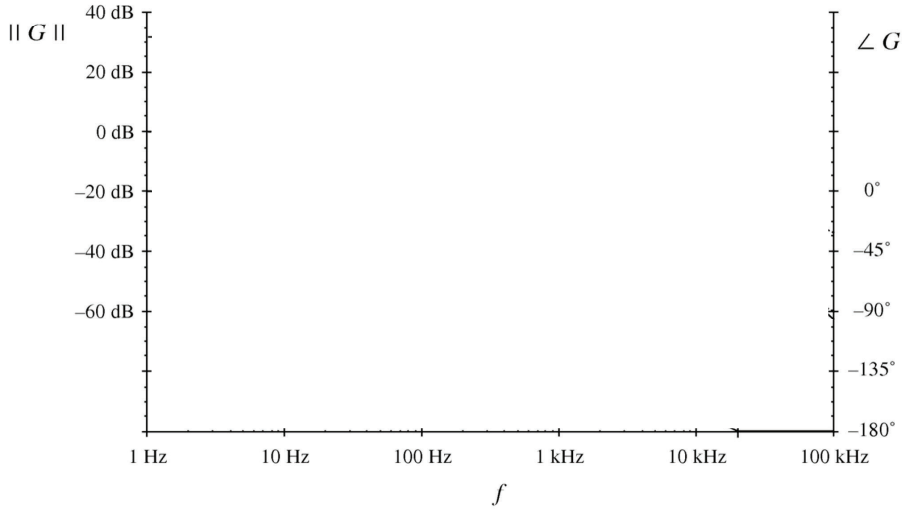
Example 1

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

with $G_0 = 40 \Rightarrow 32$ dB, $f_1 = \omega_1/2\pi = 100$ Hz, $f_2 = \omega_2/2\pi = 2$ kHz

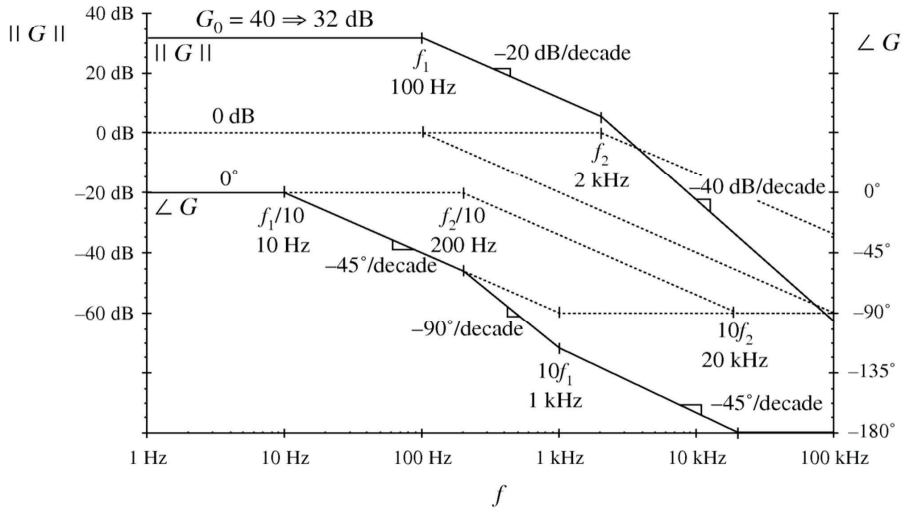


THE UNIVERSITY OF TENNESSEE KNOXVILLE

Example 1

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

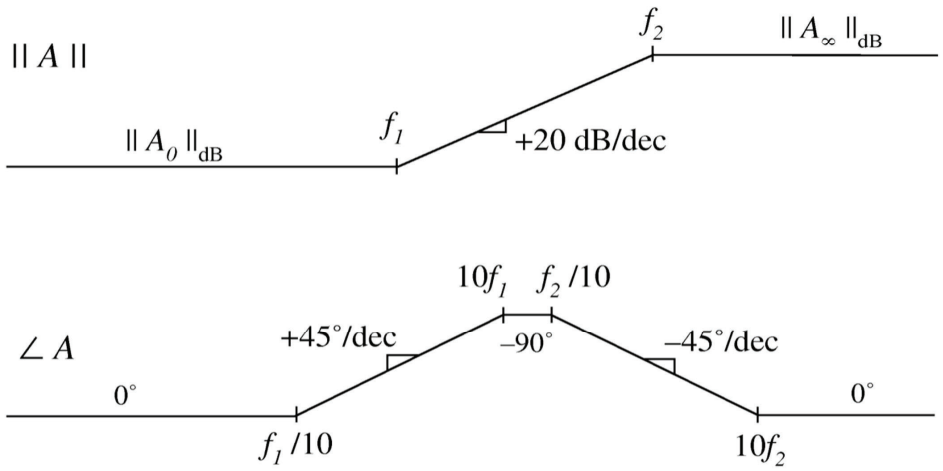
with $G_0 = 40 \Rightarrow 32$ dB, $f_1 = \omega_1/2\pi = 100$ Hz, $f_2 = \omega_2/2\pi = 2$ kHz



