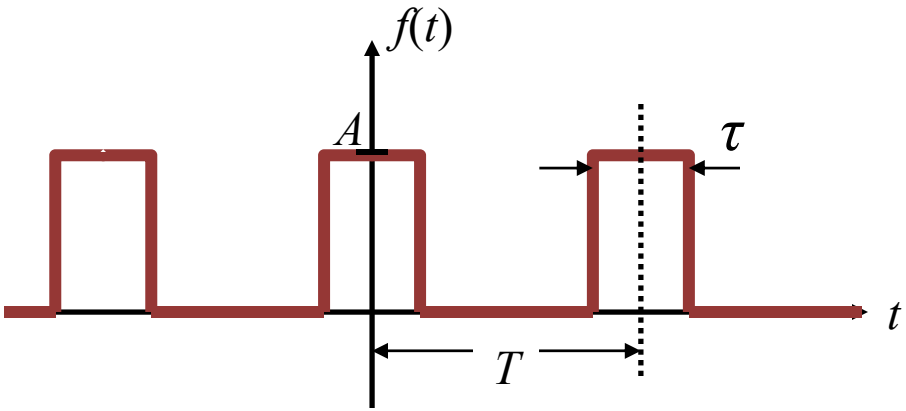
Fourier Series of a Pulse Train

$$a_0 = A\frac{\tau}{T}$$

$$b_k = 0$$

$$a_k = \frac{2A}{k\pi}\sin\left(k\pi\frac{\tau}{T}\right)$$

$$c_k = \frac{A}{k\pi}\sin\left(k\pi\frac{\tau}{T}\right) = \frac{a_k}{2}$$



Example 17.4 in book

$$\frac{1}{x}\sin(x) = \mathrm{sinc}(x)$$

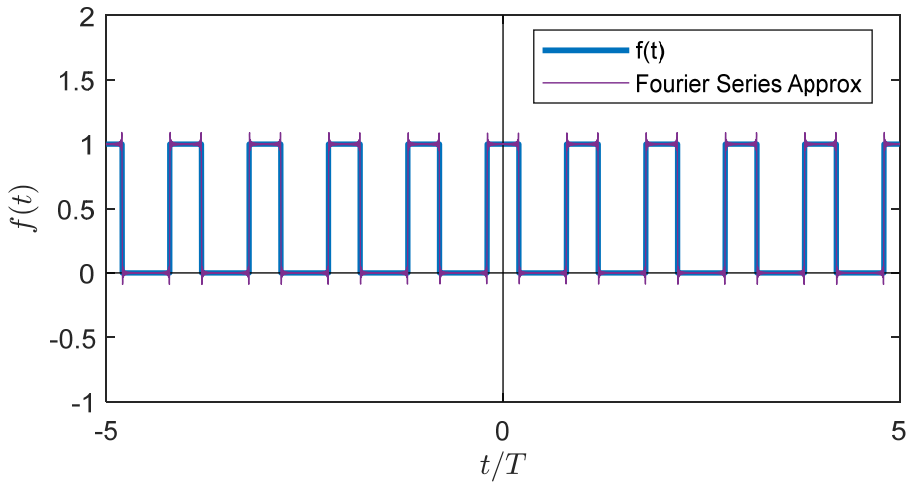
Example Matlab Calculation

