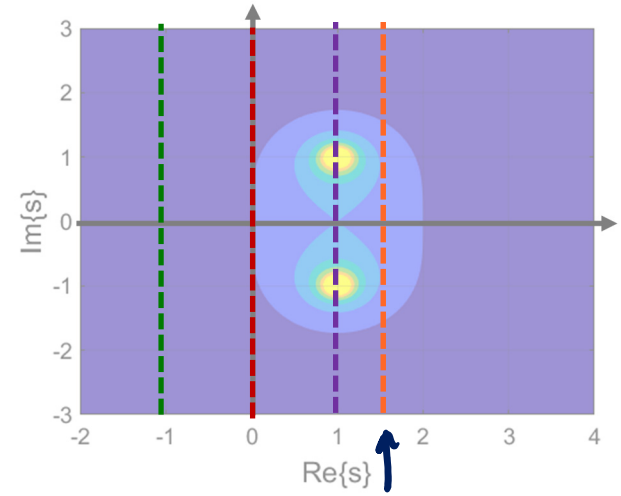
Example R.O.C.

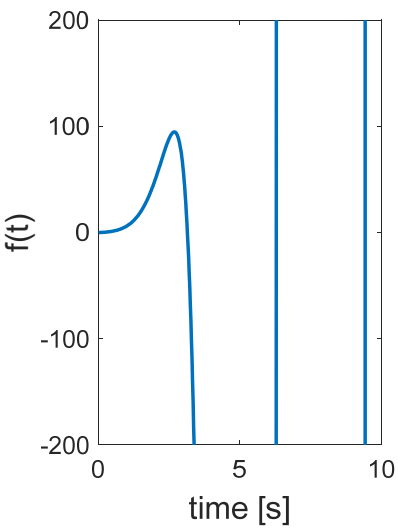
$$F(s) = \int_{0^-}^{+\infty} e^{-st} f(t)\, dt = \int_{0^-}^{+\infty} e^{-j\omega t} e^{-\sigma t} f(t) dt$$