THE UNIVERSITY OF TENNESSEE KNOXVILLE

Example 2

Determine the transfer function $A(s)$ corresponding to the following asymptotes:



THE UNIVERSITY OF TENNESSEE KNOXVILLE

Example 3

THE UNIVERSITY OF TENNESSEE KNOXVILLE

Resonant Poles

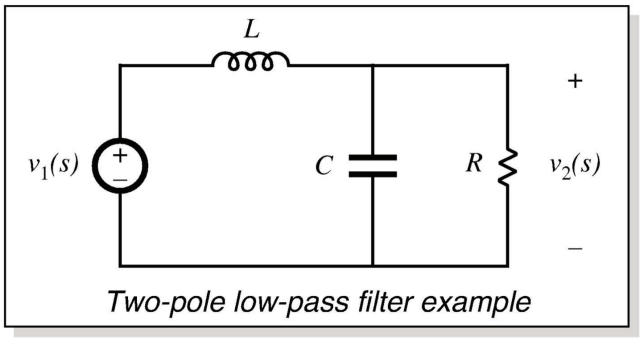
Example

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

Second-order denominator, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2}$$

with $a_1 = L/R$ and $a_2 = LC$



Two-pole low-pass filter example

How should we construct the Bode diagram?

THE UNIVERSITY OF TENNESSEE KNOXVILLE

Standard Form for Complex Poles

$$G(s) = \frac{1}{1 + 2\zeta\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

- When the coefficients of s are real and positive, then the parameters ζ, ω_0, and Q are also real and positive
- The parameters ζ, ω_0, and Q are found by equating the coefficients of s
- The parameter ω_0 is the angular corner frequency, and we can define $f_0 = \omega_0/2\pi$
- The parameter ζ is called the *damping factor*. ζ controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $\zeta < 1$.
- In the alternative form, the parameter Q is called the *quality factor*. Q also controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $Q > 0.5$.

THE UNIVERSITY OF TENNESSEE KNOXVILLE

The Q Factor

In a second-order system, ζ and Q are related according to

$$Q = \frac{1}{2\zeta}$$

Q is a measure of the dissipation in the system. A more general definition of Q, for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}$$

For a second-order passive system, the two equations above are equivalent. We will see that Q has a simple interpretation in the Bode diagrams of second-order transfer functions.

THE UNIVERSITY OF TENNESSEE KNOXVILLE

Magnitude Asymptotes

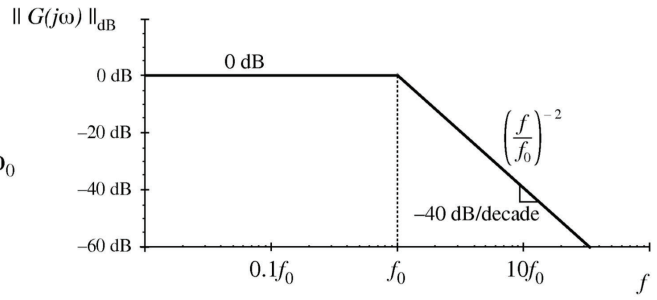
In the form $G(s) = \dfrac{1}{1 + \dfrac{s}{Q\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2}$

let $s = j\omega$ and find magnitude: $\left\| G(j\omega) \right\| = \dfrac{1}{\sqrt{\left(1 - \left(\dfrac{\omega}{\omega_0}\right)^2\right)^2 + \dfrac{1}{Q^2}\left(\dfrac{\omega}{\omega_0}\right)^2}}$

Asymptotes are

$\left\| G \right\| \rightarrow 1$ for $\omega << \omega_0$

$\left\| G \right\| \rightarrow \left(\dfrac{f}{f_0}\right)^{-2}$ for $\omega >> \omega_0$