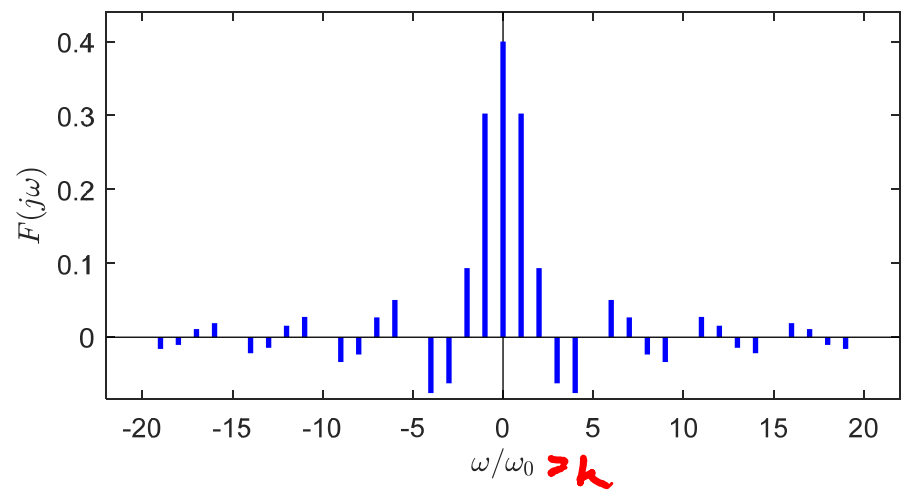
$f = 200$ Hz
$T = 5$ ms
$\tau = 2$ ms

$D = \frac{\tau}{T}$

Fourier Series Approx

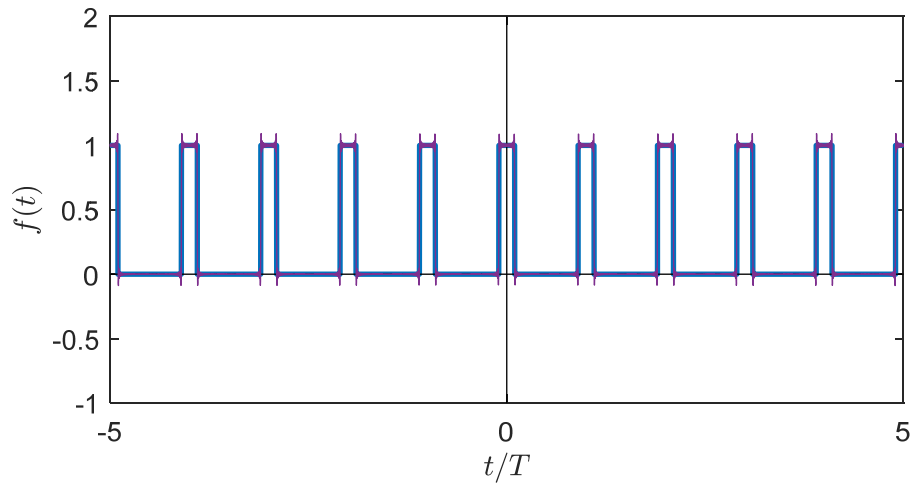
```
f = 200;
A = 1;
tau = 2e-3;

t = linspace(-1/f*5,1/f*5,100000);
a0 = A*tau*f;

sum = a0*(t./t);
kmax = 200;
for k=1:kmax
    ak(k) = 2*A/k/pi*sin(k*pi*D);
    sum = sum + ak(k)*cos(k*w0*t);
end
```

