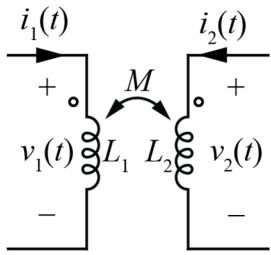


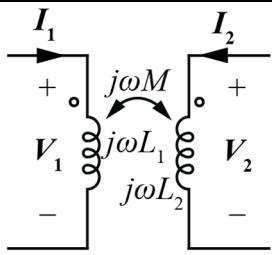
Time Domain



$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

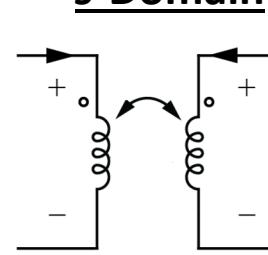
Phasor Domain



$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

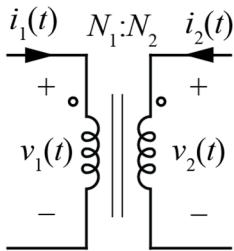
$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

s-Domain



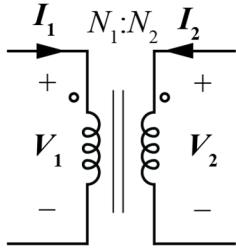
$$V_1(s) = sL_1 I_1(s) - L_1 i_1(0^-) + sMI_2(s) - M i_2(0^-)$$

$$V_2(s) = sMI_1(s) - M i_1(0^-) + sL_2 I_2(s) - L_2 i_2(0^-)$$



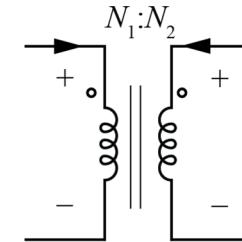
$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$

$$N_1 i_1(t) + N_2 i_2(t) = 0$$



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}$$

$$N_1 \mathbf{I}_1 + N_2 \mathbf{I}_2 = 0$$



$$\frac{V_1(s)}{N_1} = \frac{V_2(s)}{N_2}$$

$$N_1 I_1(s) + N_2 I_2(s) = 0$$

Laplace Transform of Diff EQs

N^{th} order circuit with sinusoidal input described by ($M \leq N$ for causality)

$$b_N \frac{d^N}{dt^N} v_o(t) + \cdots + b_1 \frac{d}{dt} v_o(t) + b_0 v_o(t) = a_M \frac{d^M}{dt^M} v_i(t) + \cdots + a_1 \frac{d}{dt} v_i(t) + a_0 v_i(t)$$

$$\sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t)$$

Then the Laplace transform of the circuit, neglecting initial conditions, is

$$\mathcal{L} \left\{ \sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) \right\} = \mathcal{L} \left\{ \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t) \right\}$$

$$\sum_{i=0}^N b_i s^i V_o(s) = \sum_{i=0}^M a_i s^i V_i(s)$$

Rearranging:

Solver will
be a will
polynomials of
 s

Transfer
function

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}$$

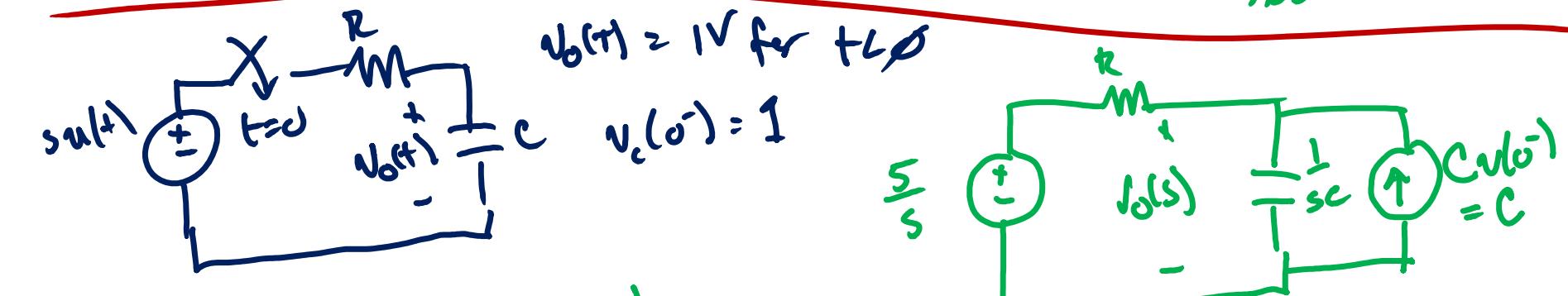
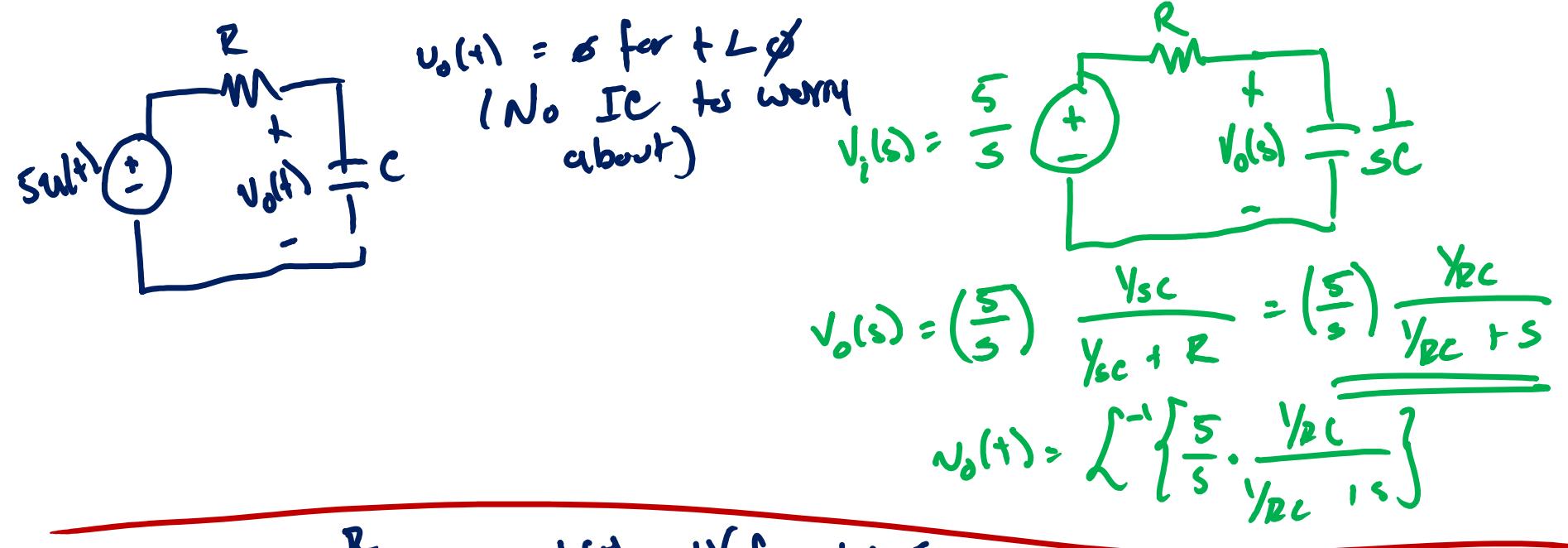
if we replace
 $s \rightarrow j\omega$
this is the
frequency response

