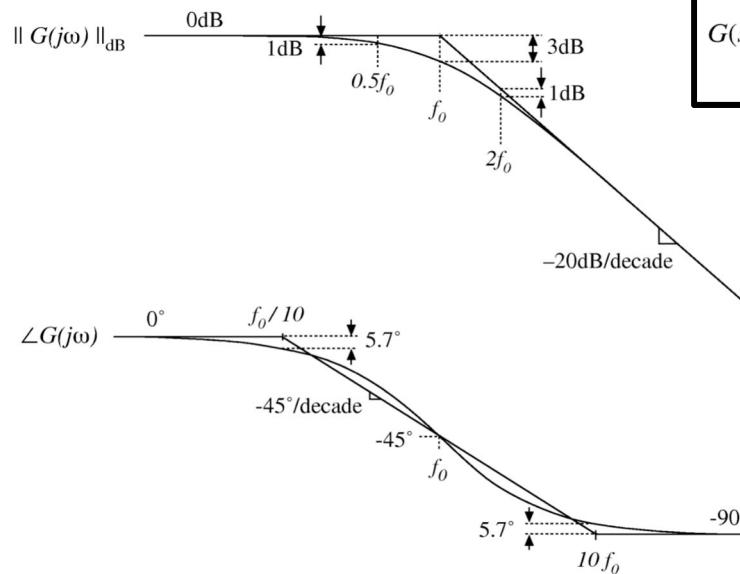


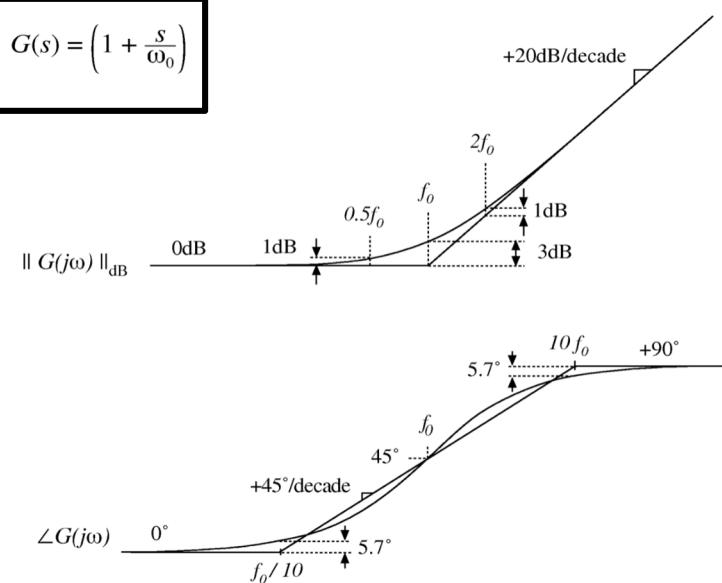
Real Pole

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$



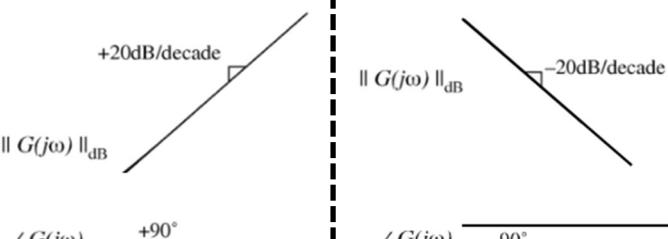
Real Zero

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$



$$G(s) = s$$

LF Zero

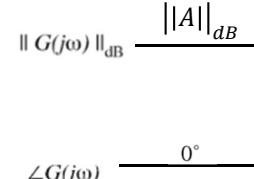