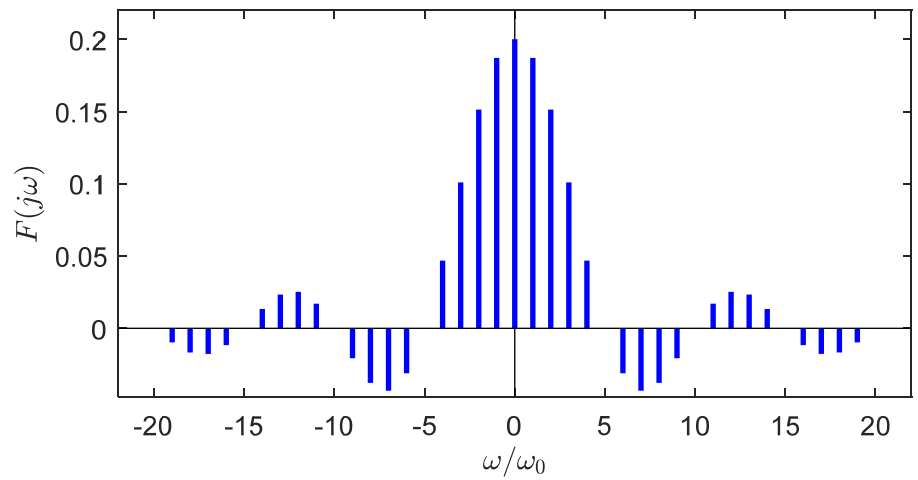




Example Matlab Calculation

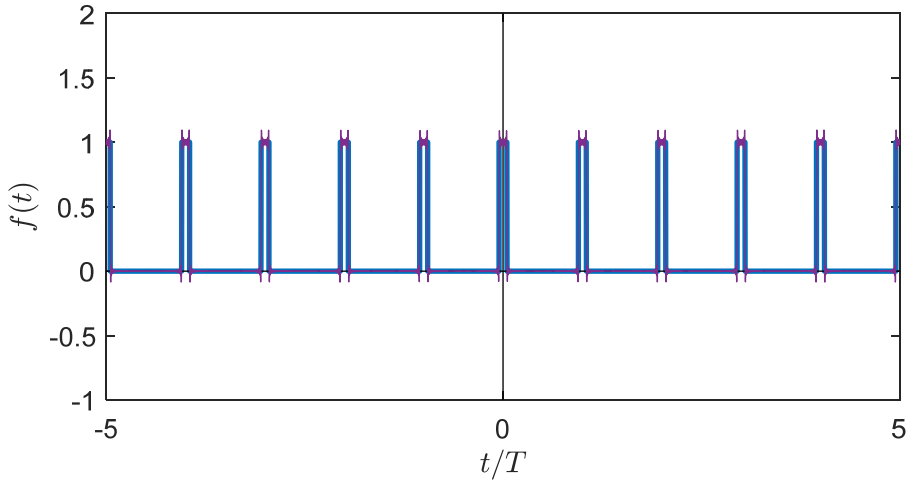
$f = 100$ Hz
$T = 10$ ms
$\tau = 2$ ms

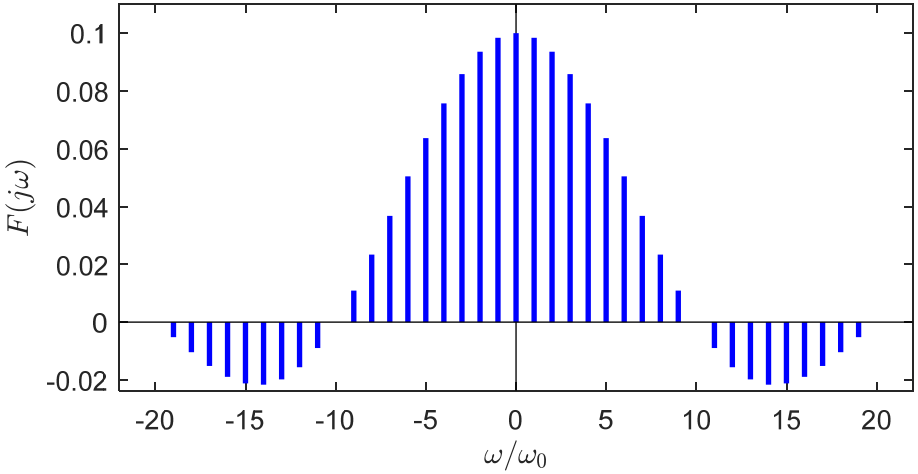




Example Matlab Calculation

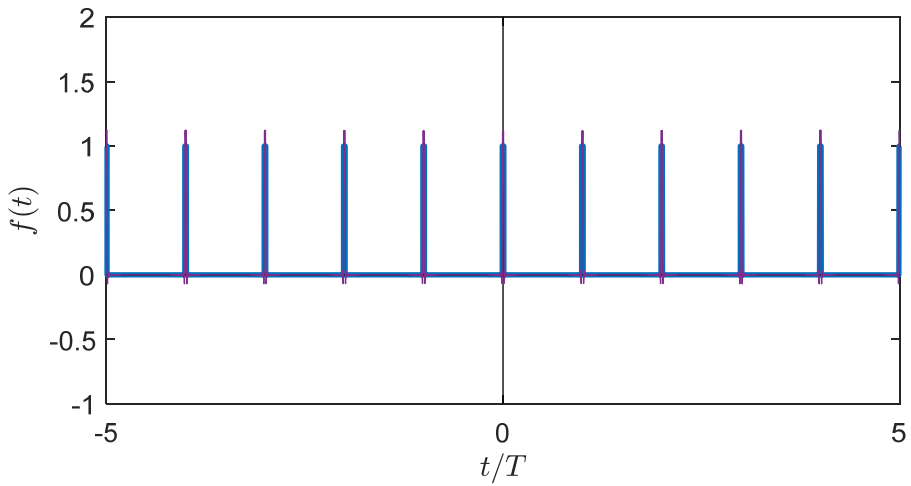
$f = 50$ Hz
$T = 20$ ms
$\tau = 2$ ms

