RMS of a sinusoid

$\omega = 2\pi f \qquad T = \frac{1}{f} = \frac{2\pi}{\omega}$

$i(t) = I_A \cos(\omega t + \varphi_I)$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left[I_A \cos(\omega t + \varphi_I)\right]^2 dt}$$

Trig Identity

$\cos^2(\theta) = \frac{1}{2}(1 + \cos 2\theta)$

$$I_{rms}^2 = \frac{\omega}{2\pi} I_A^2 \int_0^{2\pi/\omega} \cos^2(\omega t + \varphi_I)\, dt$$

$$I_{rms}^2 = \frac{\omega}{2\pi} I_A^2 \frac{1}{2} \int_0^{2\pi/\omega} 1 + \cos(2\omega t + 2\varphi_I)\, dt$$

$$I_{rms}^2 = \frac{\omega}{2\pi} I_A^2 \frac{1}{2} \left[t + \sin(2\omega t + 2\varphi_I) \frac{1}{2\omega} \right]\Bigg|_0^{2\pi/\omega}$$

$$I_{rms}^2 = \frac{\omega}{2\pi} I_A^2 \frac{1}{2} \left[\left(\frac{2\pi}{\omega} - 0\right) + \frac{1}{2\omega}(\emptyset) \right]$$

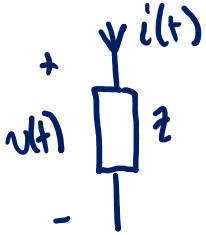
$$I_{rms}^2 = \frac{I_A^2}{2}$$

$$\boxed{I_{rms} = \frac{I_A}{\sqrt{2}}}$$

$$\boxed{X_{rms} = \frac{X_A}{\sqrt{2}}}$$

for sinusoidal signals

Power with Sinusoidal Sources



In steady-state for sinusoidal $i(t)$ & $v(t)$ @ ω, Z is some impedance

$$v(t) \rightarrow \underline{V} = V_A e^{j\varphi_V}$$
$$i(t) \rightarrow \underline{I} = I_A e^{j\varphi_I}$$
$$\underline{V} = \underline{I}\,Z$$

$$Z = \frac{\underline{V}}{\underline{I}} = \frac{V_A}{I_A} e^{j(\varphi_V - \varphi_I)} = \frac{V_A}{I_A} \measuredangle \underbrace{(\varphi_V - \varphi_I)}_{\varphi_Z}$$

power: $p(t) = i(t) \cdot v(t)$ is not LTI calculation

so go back to time domain

Trig Identity

$$2\cos\theta\cos\varphi = \cos(\theta+\varphi) + \cos(\theta-\varphi)$$

$$p(t) = I_A \cos(\omega t + \varphi_I)\, V_A \cos(\omega t + \varphi_V)$$

$$p(t) = I_A V_A \frac{1}{2}\left[\underbrace{\cos(2\omega t + \varphi_I + \varphi_V)}_{\text{sinusoidal at } 2\omega} + \underbrace{\cos(\varphi_I - \varphi_V)}_{\text{constant (not time-varying)}}\right]$$

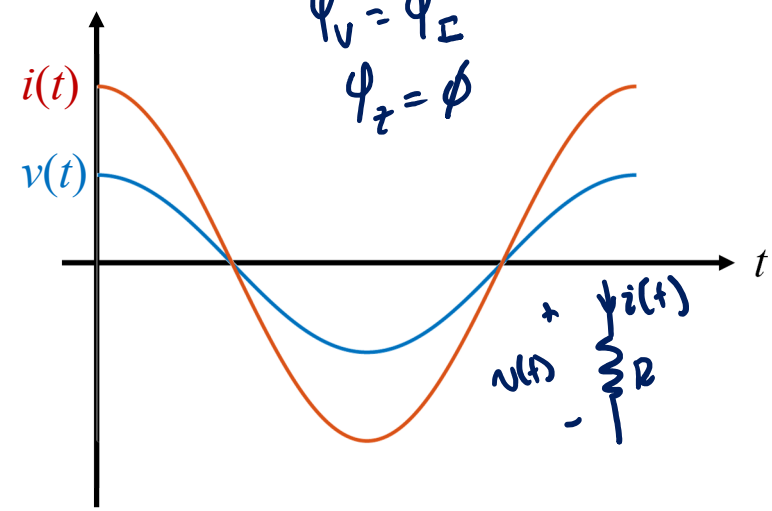
Average power:

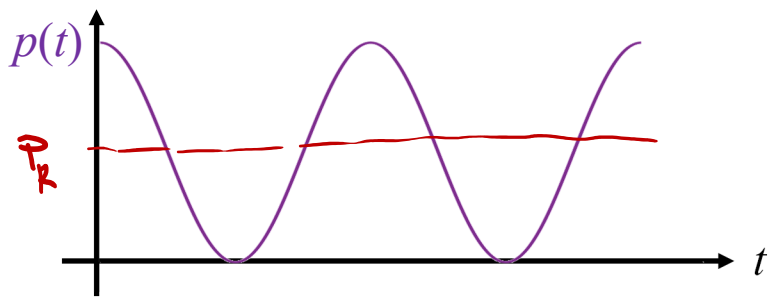
$$\boxed{P = \frac{I_A V_A}{2}\cos(\overbrace{\varphi_I - \varphi_V}^{\varphi_V - \varphi_I})} = \frac{I_A V_A}{2}\cos(\varphi_Z) = V_{rms} I_{rms} \cos(\varphi_Z)$$

