Is $an^2 + bn + d = O(n^2)$ for $a, b, d >= 1, b > d$? To prove this, we need to find a constant $c$ such that $cn^2 \geq an^2 + bn + d$. Let $c = 2a$. Now we need to find a constant $x$ such that for all $n >= x$, $2an^2 \geq an^2 + bn + d$. We'll try $x = 2b$.

Let's proceed by an inductive argument. To make our life simpler, let $f(n) = 2an^2$, and $g(n) = an^2 + bn + d$. When $n$ = 2b, $f(2b) = 8ab^2$ and $g(2b) = 4ab^2 + 2b^2 + d = (4a + 2)b^2 + d$. Since $b > d$ and $b^2 > b$, $f(x) > g(x)$.

Now, let's assume that our statement is true for all values between $x$ and $n$ for some $n$. We already know that this is true for $n = x$. Let's look at $n + 1$:

$$\begin{aligned} f(n+1) &= 2a(n+1)^2 \\ &= 2an^2 + 4an + 2a \\ &= f(n) + 4an + 2a \end{aligned}$$

$$\begin{aligned} g(n+1) &= a(n+1)^2 + b(n+1) + d \\ &= an^2 + 2an + a + bn + b + d \\ &= an^2 + bn + d + 2an + (a+b) \\ &= g(n) + 2an + (a+b) \end{aligned}$$

From our inductive hypothesis, we know $f(n) \geq g(n)$, thus:

$$f(n) + 4an + 2a \geq g(n) + 4an + 2a$$

All that we need to show is that $4an + 2a > 2an + a + b$:

$$\begin{aligned} 4an + 2a &>^? 2an + a + b \\ 2an &>^? b - a \end{aligned}$$

Since $n \geq 2b$, this means $4ab >^? b - a$, which is clearly true when $a, b \geq 1$. Thus:

$$\begin{aligned} f(n) + 4an + 2a &> g(n) + 4an + 2 \\ &> g(n) + 2an + a + b \\ f(n+1) &> g(n+1) \end{aligned}$$

Therefore, for all $n >= 2b$, $2an^2 > an^2 + bn + d$, meaning $2an^2 \geq an^2 + bn + d$, meaning $an^2 + bn + d = O(n^2)$ ■.