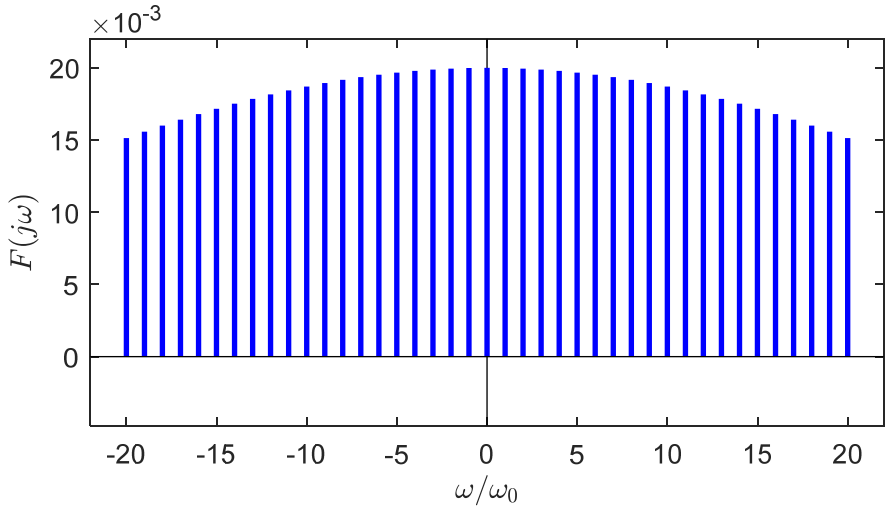




Example Matlab Calculation

$f = 10$ Hz
$T = 100$ ms
$\tau = 2$ ms



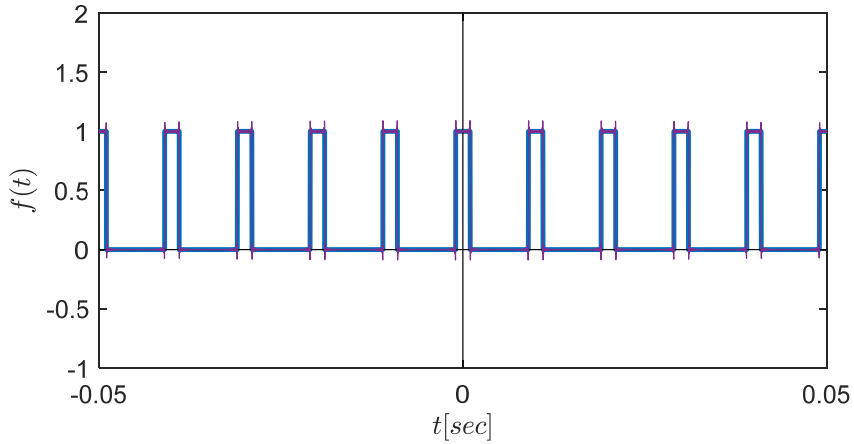


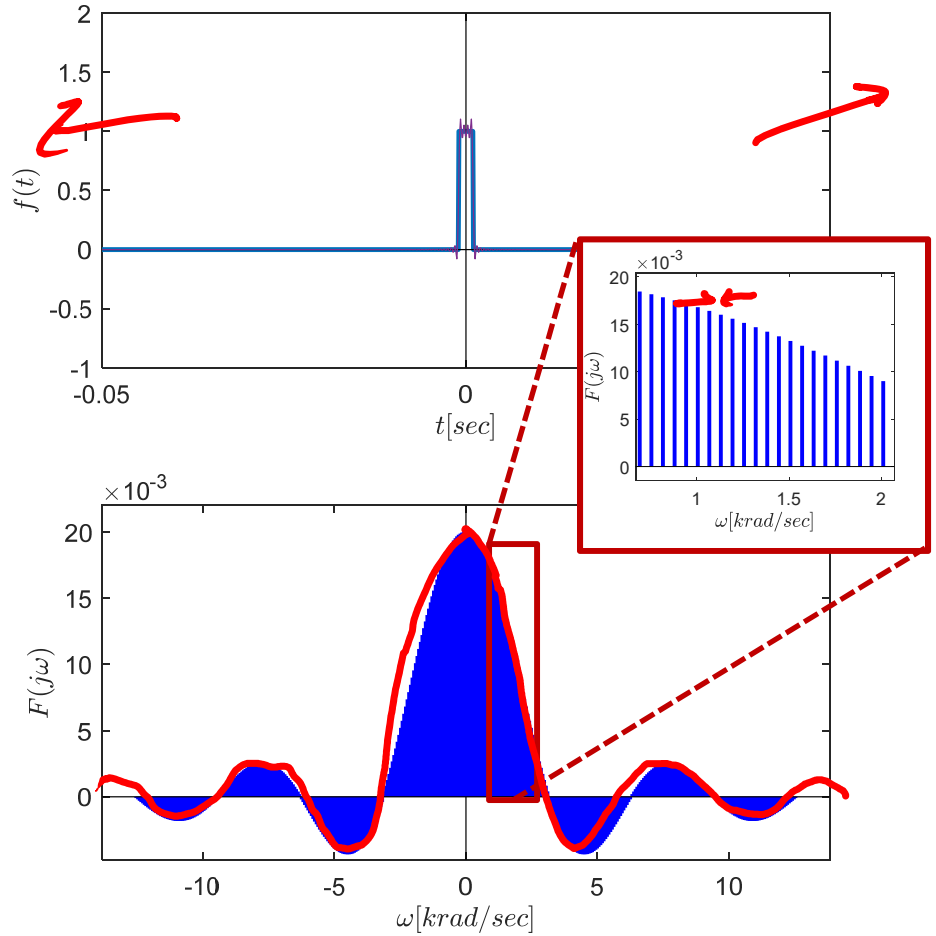
Alternate View

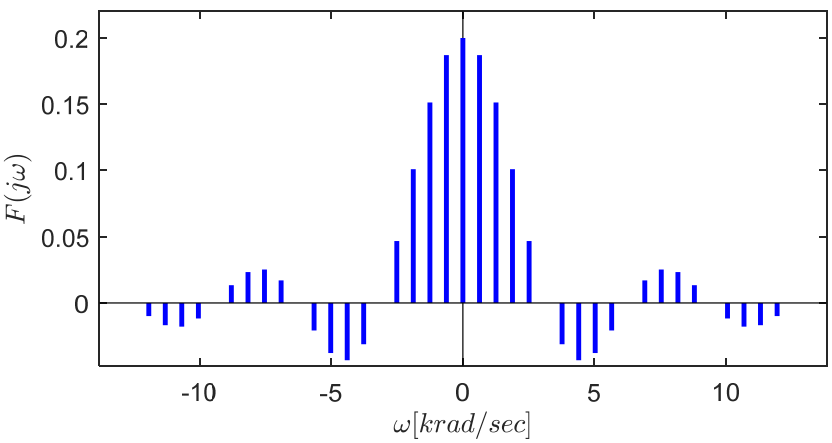
$f = 100$ Hz
$T = 10$ ms
$\tau = 2$ ms

$f = 10$ Hz
$T = 100$ ms
$\tau = 2$ ms

as $f \to 0$
$T \to \infty$







Non-periodic Waveforms: Fourier Transform

Fourier Series → only periodic waveforms

Fourier Transform → non-periodic signals

→ treat any non-periodic signal as a periodic signal with $T \to \infty$

Fourier Series: $c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jk\omega_0 t}\,dt$