Power with Sinusoidal Sources

$\varphi_V = \varphi_I$

$\varphi_Z = \phi$





$Z = Z_A e^{j\phi} = Z_A$ all real

so its a resister

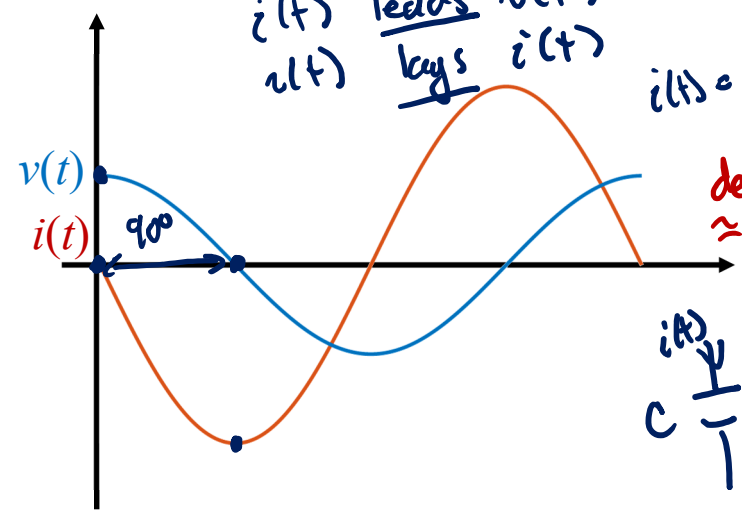
$$P_R = \frac{I_A V_A}{2}\cos(\varphi_Z) = \underbrace{\frac{I_A V_A}{2}}_{\text{for a resister}} = V_{rms} I_{rms}$$

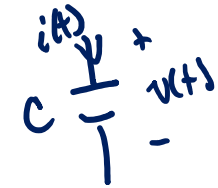
$i(t)$ leads $v(t)$

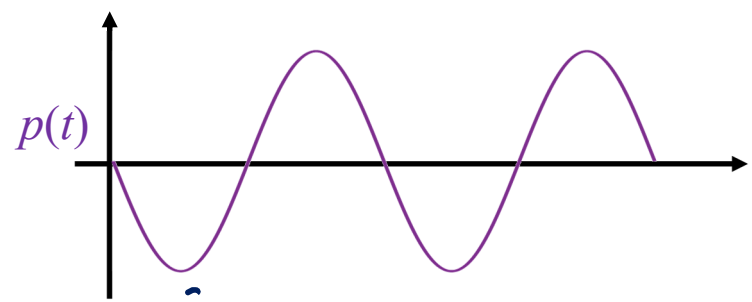
$v(t)$ lags $i(t)$

$i(t) = C\frac{dv}{dt}$

derivative $\approx 90^\circ$ lead







$Z_C = \frac{-j}{\omega C}$ $\qquad \varphi_Z = -90^\circ$

$$P_C = \frac{I_A V_A}{2}\cos(\varphi_Z) = \emptyset$$

Reactive elements (L or C) have zero average power in steady-state

Complex Power

complex conjugate

$$P = \frac{V_A I_A}{2}\cos(\varphi_V - \varphi_I) = \frac{1}{2}\mathrm{Re}\{\underline{V}\,\underline{I}^*\}$$

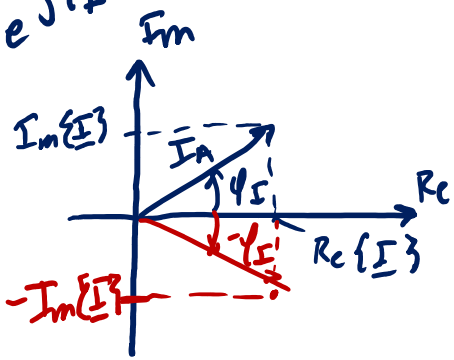
$$\underline{V} = V_A e^{j\varphi_V}$$

$$\underline{I} = I_A e^{j\varphi_I}$$

$$= \frac{1}{2}\mathrm{Re}\{V_A e^{j\varphi_V} I_A e^{j(-\varphi_I)}\}$$

$$= \frac{1}{2}\mathrm{Re}\{V_A I_A e^{j(\varphi_V - \varphi_I)}\}$$

$$= \frac{1}{2} V_A I_A \cos(\varphi_V - \varphi_I)$$



Average Power $P = \frac{V_A I_A}{2}\cos(\varphi_V - \varphi_I) = V_{rms} I_{rms}\cos(\varphi_Z) = \frac{1}{2}\mathrm{Re}\{\underline{V}\,\underline{I}^*\} = \mathrm{Re}\{\underline{V}_{rms}\,\underline{I}_{rms}^*\}$

what about the imaginary part of $\underline{V}\,\underline{I}^*$?

$$S = \frac{1}{2}\underline{V}\,\underline{I}^* = P + jQ$$

S: Complex Power [V·A] volt-amps

P: Average Power [W] watts

jQ: Reactive or Quadrature Power [V·AR] volt-amps reactive "VAR"

Apparent Power & Power Factor

"Apparent Power" $= |S| = \frac{V_A I_A}{2} = V_{rms} I_{rms}$

– maximum possible value of P = real, average power for a given V_A & I_A

"Power Factor" $PF = \frac{P}{|S|} = \frac{P}{V_{rms} I_{rms}} = \cos(\varphi_Z)$

"leading" → capacitive

"lagging" → inductive

current w.r.t. voltage

$0 \leq PF \leq 1$