$f(t) = e^{t}\sin(t)$



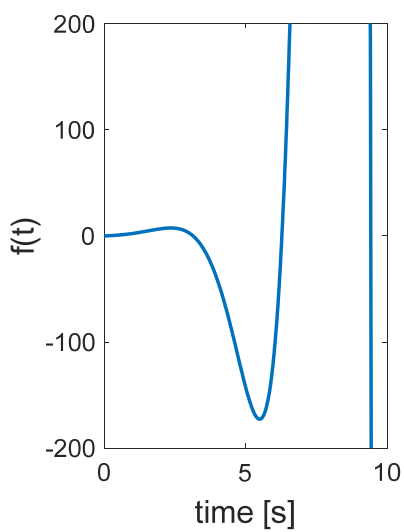
$\sigma = -1$

$e^{-(-1)t} f(t)$



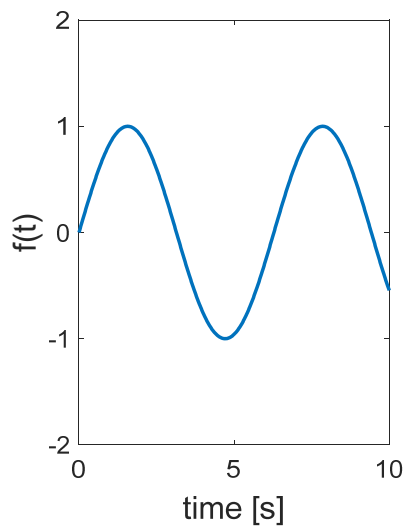
$\sigma = 0$

$f(t)$



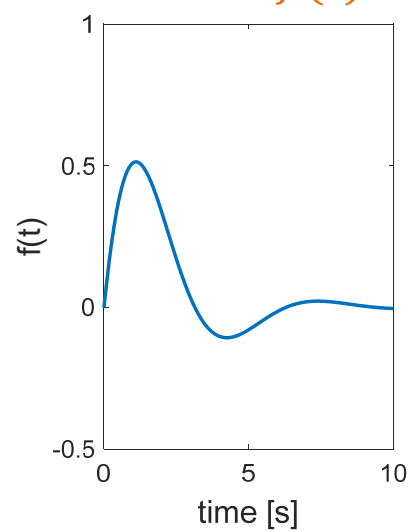
$\sigma = 1$

$e^{-(1)t} f(t)$



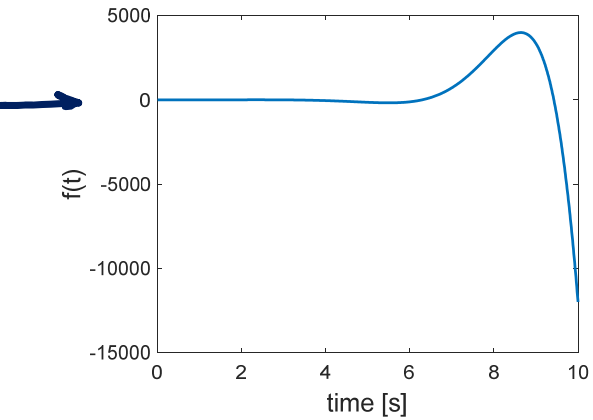
$\sigma = 1.5$

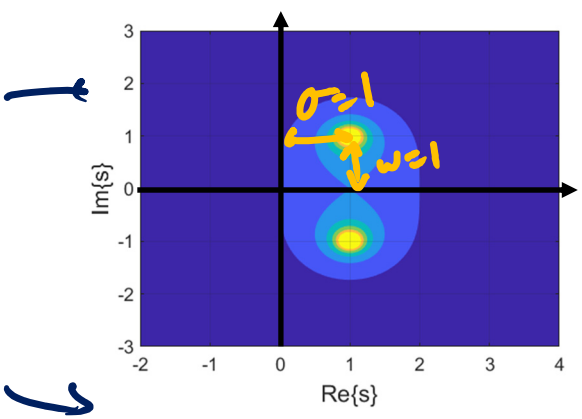
$e^{-(1.5)t} f(t)$



Example Functions

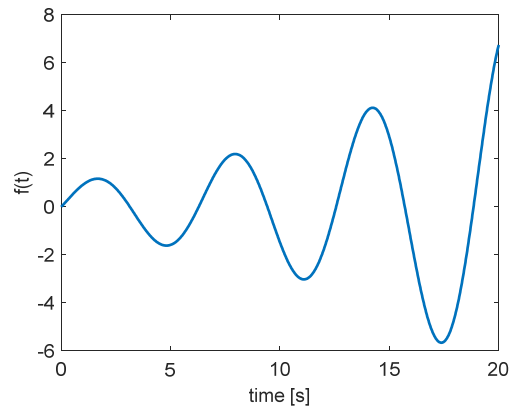
$f(t) = e^{t}\sin(t)$

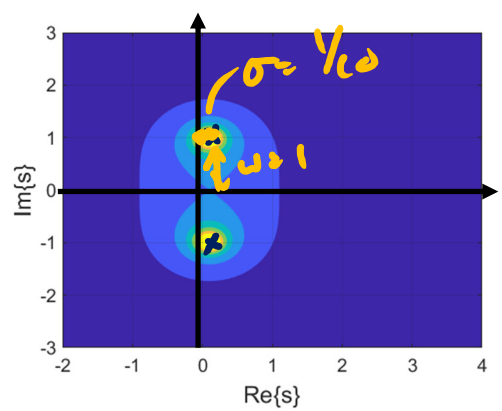




$$F(s) = \frac{1}{(s-(1+j))(s-(1-j))}$$

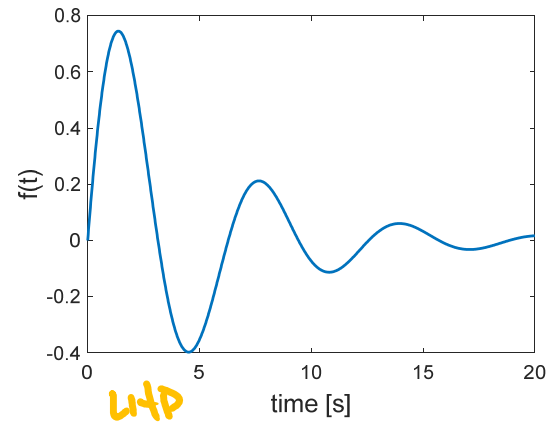
$f(t) = e^{t/10}\sin(t)$



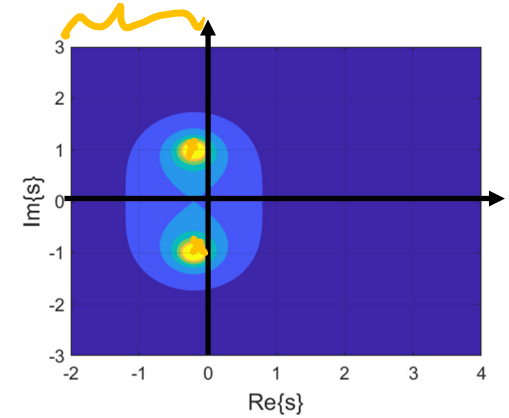