$$G(s) = \frac{1}{s}$$

LF Pole

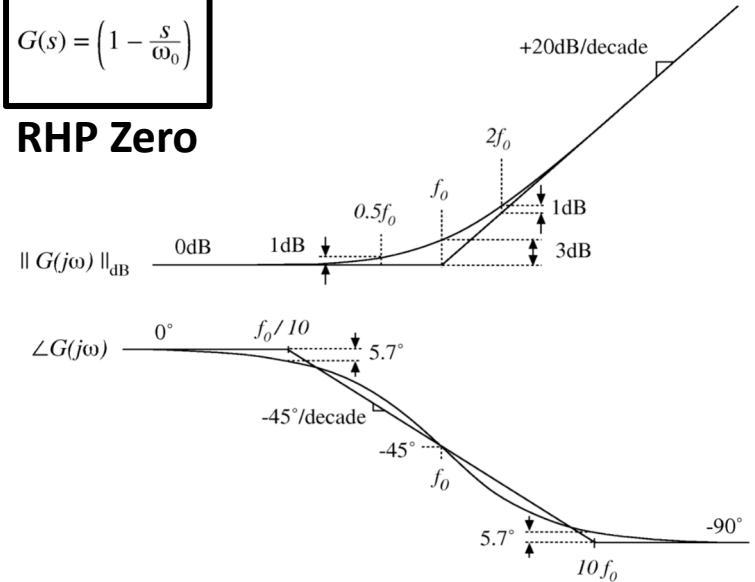
$$G(s) = A$$

Constant



$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$

RHP Zero



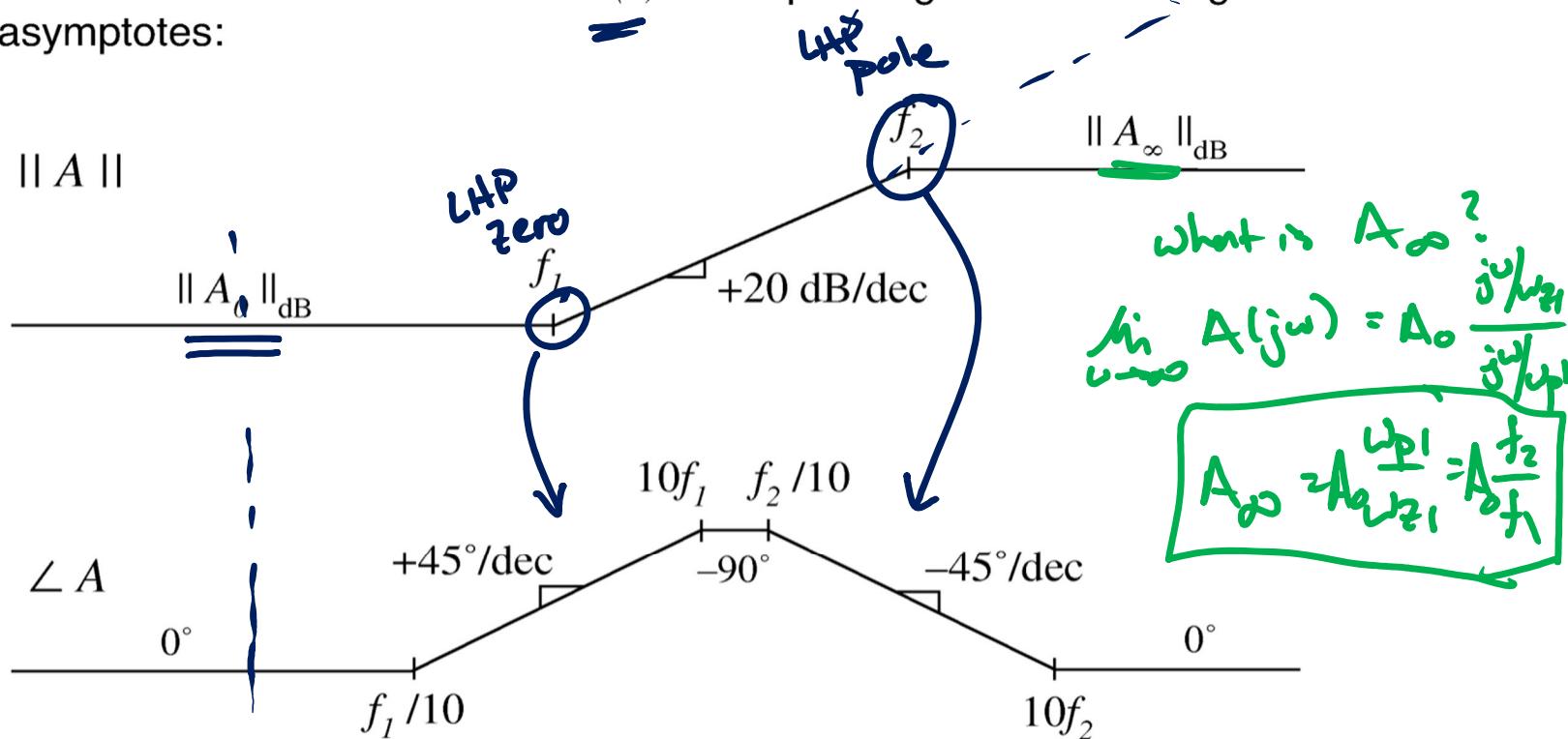
Example 2

$$A(s) = A_0 \frac{\left(1 + \frac{s}{\omega_{z1}}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)}$$

