- Stannett (1990)
- Siegelmann & Sontag (1994)
- Bournez & Cosnard (1995)

## II. Assumptions of TM Theory

**A** Historical Context

**B** Information Representation

**C** Information Processing

**D** Interpretation

**E** Theory

### Historical Context

- Questions of effective calculability in mathematics
- Rooted in perennial assumptions about knowledge and thought
- TM is a good first approximation

### Information Representation

- Finiteness, definiteness, formality
- Atomic constituents: Tokens & types
- Texts & schemata

### Information Processing

- Finite number of definite, discrete steps
- Expressible as finite number of discrete, finite rules

### Interpretation

- Formal semantics

- Well-formed formulas
- Semantics defined recursively in terms of constituent structure

### Theory

- Issues of consistency & computational completeness
- Termination of computations
- What classes of functions can it compute?
- Functions on reals are approximated
- Not very relevant to natural computation

### III. Ubiquity of Assumptions

- Taken for granted in logic & mathematics
- E.g. point-set topology & set theory
- Formal axiom systems
- Cannot be taken for granted in natural computation

### IV. Transcending Turing Computability

**A** Natural Computation

**B** What issues should a model of NC address?

**C** Directions toward a theory of NC

**D** Massively-parallel Analog Computation

### A. Natural Computation

*Natural Computation* is computation inspired by computation in nature (e.g., neural networks, genetic algorithms).

Very similar to *Biocomputation.*

### B. Some Key Issues of NC

- Real-time response
- Tolerance to noise, error, faults, damage
- Flexible response to novelty
- Continuously adaptive
- "Satisficing" vs. optimizing
- Non-general algorithms

### C. Directions Towards a Theory of NC

**1** Information Representation

**2** Information Processing

**3** Interpretation

**4** Theory

### Information Representation

- Information represented in continuous *images*
- All quantities, qualities, etc. are continuous
- Images treated as wholes
- Noise and uncertainty are always present

### Information Processing

- Continuous in real time
- Usually nonterminating
- Noise, error, uncertainty, and nondeterminacy assumed
- Continuous in dependence on states, inputs, etc.
- May be gradually adaptive
- Need not be describable by rules
- Matched to specific computational resources and requirements

### Interpretation

- Images need not represent propositions;processes need not represent inference
- Pragmatics is primary; there need not be an interpretation
- Interpretability and interpretations are continuous

### Theory

Unimportant issues:

- Termination
- Universal computation
- Asymptotic complexity & complexity classes

### Important Issues

- Generalization ability & flexibility in response to novelty
- Behavior in response to noise, error, uncertainty, etc.
- Optimizing performance subject to realtime & resource constraints
- Adapting processes to improve performance
- Power defined in terms of real-time response, flexibility, adaptability, robustness, etc.

### V. Conclusions

- Turing Machine theory is not *wrong* but *irrelevant*.
- An overstatement
- Assumptions of TM theory are not a good match to natural analog computation
- It is important to engage the traditional issues
- But it's also imperative to transcend them
- New paradigms bring new questions as well as new answers