THE UNIVERSITY OF TENNESSEE KNOXVILLE

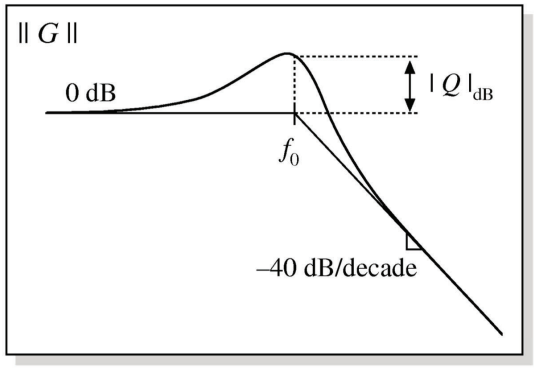
Exact Magnitude Curve

$$\| G(j\omega) \| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2}\left(\frac{\omega}{\omega_0}\right)^2}}$$

At $\omega = \omega_0$, the exact magnitude is

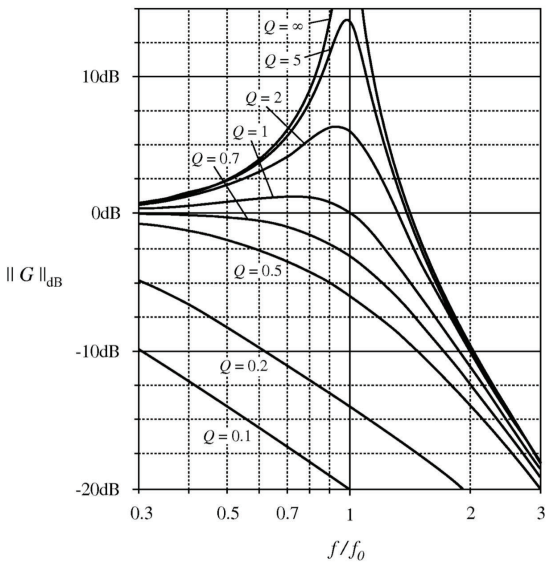
$$\| G(j\omega_0) \| = Q \qquad \text{or, in dB:} \qquad \| G(j\omega_0) \|_{dB} = | Q |_{dB}$$

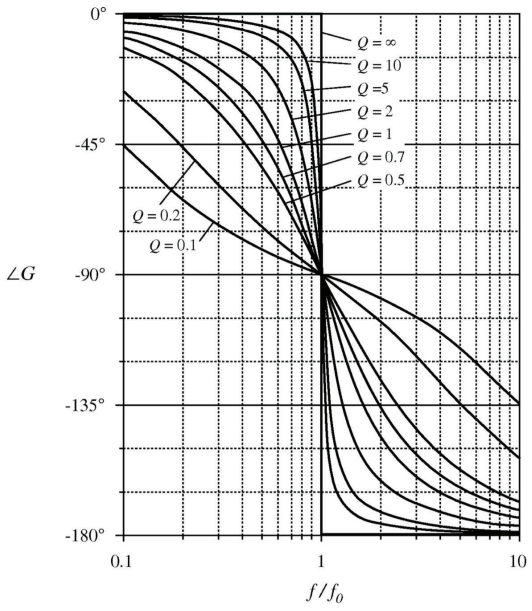
The exact curve has magnitude Q at $f = f_0$. The deviation of the exact curve from the asymptotes is $| Q |_{dB}$