$$F(s) = \frac{1}{\left(s-\left(\frac{1}{10}+j\right)\right)\left(s-\left(\frac{1}{10}-j\right)\right)}$$

$f(t) = e^{-t/5}\sin(t)$



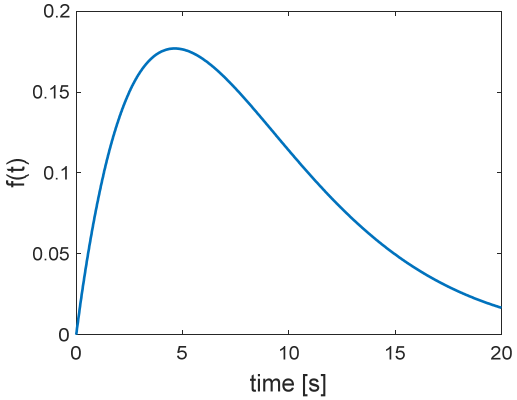
LHP

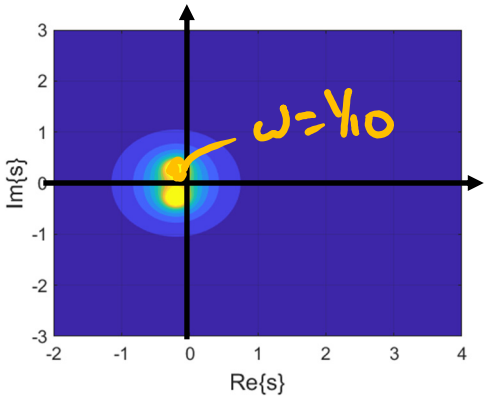


$$F(s) = \frac{1}{\left(s+\left(\frac{1}{5}+j\right)\right)\left(s+\left(\frac{1}{5}-j\right)\right)}$$

Example Functions

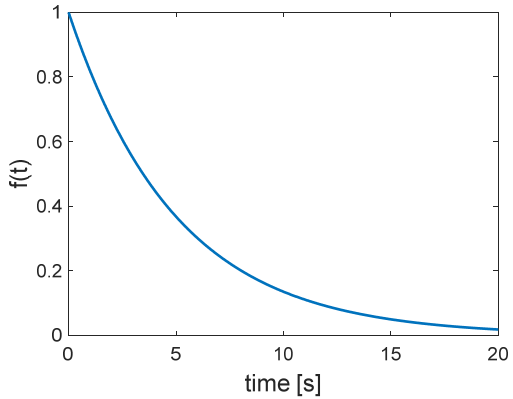
$$f(t) = e^{-t/5}\sin(t/10)$$

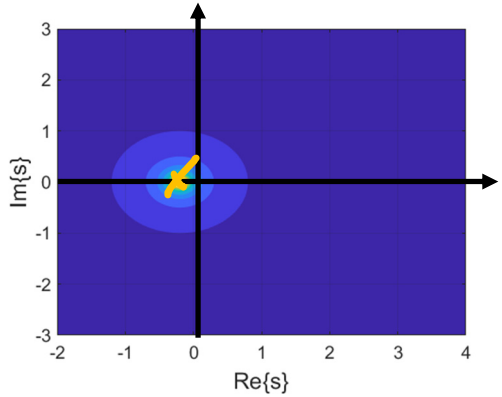




$$F(s) = \frac{1/10}{\left(s + \left(\frac{1}{5} + \frac{j}{10}\right)\right)\left(s + \left(\frac{1}{5} - \frac{j}{10}\right)\right)}$$