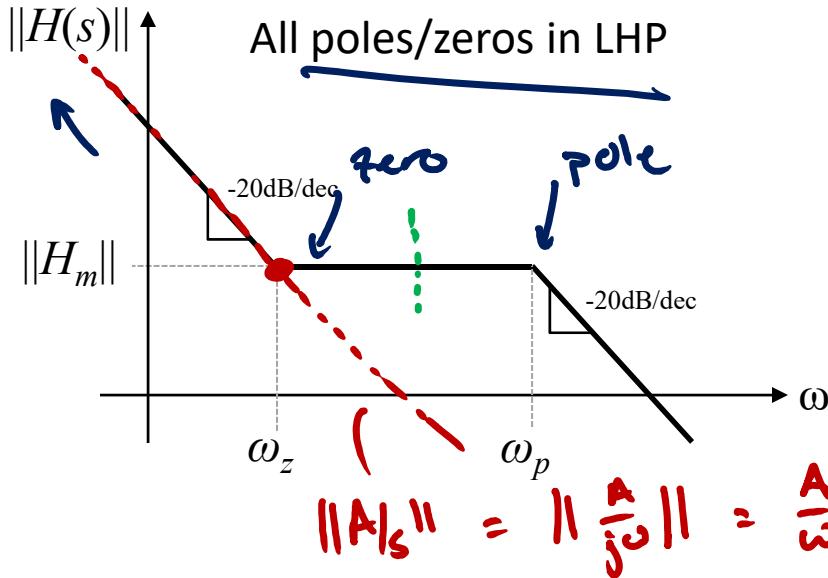
$$\omega_{z1} = 2\pi f_1$$

$$\omega_{p1} = 2\pi f_2$$

Determine the transfer function $A(s)$ corresponding to the following asymptotes:



Example 3



$$\textcircled{C} \omega_z \quad \frac{A}{\omega_z} = H_m$$

$$A = H_m \omega_z$$

find $H(s)$

$$A(s) = H_m \omega_z \frac{1}{s}$$

$$\frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

$= 1 @ \omega \rightarrow \infty$

In the midband

$$H(j\omega) \approx H_m \omega_z \frac{\frac{1}{s}}{1}$$

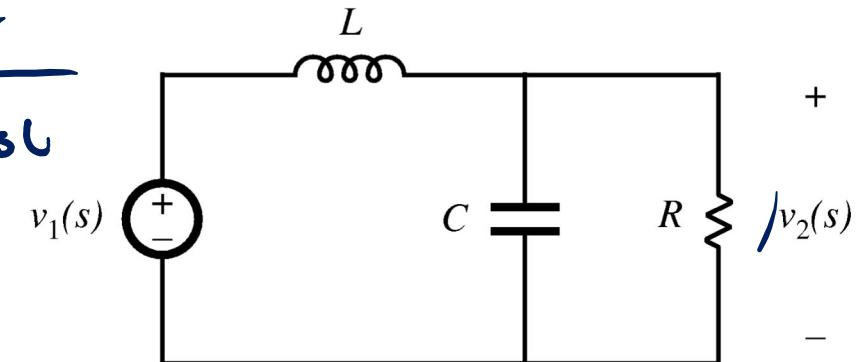
$$= H_m$$

for $\omega_z \ll \omega \ll \omega_p$

Complex Poles

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{R \parallel \frac{1}{sC}}{R \parallel \frac{1}{sC} + sL} = \frac{\frac{R}{1+Rsc}}{\frac{R}{1+Rsc} + sL}$$

$$= \frac{1}{s^2LC + s\frac{L}{R} + 1}$$



Two-pole low-pass filter example

Standard form for 2nd order pole pair:

$$= \frac{1}{(\frac{s}{\omega_0})^2 + \frac{s}{Q\omega_0} + 1}$$

find ω_0 & Q by coefficient matching
for this circuit

$$\frac{1}{\omega_0^2} = LC \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{Q\omega_0} = \frac{L}{R} \rightarrow Q = \frac{R}{L\omega_0} = \frac{R}{L\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$

$$= \frac{R}{R_0} \quad R_0 = \sqrt{4L}$$

Standard Form for Complex Poles

$$H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

$$H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta \frac{s}{\omega_0} + 1}$$

$$\zeta = \frac{1}{2Q} \quad Q = \frac{1}{2\zeta}$$

- ω_0 , Q , & ζ are all real for real systems & found by coefficient matching
- ω_0 is the angular/resonant frequency
- Q is the "Quality factor". Roots are complex when $Q > \frac{1}{2}$
if $Q < 0.5 \rightarrow$ factor into two real poles
- ζ is the "damping factor". Roots are complex when $\zeta < 1$

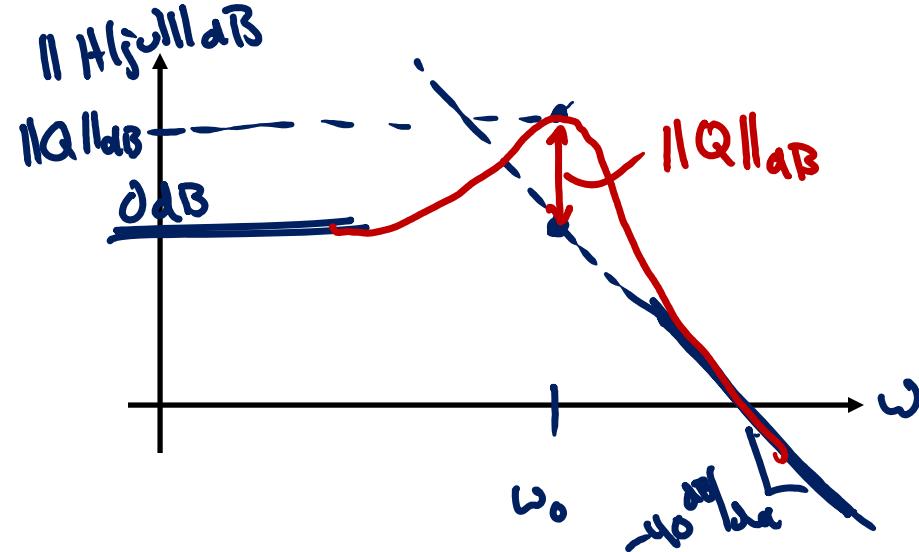
$$Q = 2\pi \frac{\text{Max stored energy}}{\text{energy lost per cycle}}$$