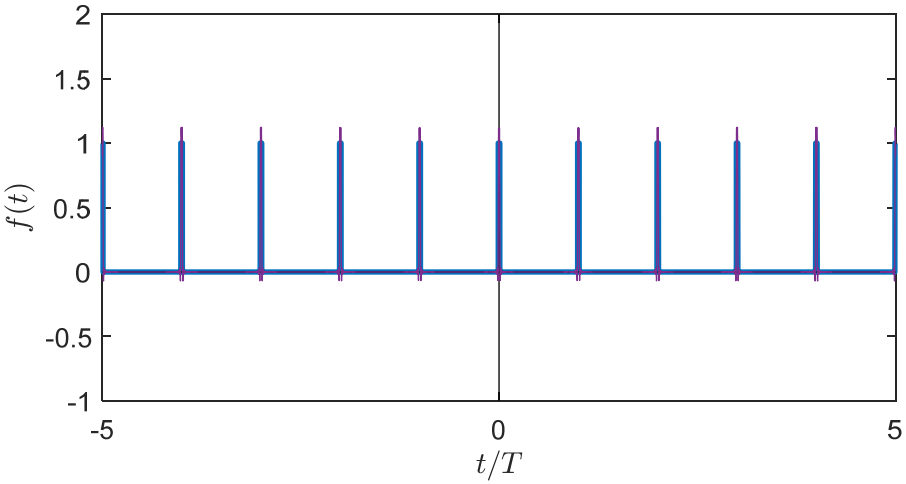
Fourier Transform: $Tc_k = \boxed{\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt = F(\omega)}$

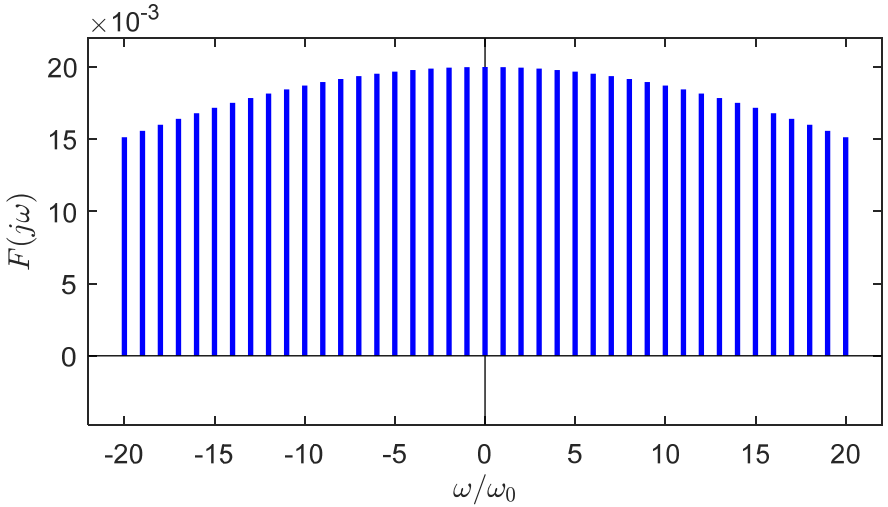
Fourier Series: $f(t) = c_0 + \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

Fourier Inverse Transform: $\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,dt}$