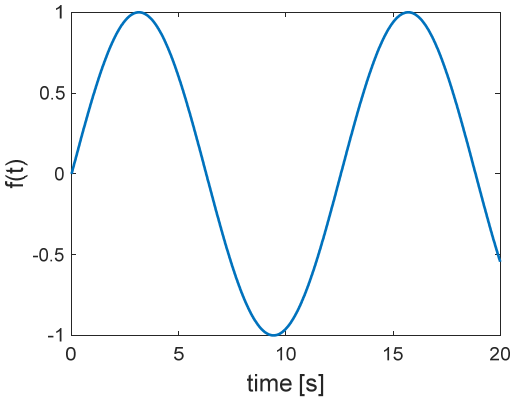
$$f(t) = e^{-t/5}$$

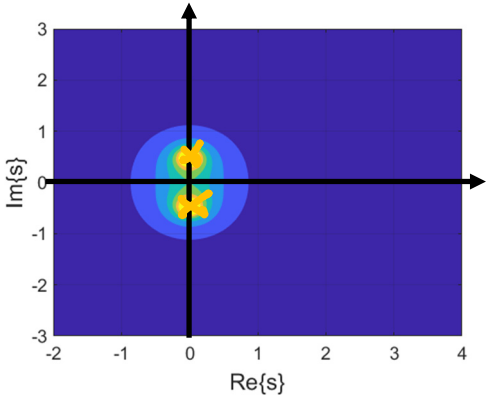




$$F(s) = \frac{1}{\left(s + \frac{1}{5}\right)}$$

$$f(t) = \sin(t/2)$$

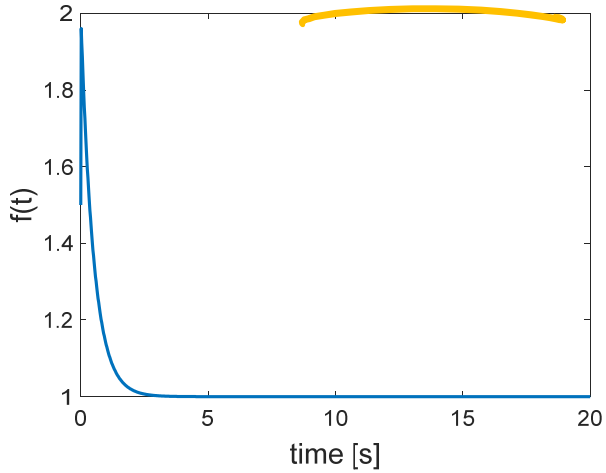


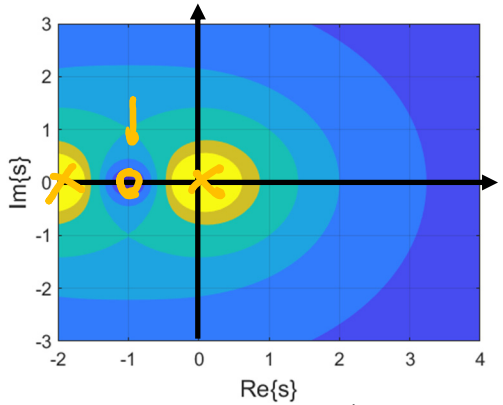


$$F(s) = \frac{1/2}{\left(s + \frac{j}{2}\right)\left(s - \frac{j}{2}\right)}$$

Example Functions

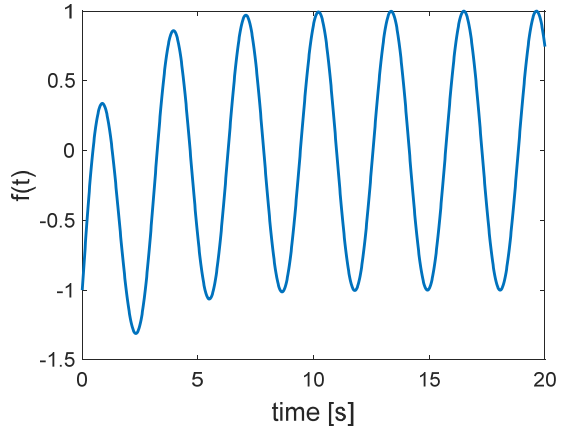
$$f(t) = -e^{-t/2} + u(t)$$





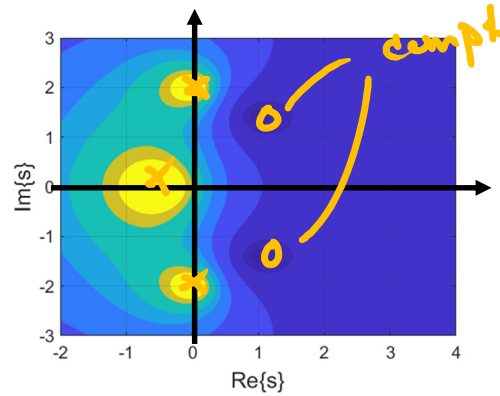
$$F(s) = 2\frac{s+1}{s(s+2)}$$