THE UNIVERSITY OF TENNESSEE KNOXVILLE

Curves for Varying Q

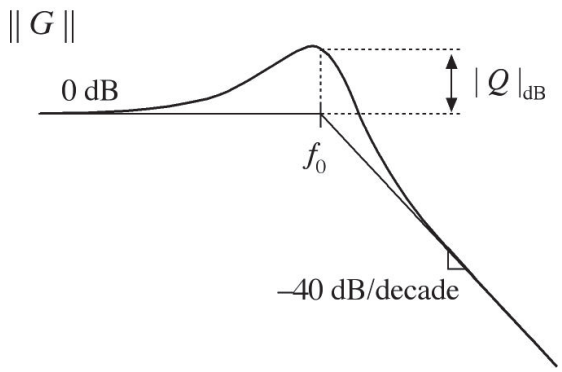




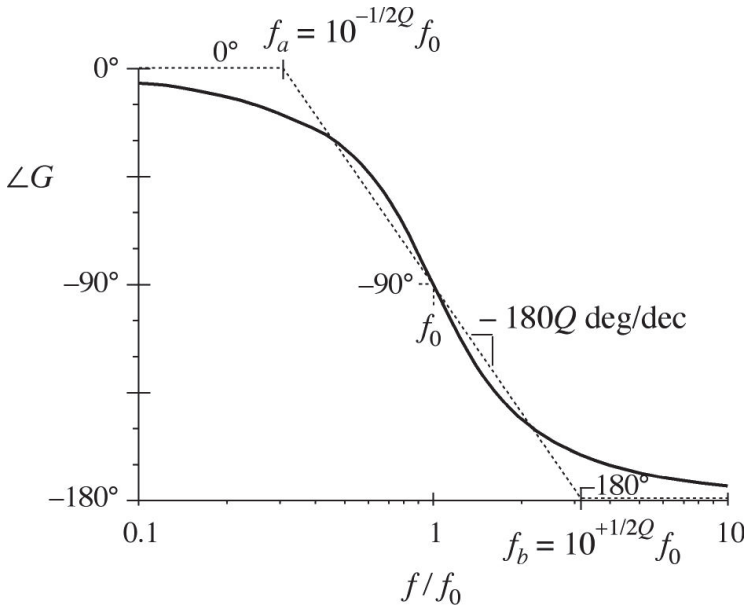
THE UNIVERSITY OF TENNESSEE KNOXVILLE

Asymptotes for Complex Poles, Q>0.5

Magnitude



Phase



Low Q Factorization

Given a second-order denominator polynomial, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \qquad \text{or} \qquad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

When the roots are real, i.e., when $Q < 0.5$, then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

This is a particularly desirable approach when $Q << 0.5$, i.e., when the corner frequencies ω_1 and ω_2 are well separated.

$$\omega_1 = \frac{\omega_0}{Q}\,\frac{1 - \sqrt{1 - 4Q^2}}{2} \qquad \omega_2 = \frac{\omega_0}{Q}\,\frac{1 + \sqrt{1 - 4Q^2}}{2}$$

Corner Frequency ω_1

$$\omega_1 = \frac{\omega_0}{Q}\,\frac{1-\sqrt{1-4Q^2}}{2}$$

can be written in the form

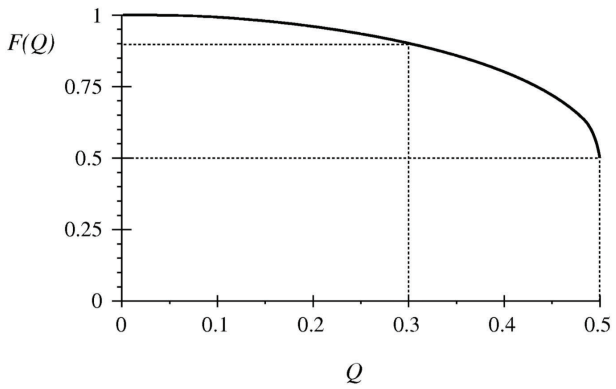
$$\omega_1 = \frac{Q\,\omega_0}{F(Q)}$$

where

$$F(Q) = \frac{1}{2}\left(1+\sqrt{1-4Q^2}\right)$$

For small Q, $F(Q)$ tends to 1. We then obtain

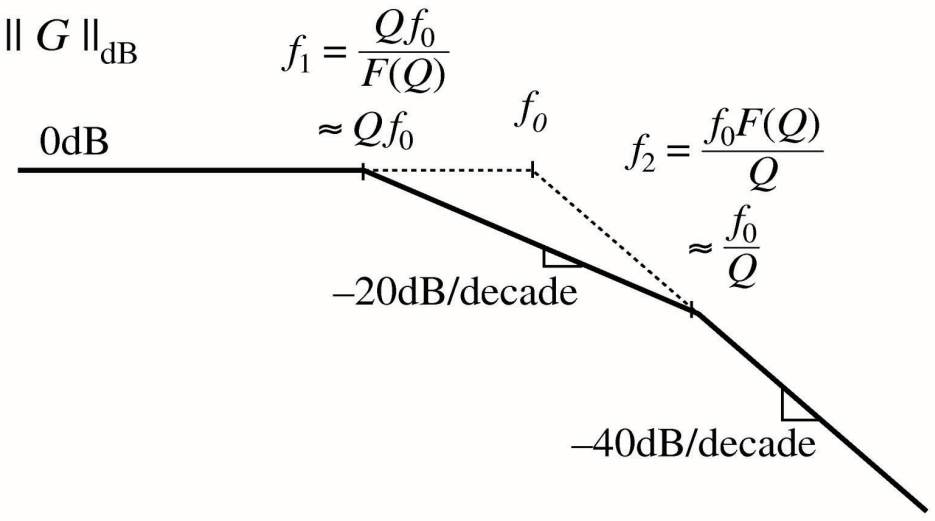
$$\omega_1 \approx Q\,\omega_0 \quad \text{for} \quad Q << \frac{1}{2}$$



For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.

THE UNIVERSITY OF TENNESSEE KNOXVILLE

The Low-Q Approximation



THE UNIVERSITY OF TENNESSEE KNOXVILLE