Fourier Series of Impulse Train

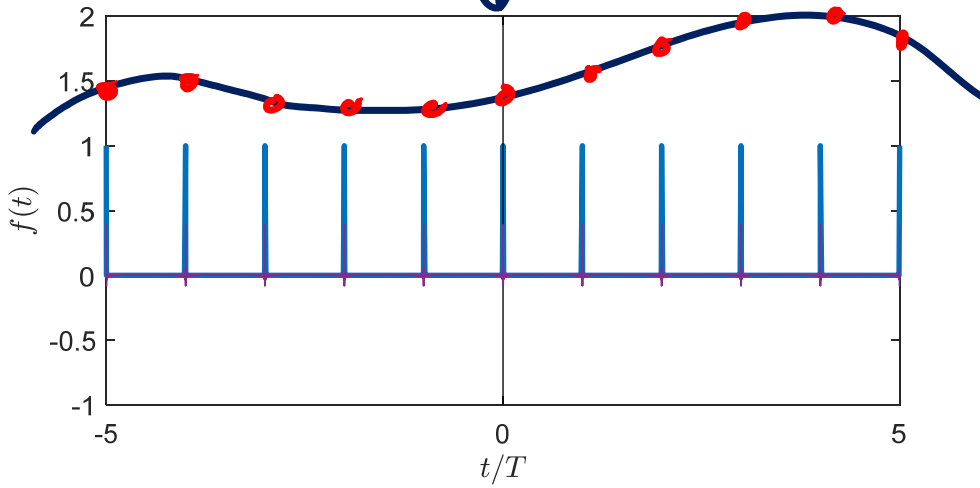
$f = 10$ Hz
$T = 100$ ms
$\tau = 2$ ms

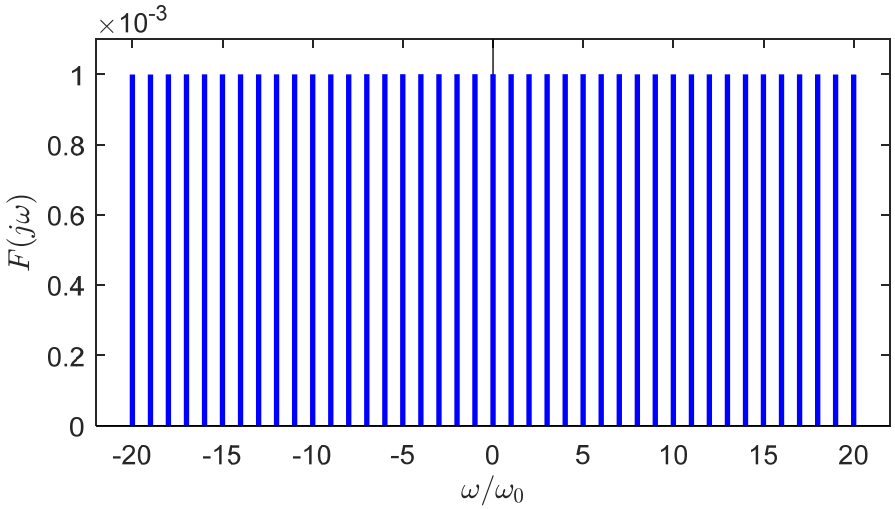




$f = 1000$ Hz
$T = 1$ ms
$\tau = .02$ ms

$\lim_{\tau \to \phi}$





Applications of Fourier Transform

- Imaging
  - Spectroscopy, x-ray crystallography
  - MRI, CT Scan
- Image analysis
  - Compression
  - Feature extraction
- Signal processing
  - Audio filtering
  - Spike detection
  - Modeling sampled systems (A/D & D/A)
  - Understanding aliasing
  - Speech recognition
- RF Communications
  - AM & FM Encoding