Magnitude Asymptotes

for $Q > 0.5$

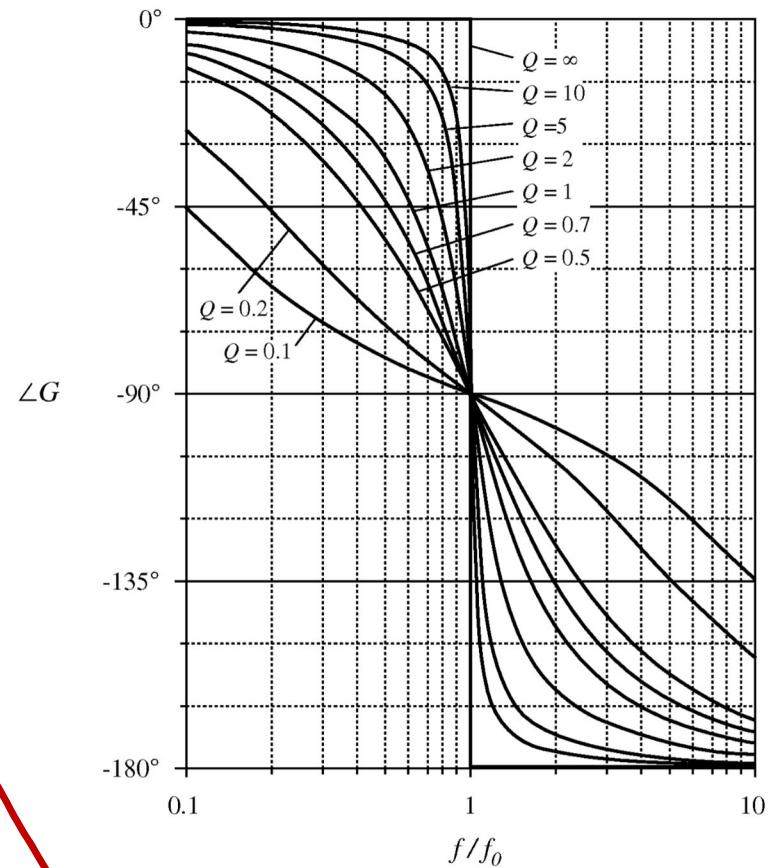
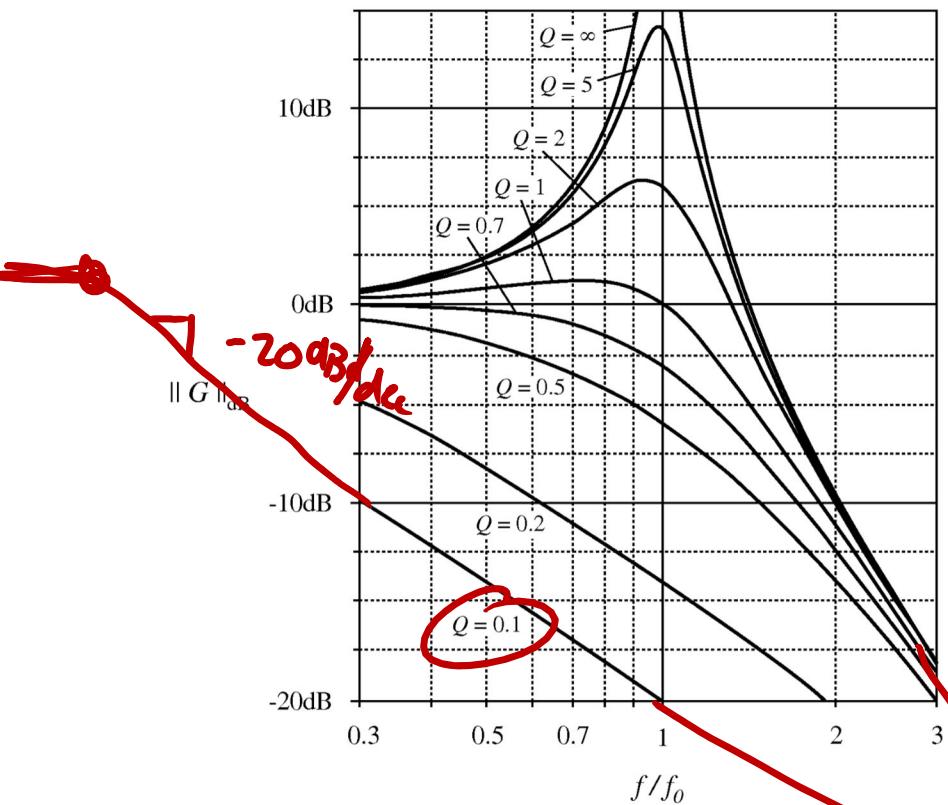
$$H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

$$\|H(j\omega)\| = \sqrt{\left(\frac{\omega}{Q\omega_0}\right)^2 + \left(1 - \frac{\omega}{\omega_0}\right)^2}$$



$$\left\{ \begin{array}{l} \|H(j\omega)\|_{dB} \\ \text{---} \\ \text{---} \end{array} \right. \begin{array}{l} 0 dB, \omega \ll \omega_0 \\ Q \rightarrow 20 \log(Q), \omega = \omega_0 \\ \frac{1}{(\omega/\omega_0)^2} = 40 \log(\omega_0) - 40 \log(\omega), \omega \gg \omega_0 \end{array}$$