Laplace Circuit Solution Algorithm

1. Transform all sources, signals into Laplace Domain
2. Transform circuit components (including initial conditions) into Laplace Domain
3. Solve the circuit using 201 techniques

$$V_o(s) = H(s)V_i(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}$$

4. Inverse Laplace Transform to get back to time domain



Apply superposition

$$v_o(s) = \frac{5}{s} \frac{\frac{1}{sC}}{s + \frac{1}{sC}} + C \left[R \parallel \frac{1}{sC} \right]$$

$$v_o(s) = \frac{5}{s} \frac{\frac{1}{sC}}{s + \frac{1}{sC}} + \frac{1}{s + \frac{1}{sC}}$$

$$v_o(t) \approx \mathcal{L}^{-1}\left\{\frac{5}{s} \frac{\frac{1}{sC}}{s + \frac{1}{sC}}\right\} + e^{-\frac{1}{sC}t} v_o(0^+)$$

Inverse Transforms

solve circuit to get

$$V_o(s) = H(s)V_{in}(s)$$

\uparrow

s-domain
circuit solution

laplace transform of
input

$$V_o(t) = L^{-1}\{V_o(s)\} + L^{-1}\{H(s)V_{in}(s)\}$$

Generally, we'll need to factor
the polynomials

will usually look like

$$V_o(t) = L^{-1}\left\{\frac{\sum a_i s^i}{\sum b_i s^i}\right\}$$

ex/

$$F(s) = \frac{10}{s^2 + 4s + 4} = \frac{10}{(s+2)(s+2)} = \frac{10}{(s+2)^2}$$

find roots by quadratic formula

from tables

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$f(t) = L^{-1}\{P(s)\} \Rightarrow 10t e^{-2t} u(t)$$

ex/

$$F(s) = \frac{5s+1}{s+1} \rightarrow \text{if } M > N, \text{ use long division}$$

$$= 5 + \frac{-4}{s+1}$$

$$f(t) = 5\delta(t) - 4e^{-t} u(t)$$

$$\begin{array}{r} 5 \\ s+1 \longdiv{5s+1} \\ \underline{-5s-5} \\ 4 \end{array}$$

Transfer Functions

$$H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} \stackrel{\text{factoring}}{=} \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}$$

polynomial form factored pole-zero form

roots of numerator, z_i , are called "zeros"
 - values of s where $H(s) = \emptyset$

- values of s where $H(s) \rightarrow \infty$

if all a_i are real, all z_i are either real or complex-conjugate pairs

"a bit & in all pi are .."

"bits" in "all pi" are "poles define" form of $f(t)$, zeros will define coefficients of terms \sum constant terms from $H(s)$

$$V(s) = H(s) V_F(s) \quad \{ \text{ } \beta \text{ terms from } V_F(s)$$

Partial Fraction Expansion / Decomposition

$$\frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)} = \frac{k_1}{(s - p_1)} + \frac{k_2}{(s - p_2)} + \cdots + \frac{k_N}{(s - p_N)}$$

Factor *factored pole/zeros* *PFE* *Partial Fraction Expansion*
polynomial form

$$\frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)} = \frac{k_1}{(s - p_1)} + \frac{k_2}{(s - p_2)} + \cdots + \frac{k_N}{(s - p_N)}$$