https://youtu.be/r6sGWTCMz2k?t=1310

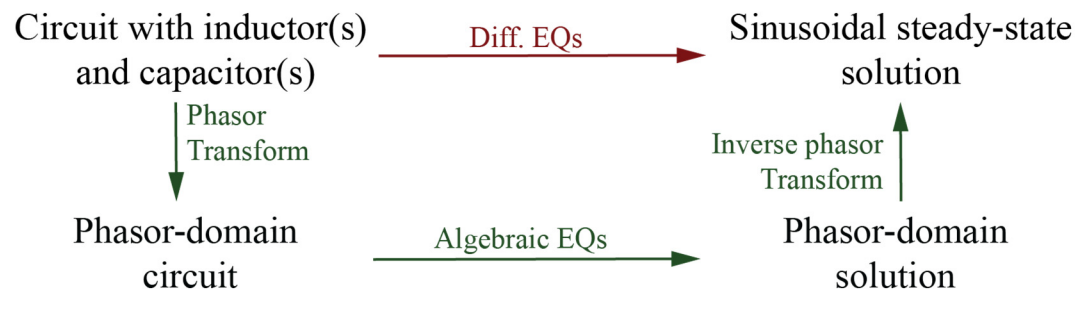
Chapter 14

S-DOMAIN CIRCUIT ANALYSIS

Transform Domains

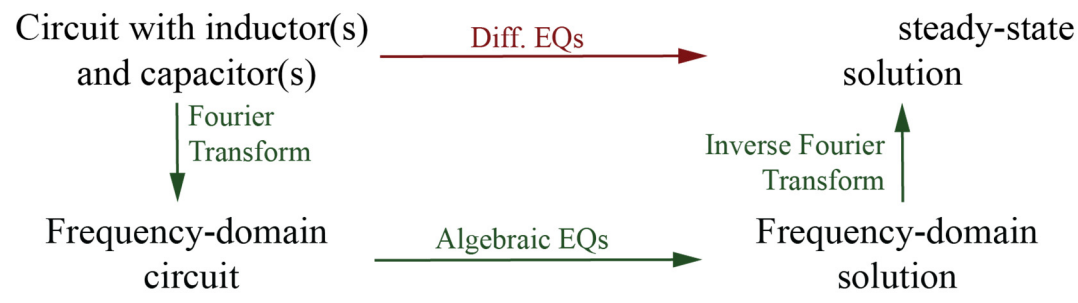
Phasor Transform

$$u(t) = A\cos(\omega_0 t + \varphi)$$



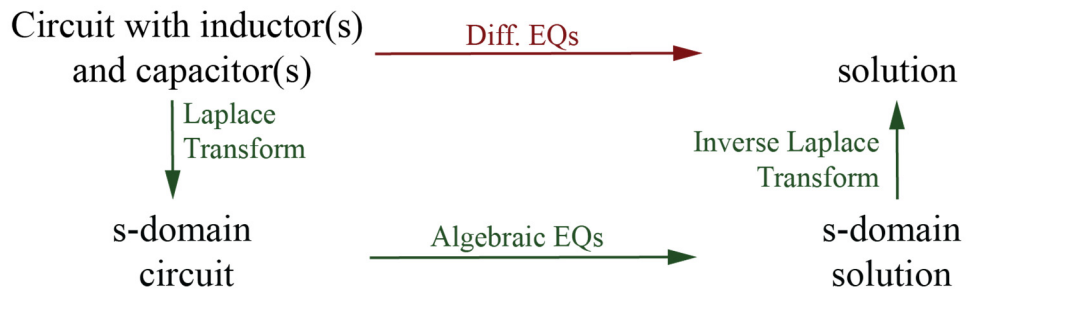
Fourier Transform

$$u(t) = \sum C_n \cos(n\omega_0 t + \varphi_n)$$



Laplace Transform

$$u(t) = \sum K_n e^{s_n t}$$



The Laplace Transform

Take Fourier Transform & replace $(j\omega)$ w/ $s = \sigma + j\omega$

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\, dt \qquad f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s) e^{st}\, ds$$

Usually (always in ELE 202) we'll use the unilateral Laplace transform

$$F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\, dt \qquad f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s) e^{st}\, ds$$

Short-hand:

$$F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{f(t)\} \qquad f(t) \longrightarrow F(s)$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{F(s)\}$$

time-domain		Frequency/Fourier Domain		Laplace/s/complex freq Domain	
$f(t)$	ODEs	$F(j\omega)$	$H(j\omega)$	$F(s)$	$H(s)$
signals	systems	signals	systems	signals	systems