Curves for Varying Q



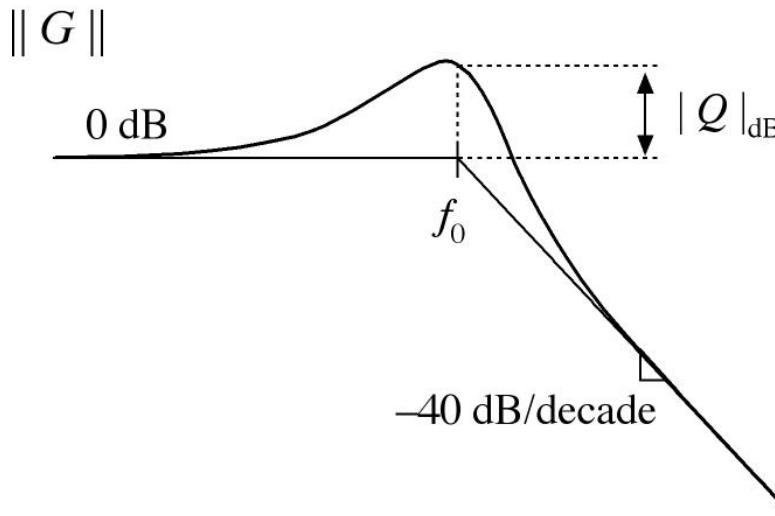
Fundamentals of Power Electronics

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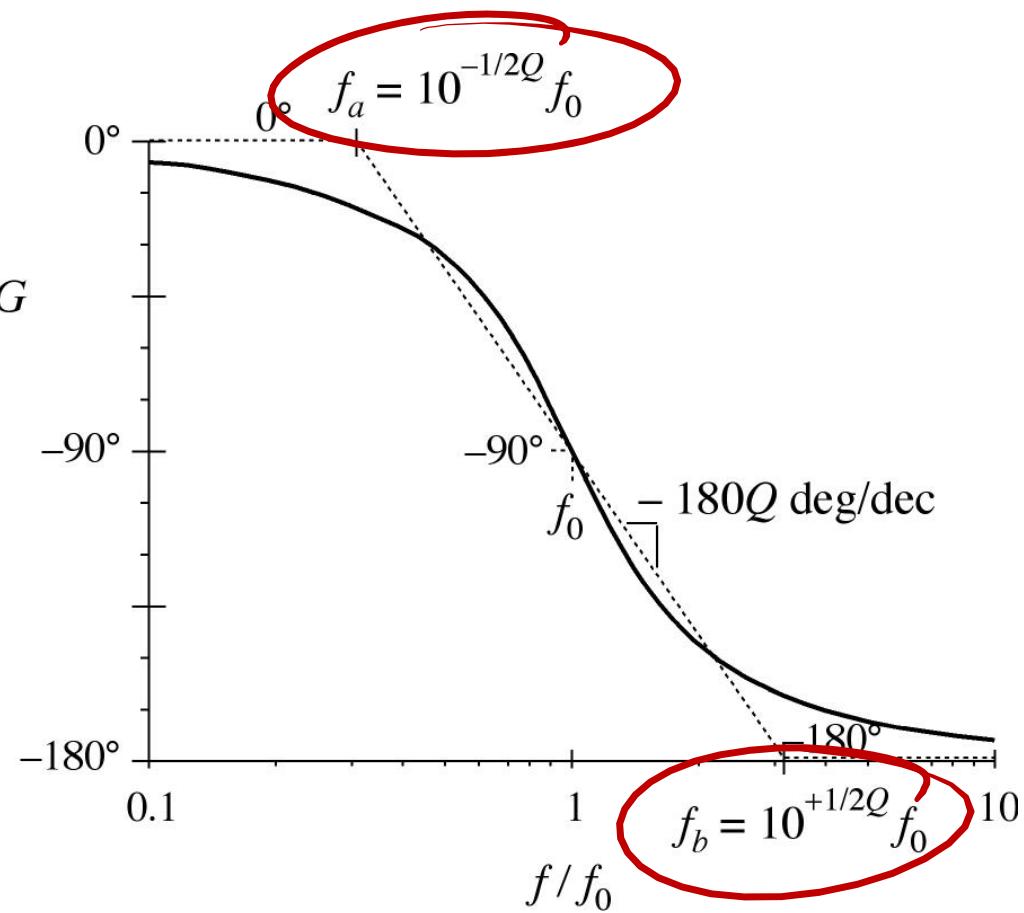
Chapter 8: Converter Transfer Functions

Asymptotes for Complex Poles, $Q>0.5$

Magnitude



Phase



MATLAB

```
A = 1000;  
wz1 = 100;  
wz2 = 10e3;  
w0 = 1e3;  
Q = 10;  
wp = 100e3;
```

s = tf('s');

```
H = A*(1+s/wz1)*(1+s/wz2)/...  
      (s*(1+s/wp)*((s/w0)^2+s/Q/w0+1));
```

bode(H)