$$f(t) = -e^{-t/2} + \sin(2t)$$



RHP poles = unstable/unbounded

RHP zeros = bad, but not unstable

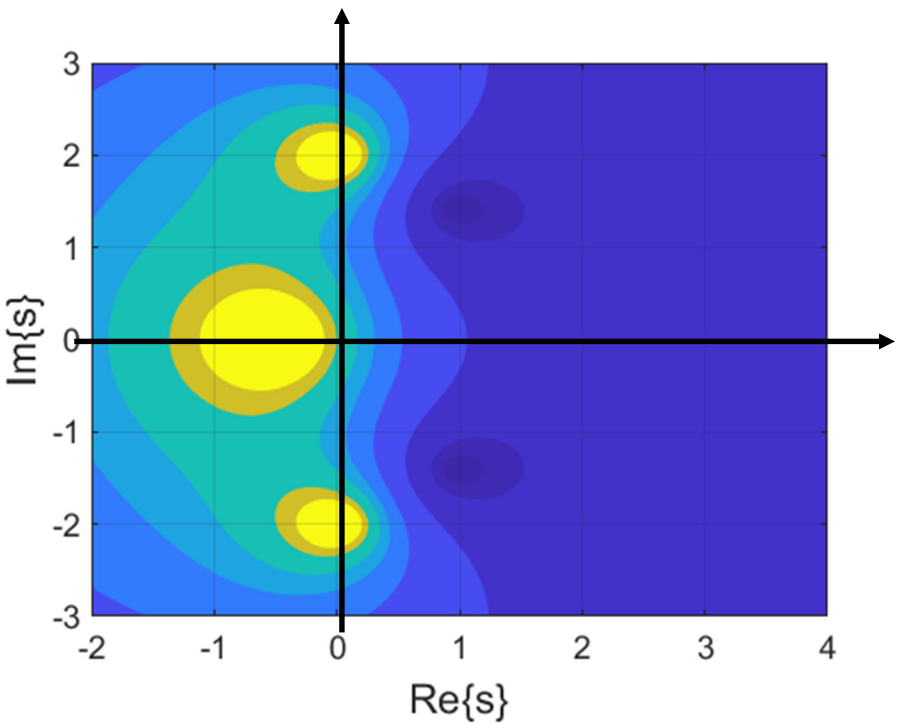


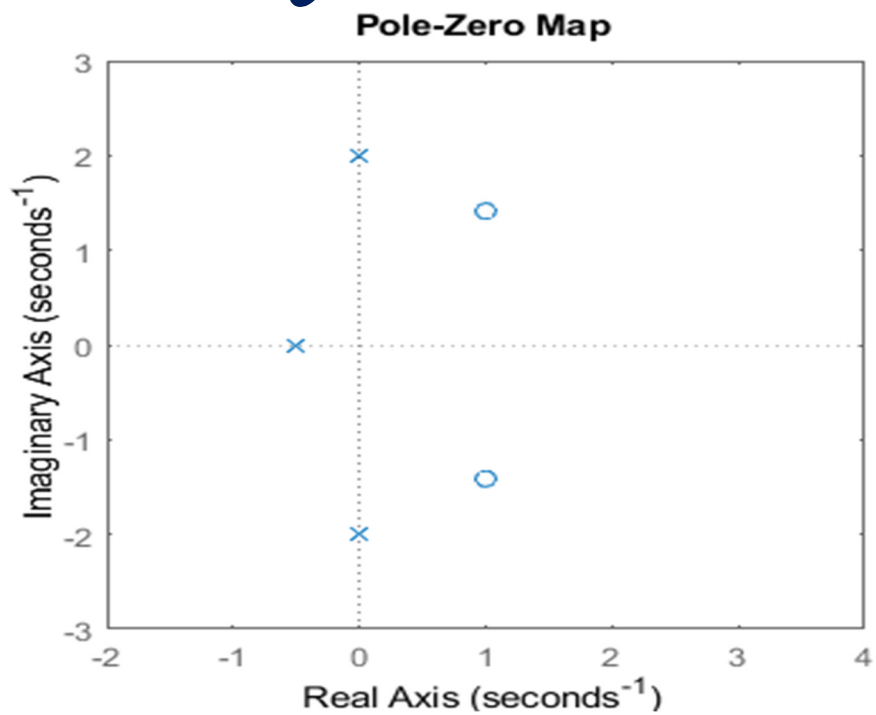
complex pair of zeros

$$F(s) = -\frac{\left(s + \left(-1 + j\sqrt{2}\right)\right)\left(s + \left(-1 - j\sqrt{2}\right)\right)}{\left(s + {}^{1}/_{2}\right)(s + j2)(s - j2)}$$

Pole-Zero Map

x = pole
o = zero



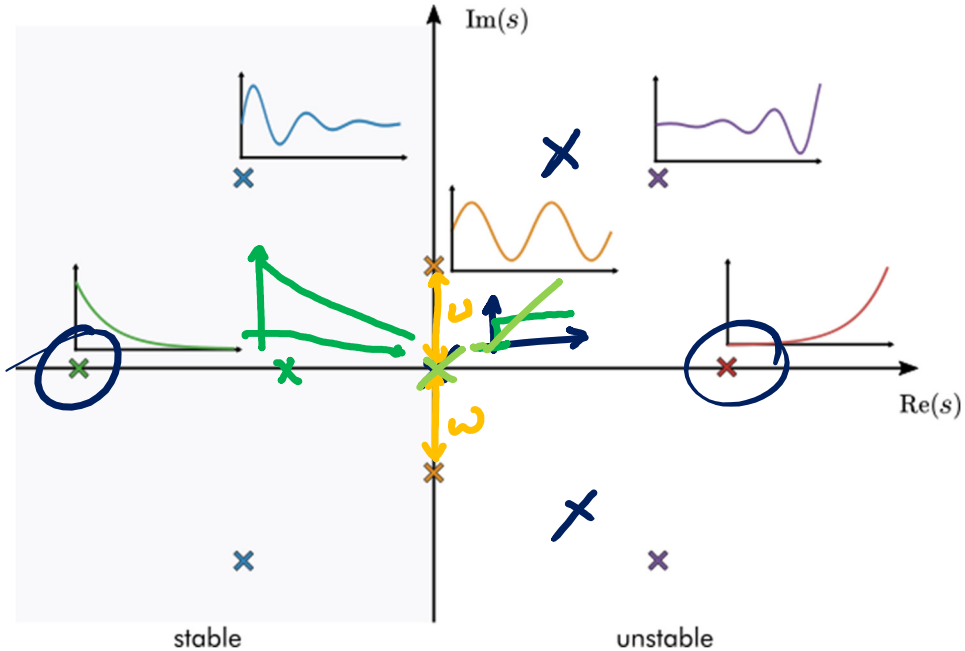


$$F(s) = -\frac{\left(s + \left(-1 + j\sqrt{2}\right)\right)\left(s + \left(-1 - j\sqrt{2}\right)\right)}{\left(s + {}^{1}/_{2}\right)(s + j2)(s - j2)}$$

<u>MATLAB:</u>

```
s = tf('s');
Fs = 2/(s^2 + 4) - 1/(s + 1/2);
pzmap(Fs);
```

Poles-Zero Plot



Takeaways

- complex poles/zeros always show up in conjugate pairs (required for real-valued time-domain function)

- If all poles in open LHP system is stable / signal is bounded

https://www.mathworks.com/matlabcentral/fileexchange/94635-transfer-function-analysis-of-dynamic-systems